Shear-wave splitting in cross-hole surveys: Modeling

Enru Liu*, Stuart Crampin‡, and David C. Booth‡

ABSTRACT
Shear-wave splitting, diagnostic of some form of effective seismic anisotropy, is observed along almost all near-vertical raypaths through the crust. The splitting is caused by propagation through distributions of stress-aligned vertical parallel fluid-filled cracks, microcracks, and preferentially oriented pore space that exist in most crustal rocks. Shear waves have severe interactions with the free surface and may be seriously disturbed by the surface and by near-surface layers.

In principle, cross-hole surveys (CHSs) should be free of much of the near-surface interference and could be used for investigating shear waves at higher frequencies and greater resolution along shorter raypaths than is possible with reflection surveys and VSPs. Synthetic seismograms are examined to estimate the effects of vertical cracks on the behavior of shear waves in CHS experiments. The azimuth of the CHS section relative to the strike of the cracks is crucial to the amount of information about seismic anisotropy that can be extracted from such surveys. Interpretation of data from only a few boreholes located at azimuths chosen from other considerations is likely to be difficult and inconclusive.

Application to interpreting acoustic events generated by hydraulic pumping is likely to be more successful.

INTRODUCTION
Shear-wave splitting has recently been observed in many shear-wave reflection surveys (surface-to-surface) in sedimentary basins across North America (Alford, 1986; Lynn and Thomsen, 1986; Willis et al., 1986), in shear-wave vertical seismic profiles (VSPs) (surface-to-subsurface) in sedimentary rocks (Johnston, 1986; Becker and Perelberg, 1986; Crampin et al., 1986) and in mixed metamorphic regimes (Majer et al., 1985; Leary and Li, 1986; Li et al., 1986), and above small earthquakes in many seismic areas around the world (reviewed by Crampin, 1987a). It appears that shear-wave splitting is characteristic of almost all shear-wave propagation in at least the upper 20 km of the crust. The splitting is principally caused by extensive-dilatancy anisotropy, or EDA: the distributions of stress-aligned parallel vertical fluid-filled microcracks, cracks, and preferentially oriented pore space which pervade most rocks in the crust (Crampin et al., 1984; Crampin, 1985a, 1987a). We shall refer to these fluid-filled inclusions as EDA cracks.

At present, all published records of shear-wave splitting involve shear waves generated, recorded, or both generated and recorded at the free surface. Shear waves, however, may suffer severe scattering at the free surface and by irregular topography within a wavelength or two of the recording site (Evans, 1984; Booth and Crampin, 1985). In principle, cross-hole surveys (CHSs), where both source and receiver are subsurface, should be free of many of the difficulties associated with long raypaths and near-surface interference when shear waves are either generated or recorded at the surface (Fehler and Pearson, 1984). CHSs should allow shear-wave splitting to be monitored along shorter raypaths at higher frequencies; the resulting shorter wavelengths would increase the resolution with which we could specify the effective anisotropy of EDA cracks within the rock mass. Such information might not be of direct use in discovering new reservoirs but should enable fractured beds and the structure of EDA cracks to be identified in known reservoirs and some of the parameters to be estimated so that the internal structure could be evaluated. The orientations of the in-situ stress-aligned microcracks are expected to be directly related to the orientations of hydraulic fractures and preferred directions of flow in hydrocarbon reservoirs.

The major surveying distinction is that the raypaths for VSPs and reflection surveys are usually within ±45° of the vertical (often much closer to vertical), whereas the raypaths for CHSs are usually within ±45° of the horizontal. This difference requires different field techniques and different schemes of analysis when surveying vertically oriented cracks. Below, we examine the behavior of shear waves propagating through cracked rock by analyzing shear-wave splitting on synthetic seismograms along horizontal and nearly horizontal raypaths.
EDA cracks are aligned by stress relationships similar to those orienting hydraulic fractures in intact rock (Crampin, 1987a). Consequently, where the vertical stress is greater than the minimum horizontal compression, which is usually the case below the immediate surface layers, fluid-filled microcracks are aligned nearly vertically and perpendicular to the direction of minimum compression (Crampin, 1987a). We consider cracked rock where the dimensions of the cracks are several times smaller than the wavelengths of the shear waves. This is little restriction, since the minimum wavelength of observed shear waves is usually measured in meters (often many tens of meters) and EDA cracks are expected to be principally microcracks with dimensions less than a few millimeters, or at most open fractures of one or two meters (Crampin, 1987a). Such aligned cracks are effectively anisotropic to seismic waves (Crampin, 1984). A rock containing parallel vertical cracks in an isotropic matrix is transversely isotropic with a horizontal axis of cylindrical symmetry perpendicular to the face of the cracks.

