Converted-wave seismology in anisotropic media revisited
Part I: Theory

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We present some recent developments in converted-wave (C-wave) imaging in anisotropic, inhomogeneous media. These developments are extensions of those in Thomsen (1999), but describe the converted-wave signature more accurately for large offsets, and enable the use of standard procedures such as semblance analysis and Kirchhoff summation to implement anisotropic imaging with high accuracy and low cost. Re-processing the Alba, Valhall and Mahogany datasets has resulted in substantial improvements to the converted-wave images. Here we present these new developments in two parts: basic theory and application to velocity analysis and C-wave imaging. This part deals with the basic theory, including both conversion-point calculation and C-wave reflection moveout analysis.

Determining the conversion point and calculating the reflection moveout are two basic steps in C-wave processing. The existing higher-order Taylor expansion of Thomsen (1999) for calculating the conversion point is derived for layered isotropic media, and it is strictly limited to offsets about half the reflector depth (an offset-depth ratio, $x/z$, of 0.5), when applied to layered VTI media. The corresponding C-wave moveout equations are limited to offsets about equal to the reflector depth ($x/z=1.0$).

The new developments are based on the Taylor-series expansion of the anisotropic ray equation. The C-wave signature in layered VTI media is determined by five parameters: $\gamma_0$, $V_{P2}$, $V_{S2}$, $\eta_{\text{eff}}$ and $\zeta_{\text{eff}}$, where,

$$\eta_{\text{eff}} = \frac{1}{8t_{P0}V_{P2}} \left[ \sum_{i=1}^{n} V_{P2i}^4 \Delta t_{P0}(1 + 8\eta_i - t_{P0}V_{P2}^4) \right] ; \quad \zeta_{\text{eff}} = \frac{-1}{8t_{S0}V_{S2}^4} \left[ \sum_{i=1}^{n} V_{S2i}^4 \Delta t_{S0}(1 - 8\zeta_i - t_{S0}V_{S2}^4) \right].$$

(1)

and $\eta_i = (e_i - d_i)/(1 + 2\delta_i)$, $\zeta_i = \gamma_i(1 + 2\sigma_i)/(1 + 2\gamma_i)$. All parameters have the same meaning as in Thomsen (1999) except parameters $\eta_{\text{eff}}$ and $\zeta_{\text{eff}}$. $\eta_{\text{eff}}$ is the Alkhalifah $\eta$, describing P-wave anisotropy, and $\zeta_{\text{eff}}$ is a new parameter describing S-wave anisotropy. These five parameters ($\gamma_0$, $V_{P2}$, $V_{S2}$, $\eta_{\text{eff}}$ and $\zeta_{\text{eff}}$) control the C-wave diffraction curve for Kirchhoff summation, and define the C-wave prestack time migration model.

The C-wave reflection moveout is determined by four parameters: $\gamma_0$, $V_{C2}$, $\gamma_{\text{eff}}$ and $\chi_{\text{eff}}$, which define the C-wave stacking velocity model. The C-wave conversion point is also determined by four parameters: $\gamma_0$, $V_{C2}$, $\gamma_{\text{eff}}$ and $\kappa_{\text{eff}}$, which define the C-wave binning velocity model. Here $\chi_{\text{eff}}$ and $\kappa_{\text{eff}}$ are new parameters,

$$\chi_{\text{eff}} = \eta_{\text{eff}} \gamma_{\text{eff}}^2 - \zeta_{\text{eff}} \gamma_{\text{eff}} \chi_{\text{eff}} + \zeta_{\text{eff}} , \quad \kappa_{\text{eff}} = \eta_{\text{eff}} \gamma_{\text{eff}} \chi_{\text{eff}}^2 + \zeta_{\text{eff}} \chi_{\text{eff}} ,$$

(2)

describing the C-wave anisotropic behaviour in the moveout and conversion-point signatures, respectively. The resulting new equations for calculating the conversion-point extend into offsets about three-times the reflector depth ($x/z=3.0$), while those for calculating the C-wave traveltime extend into offsets twice the reflector depth ($x/z=2.0$). With the improved accuracy, the equations can help in C-wave data processing and parameter estimation in anisotropic, inhomogeneous media (Part II).

Reference
Thomsen, L., 1999, Converted-wave reflection seismology over inhomogeneous, anisotropic media: Geophysics, 64, 678-690.