Shear waves propagating through aligned cracks generally split into two components with different vector displacements (polarizations) traveling at different velocities, where both velocities and displacements are fixed for the particular raypath through the cracked rock (Crampin, 1981, 1984). This phenomenon is known as shear-wave splitting or shear-wave birefringence. Figure 1 shows a schematic illustration of shear-wave splitting. A shear wave propagating nearly vertically through EDA cracks splits into two phases with polarizations parallel and perpendicular to the face of the cracks. The phase with polarization parallel to the cracks meets less acoustic impedance, travels faster, and is less attenuated than the phase with polarization normal to the crack face. The splitting has inserted into the three-dimensional (3-D) particle motion characteristic waveforms, which are preserved for any subsequent propagation through isotropic rock. Note that splitting does not occur when the incident shear wave is polarized parallel (or perpendicular) to the crack face, so that only the faster (or slower) phase is excited. When the slower shear wave is excited, additional motion orthogonal to the expected polarizations occurs, behavior which has been observed on many occasions. Wave propagation through rock containing such aligned cracks may be simulated by propagation through a homogeneous, purely elastic anisotropic solid that has the same patterns of velocity (and attenuation) as the cracked rock (Crampin, 1978).

Figure 2 shows the velocity variations of body waves propagating through distributions of thin parallel liquid-filled cracks with two crack densities. The crack densities are specified by \( CD = Na^2/r \), where \( N \) is the number of cracks of radius \( a \) in volume \( r \). Figure 2a shows the velocity variations for \( CD = 0.1 \), where the velocity anisotropy is large enough for the group and phase velocities to be clearly separated; Figure 2b shows the velocity variations for \( CD = 0.04 \), which is a crack density commonly found in the Earth in sedimentary (Crampin et al., 1986), metamorphic (Crampin and Booth, 1985), and igneous rocks (Roberts and Crampin, 1986). A crack density of 0.04 is equivalent to a crack with a diameter less than 0.7 in each unit cube. The three body waves are a quasi P-wave \( qP \) with nearly longitudinal displacement, and two quasi shear waves \( qSP \) and \( qSR \) polarized (P)arallel and (R)ight angle to the cracks. The solid lines are the phase velocities and broken lines are the group velocities which are joined to the equivalent phase velocity at every 10 of phase velocity. Arrowheads mark directions where the two velocity surfaces intersect in line singularities. The crack densities are (a) \( CD = 0.1 \); and (b) \( CD = 0.04 \).
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at (R)ight angles, respectively, to the plane through the crack normals. Figure 2 was calculated using Hudson’s (1980, 1981) theoretical formulations following Crampin (1984). Note that Hudson’s formulations also model velocity and attenuation due to cracks for specified aspect ratios and both gaseous and viscous fluid contents. These refinements generally produce only second-order differences to the effects of thin liquid-filled cracks on shear-wave propagation; for simplicity, they are not considered in this paper.

The behavior of shear-wave splitting along nearly vertical raypaths can be conveniently specified by mapping the polarizations and delays between the split shear waves in equal-area projections (polar maps) over an upper or lower hemisphere of directions. Thus, Figure 3 shows a polar map of the horizontal strike of the polarization of the leading (faster) shear wave for a hemisphere of directions of plane waves propagating through parallel vertical liquid-filled cracks. The cracks strike east-west and have the same crack density as for Figure 2b. Figure 3a shows that the polarization of the leading shear wave is parallel to the strike of the cracks for a broad band of directions across the center of the projection, as suggested by Figure 1. The abrupt change in polarization on either side of the central band is caused by the intersection of the velocity curves of the two shear-wave polarizations at 60° from the crack normal (30° from the vertical) marked by an arrowhead in Figure 2. Figure 3b shows contoured delays between the split shear waves for a normalized path length. Such polar projections, although suitable for specifying the behavior of shear waves along raypaths within ±45° of the vertical, are not appropriate for describing the behavior of shear waves along more nearly horizontal raypaths expected in CHSs.

The remarkable feature of shear-wave splitting in parallel vertical cracks displayed in these polar projections is that the faster shear wave is polarized parallel to the strike of the vertical cracks for a broad band of directions across the center of the projection, including almost the whole of the shear-wave window (Booth and Crampin, 1985). This diagnostic feature is seen in almost all observations of shear waves along nearly vertical raypaths in the crust (Crampin, 1987a). The time delays between the split shear waves reach maximum values in the same broad band. We show that CHS experiments in similar crack distributions do not display such diagnostic phenomena.

THE BEHAVIOR OF SHEAR WAVES IN CYLINDRICAL PROJECTIONS

We display the behavior of shear-wave splitting in CHSs by cylindrical projections of the polarizations and delays over a full range of raypaths (360° of azimuth and dips from +90° downward to −90° upward). Figure 4 shows Plate Carée cylindrical projections (equal steps of latitude and longitude) of the particle polarizations of the leading split shear wave to subsurface geophones in (a) horizontal (R)adial and (T)ransverse and (b) (V)ertical and (T)ransverse cross-sections for (CD = 0.1. Thus, Figure 4 shows the polarizations of the leading shear-wave arrivals radiating from a point source as seen by (a) horizontal instruments and (b) vertical and transverse

Fig. 3. Polar equal-area projections over a hemisphere of directions of the (a) polarizations in the (R)adial-(T)ransverse plane and (b) time delays of plane split shear waves propagating at the group (ray) velocity through the thin parallel liquid-filled cracks of Figure 2b (CD = 0.04) aligned vertically and striking east-west. The inner circles mark the shear-wave windows at the free surface at arcsin (V_s/V_p) = 35.26° and are marked as a scale. The bars in (a) are the horizontal components of the displacements of the leading (faster) split normalized shear wave, and the time delays between the split shear waves in (b) are contoured in milliseconds for a normalized pathlength of 1 km. A north-south section of the delays is to the left of the contour plot. Values for vertical directions are circled, and values for horizontal north and horizontal east are marked with triangles.
Figure 4. Cylindrical projections of the polarizations and time delays of the split shear waves propagating through the thin parallel liquid-filled cracks of Figure 2a (CD = 0.1) aligned vertically and striking east-west, for the full range of raypath directions from upward (−90°) to downward (90°) to a geophone from azimuths of 0° to 360° east of north (clockwise from north). Polarizations of the leading split shear wave are projected onto (a) horizontal, marked (R)adial and (T)ransverse and (b) (V)ertical and (T)ransverse cross-sections for a fixed amplitude of displacement. The length of the symbol indicates the amplitude of a normalized leading split shear wave for the appropriate direction. Values for horizontal north and horizontal east are marked with triangles corresponding to the triangles in Figure 3. Values for vertical directions (circled in Figure 3) lie along the −90° dip coordinates in Figure 4. Time delays in (c) are contoured in milliseconds for a normalized pathlength of 1 km, and the cross-sections of the contours in (d) are at the five specified azimuths in (c).

Instruments on the walls of a cylinder. The cylinder has then been opened out. (Figure 4 is a cylindrical map of the radiation in all directions from a point source, whereas Figure 3 is a polar map of one hemisphere.) Figure 4c shows contours and Figure 4d, sections of delays between the split shear waves for plane waves propagating at the group (ray) velocity through the same parallel vertical liquid-filled cracks striking east-west with a crack density of CD = 0.1 as in Figure 2a.

Figure 5 shows the same variations as Figure 4 for the smaller crack density (CD = 0.04) in Figure 2b. The principal effect of the reduced crack density is the smaller time delays in Figures 5c and 5d. There are also minor differences between the shapes of the contours caused by the differences between the variations of group velocity seen in Figures 2a and 2b. Figure 6 shows polarizations and delays of shear waves propagating through the same cracks as Figure 5 but with the plane of the cracks dipping at 70°.

The variations with direction of the polarizations and delays in Figures 5 and 6 show distinctive patterns in which the orientations of the cracks and relative crack densities can be easily evaluated, given observations from a sufficient range of directions. However, the patterns lack any strongly diagnostic features, such as the pattern of parallel polarizations in the polar projections in Figure 3. In practice, CHS observations are usually confined to raypaths between a limited number of approximately vertical boreholes usually at relative azimuths which have been fixed by other considerations. Thus, in most CHS surveys the behavior of shear-wave splitting can be examined only along a few vertical stripes at arbitrary azimuths in cylindrical projections. The interpretation of the polarizations and delays in terms of crack orientations and crack densities from a few vertical stripes is possible in noise-free conditions for an appropriate choice of azimuths and range of dips, but the interpretation of a few vertical stripes at arbitrary azimuths, particularly where irregularities in the rock may cause scatter in the observations, is likely to be difficult and inconclusive.

SYNTHETIC SEISMOGRAMS FROM ADJACENT BOREHOLES

The principal effect of shear-wave splitting is to introduce subtle phase and amplitude changes into the different components of motion. These may be observed by meticulously comparing the relative displacements of parallel time series, or by easily recognizable patterns in polarization diagrams.
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Fig. 5. Cylindrical projections for shear waves propagating through the thin parallel vertical cracks of Figure 2b for a crack density of $CD = 0.04$. The circles mark the directions of propagation of the synthetic seismograms in Figure 7. Notation as in Figure 4.

Fig. 6. Cylindrical projections for the same cracks as Figure 5 but dipping at 70° to the north. The circles mark the directions of propagation of the synthetic seismograms in Figure 8. Notation as in Figure 4.
Figure 7. Seismograms and polarization diagrams of shear waves through the same uniform space as Figure 5. Six three-component geophones in a vertical borehole are arranged at depths to give the raypaths identified in Figure 5 (dips of -50° to 50°) relative to vertical point forces (dominant frequency of 80 Hz) in vertical boreholes offset 200 m at azimuths of (a) N 90° E, (b) N 110° E, and (c) N 130° E. Upper diagrams are three-component synthetic seismograms aligned (V)ertical, (R)adial and (T)ransverse to the azimuth of arrival. Lower diagrams are corresponding polarization diagrams for horizontal and vertical-transverse cross-sections of the particle displacements, and labeled (V)ertical, (R)adial, and (T)ransverse. Arrowheads indicate the initial directions of the first motions of the shear waves corresponding to the polarizations identified in Figure 5. Seismograms and polarization diagrams show the true relative amplitudes in each vertical column.
FIG. 8. Seismograms and polarization diagrams for synthetic seismograms through the same uniform space as Figure 6 (cracks dipping 70° to the north) along the marked raypaths. Geometry of paths and notation is the same as in Figure 7.
(Crampin, 1985b). (Polarization diagrams, also known as hodograms, are orthogonal cross-sections of the particle displacements for short time intervals along the wavetrains.) The patterns are characteristic of the particular phase and amplitude differences between the different shear-wave phases (Crampin, 1985b). Numerous observations suggest that the patterns are stable and can be identified even in the presence of considerable noise. Figure 7 shows synthetic seismograms and polarization diagrams for shear waves from a point source propagating through a uniform space containing the thin parallel vertical cracks of Figure 2b striking east-west, giving the same structure as used for Figure 5. The synthetic seismograms have been calculated by the ANISEIS program package. Synthetic seismograms are shown at six threecomponent geophones placed in a vertical borehole at depths to give relative dips of \(-50\), \(-30\), \(-10\), \(10\), \(30\), and \(50\) from vertical point forces, with offsets from the borehole at 200 m, at azimuths N 90 E, N 110 E, and N 130 E corresponding to the circled arrivals in Figure 5. This geometry gives signals that can be compared directly with the polarizations and delays in Figure 5.

The arrowheads in the polarization diagrams in Figure 7, marking the initial directions of motion of the leading split shear waves radiating from a point source, correspond to the polarizations in the marked directions in Figure 5. The places where arrowheads are omitted are where there is no splitting either because the radiated shear wave is polarized very close to one of the fixed polarizations through the anisotropic rock so that the other split shear wave is not excited, as in Figure 7a, or because the time delays between the split shear waves are too small to cause significant splitting at the dominant period of the signal, as elsewhere in Figure 7.

Figure 8 shows synthetic seismograms and polarization diagrams for shear waves propagating through the same structure as Figure 7 but with cracks dipping 70\(^\circ\) to the north, corresponding to the marked raypaths in the cylindrical projection in Figure 6. The notation is the same as in Figure 7.

The polarization diagrams in Figures 7 and 8 display patterns of particle displacements with the abrupt changes in direction typical of impulsive single-cycle shear waves propagating through cracked rock (Crampin, 1985b, 1987b). The polarizations of the initial motion of the leading split shear waves with curved wavefronts are very similar to the polarizations of the plane waves along the group velocity (ray) directions in Figures 5 and 6, respectively. The measured inconsistencies are less than 3\(^\circ\) and are caused by the different behavior of group velocity for curved and plane wavefronts in anisotropic rocks. (The point source is about seven wavelengths from the geophone borehole.) A plane wave travels at the phase velocity and the two polarizations of the shear waves are strictly orthogonal, whereas a ray from a point source (with a curved wavefront) travels at the group velocity and, in general, will have different polarizations from the plane wave at the same angle of incidence. Consequently, for a point source the two split shear waves will not be strictly orthogonal.

**DISCUSSION**

There are at least four parameters of EDA cracks of interest to the reservoir engineer that can be extracted from seismic observations of shear-wave splitting. These are the crack geometry, particularly the strike and dip of the cracks, the aspect ratio of the cracks, and the crack dimensions.

**The strike of parallel cracks**

The polarization of the leading split shear wave along nearly vertical raypaths gives estimates of the strike of the nearly vertical parallel cracks. This type of polarization is observed in many different circumstances in the Earth (see the review by Crampin, 1987a). There is no such distinctive behavior in CHSs. The strike could be identified by the symmetrical behavior at a range of azimuths spanning the direction of strike, but, except by chance, observations between suitable boreholes are unlikely to be available. However, determination of strike might be possible for a range of sources from a horizontal borehole or tunnel with three-component geophones at some distance away from the line of the tunnel.

**The dip of parallel cracks**

Dip is difficult to identify from nearly vertical raypaths unless observations are available from a range of azimuths and angles of incidence in an appropriate range of directions. CHSs display the effects of dip as asymmetries in the polarization patterns between upward and downward propagating waves, as in Figure 8, where the cracks dip at 70\(^\circ\), in contrast to Figure 7, where the cracks are vertical. At azimuths parallel to the strike of the cracks, as in Figure 8, the dip can be read directly from the dip of the polarization of the leading shear wave.

**The aspect ratio of the cracks**

Changes in aspect ratio change the directions where the two split shear waves intersect (see Figure 2 in Crampin, 1987b) and change the position of the line of transition between the nearly orthogonal polarizations in polar and cylindrical projections in Figures 3 to 6. A larger aspect ratio increases the width of the broad band of parallel polarizations in polar projections and increases the diameter of the circular features in the cylindrical projections. Such changes in aspect ratio have been identified along nearly vertical raypaths in seismic gaps where the stress is changing before earthquakes (Peacock et al., 1988). It does not seem likely that the positions of these transition zones can be easily identified in CHSs.

**Crack dimensions**

The dimensions of EDA cracks may range from submicrometer to a few millimeters in intact rock and up to a few meters in fractured bedies (Crampin, 1987a). The elastic constants, and hence the velocity variations and shear-wave splitting, are more sensitive to the dimensionless crack density \(N_d\) than the crack dimensions (see the theoretical formulations by Hudson, 1980, 1981, or Crampin, 1984). It is likely that attenuation will be more sensitive than velocity variations to the dimensions of the cracks. If the cause of attenuation in cracked rock can be established, measurement of attenuation is likely to be a particularly valuable technique, because with a known source polarization, the relative attenuations of
the split shear waves can be compared directly because they will have propagated along very similar raypaths.

Note that the interpretation in this paper is based on the effects of a single parallel vertical crack set. We believe assuming a single crack set is justified. There are now observations of shear-wave splitting from over 50 different locations (see Crampin, 1987a). Relatively few of the data show scatter and are difficult to interpret, and the majority show clear patterns of 3-D variation; wherever a pattern can be seen, it suggests vertical cracks striking perpendicular to the minimum horizontal stress, as illustrated in Figure 1. To our knowledge, no shear-wave polarizations anywhere suggest other than nearly parallel vertical cracks. The physical reasons for this have been discussed elsewhere (Crampin, 1987a).

CONCLUSIONS

The theoretical and numerical examples presented here suggest that information about the internal structure causing shear-wave splitting is unlikely to be extracted easily from CHS experiments unless sufficient observations can be made at a range of azimuths. A large number of boreholes at suitable azimuths or a horizontal borehole are not expected to be commonly available. Note, however, that the dip of near-vertical parallel cracks can be estimated from polarization diagrams at a specific range of CHS azimuths.

As always with shear-wave splitting observed in the subsurface (away from the severe interactions with the free surface), detailed interpretation is possible with synthetic seismograms. However, this type of interpretation will be more difficult for CHSs than for VSPs, because CHSs appear to give less easily recognized information about the parameters of the cracks; and there will be less control over the initial parameters for the modeling procedure.

Note that we have only modeled synthetic seismograms from borehole shear-wave sources that radiate SV waves, reflecting current technology. A source of SH waves would produce different patterns of polarization, for example, by exciting the second slower split shear wave with orthogonal polarizations in Figure 7a; but the conclusions of this paper are unlikely to be changed significantly.

Shear-wave CHS surveys will be expensive and consequently rarely attempted. We suggest that the major applications of the results of this paper are likely to be in interpreting acoustic events induced by hydraulic pumping. Interpreting acoustic events recorded by down-well three-component geophones should yield unique information about the initial stress distribution and the developing system of cracks.

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