CONVERTED-WAVE IMAGING IN INHOMOGENEOUS, ANISOTROPIC MEDIA: PART I – PARAMETER ESTIMATION

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Summary

In transversely isotropic media with a vertical symmetry axis (VTI), the converted-wave (C-wave) moveout over middle-to-long offset range is determined by four parameters. These are the C-wave stacking velocity \( V_{C2} \), vertical and effective velocity ratios \( \gamma_0 \) and \( \gamma_{\text{eff}} \), and anisotropic parameter \( \chi_{\text{eff}} \). Detailed numerical analysis is performed to evaluate the sensitivity and error propagation during C-wave moveout inversion. The results show that the two velocity ratios cannot be resolved by semblance analysis with sufficient resolution and accuracy, even for exact inputs of \( V_{C2} \) and \( \chi_{\text{eff}} \). However, \( V_{C2} \) and \( \chi_{\text{eff}} \) can be determined by a double scanning procedure even for inputs of \( \gamma_0 \) and \( \gamma_{\text{eff}} \) that may have a 10% error margin. If a single-scanning semblance analysis is used to determine \( \chi_{\text{eff}} \), an accurate \( V_{C2} \) is required with errors less than 2%, and a practical procedure is presented for this purpose.

Introduction

Anisotropic parameter estimation becomes increasingly important for converted-wave (C-wave) processing. Four Thomsen’s (1986) parameters are responsible for P- and C-wave propagation in transversely isotropic media with a vertical symmetry axis (VTI, or polar anisotropy). The key issue is how to estimate these four parameters from reflection data and build an accurate anisotropic velocity model. The problem lies in that various parameters may be derived from different combinations of the four Thomsen parameters, each of which may leave an imprint in the reflection moveout, depending on how one parameterizes the moveout. For C-wave moveout, various velocities and velocity ratios, and anisotropic parameters have been used in the literature. For example, there are stacking velocities for P-, S- and C-waves, \( V_P^2 \), \( V_S^2 \), and \( V_{C2} \), respectively, vertical and effective velocity ratios \( \gamma_0 \) and \( \gamma_{\text{eff}} \), and anisotropic parameters \( \eta \) and \( \sigma \), etc. Consequently, which parameters can be recovered from the moveout data and how to retrieve them robustly has not been fully understood. Here, we fill this gap by examining the Taylor series expansion of the C-wave moveout. We first investigate the parameter dependencies and error propagation during parameter inversion, then discuss the accuracy and limitation of current procedures, and finally present new and robust methods for parameter estimation.

Parameterization of the moveout signature

For vertically inhomogeneous VTI media, we use the Taylor series expansion in Yuan et al. (2001),

\[
t_c^2 = t_{C0}^2 + \frac{x^2}{V_{C2}^2} + \frac{A_1 x^4}{1 + A_3 x^2},
\]

\[
A_1 = -\left(\gamma_0 \gamma_{\text{eff}} - 1\right)^2 + 8(1 + \gamma_0)\gamma_{\text{eff}} + \frac{A_1 V_{C2}^2 (1 + \gamma_0) \gamma_{\text{eff}}}{4 \gamma_0 (1 + \gamma_{\text{eff}})^2},
\]

\[
A_3 = \frac{A_1 V_{C2}^2 (1 + \gamma_0) \gamma_{\text{eff}} \left[ (\gamma_0 - 1) \gamma_{\text{eff}} + 2 \chi_{\text{eff}} \right]}{(\gamma_0 - 1) \gamma_{\text{eff}} (1 - \gamma_0 \gamma_{\text{eff}}) - 2(1 + \gamma_0) \gamma_0 \gamma_{\text{eff}} \chi_{\text{eff}}},
\]

where \( t_{C0} \) is the C-wave vertical two-way time, and \( \chi_{\text{eff}} \) is a new anisotropic parameter. In a single layer, \( \chi = (\gamma_0 - 1) \gamma_{\text{eff}} \eta \). \( \eta \) is an anisotropic parameter defined by Alkhalifah (1997) anisotropic parameter. Thus, the C-wave moveout is fully controlled by four parameters: \( V_{C2} \), \( \gamma_0 \), \( \gamma_{\text{eff}} \) and \( \chi_{\text{eff}} \). Equation (1) is
accurate up to an offset-depth ratio of $x/z=2.0$ (Yuan et al., 2001). This forms the basis for parameter estimation.

**Parameter dependencies**

The above parameterization isolates $V_{C2}$ from the other three parameters. $V_{C2}$ acts on the quadratic term and controls the near-offset moveout, whilst the other three parameters all act on the quartic term. Thus, $V_{C2}$ can be retrieved reliably, similar to $P$-wave moveout analysis. Numerical analysis is performed over the other three parameters to understand the parameter dependencies. The findings are:

1) The moveout signature is insensitive to the variation in $\gamma_0$. The influence of $\gamma_0$ decreases as offset increases. When other parameters are fixed, changes in $\gamma_0$ up to 15% still only have a very small effect on the moveout, and inversion for $\gamma_0$ from $C$-wave reflection moveout shows poor resolution (Figures 1a and 1b).

2) Inversion for $\gamma_{eff}$ by semblance analysis when other parameters are fixed also shows poor resolution (Figure 1b).

3) In contrast, given $\gamma_0$ and $\gamma_{eff}$, $\chi_{eff}$ can be inverted by a double-scanning procedure with sufficient accuracy and resolution (Figure 1c).

4) The magnitude of $C$-wave non-hyperbolic moveout increases sharply beyond near offsets, compared with that of $P$-wave. This may introduce a significant error in hyperbolic velocity analysis. At offsets with $x/z=1.0$, a 3% error is observed in $V_{C2}$, even for noise-free data (Figures 2a and 2b).

To sum up, although $C$-wave reflection moveout is controlled by four parameters, only two ($V_{C2}$ and $\chi_{eff}$) are likely to be recoverable from the moveout data with sufficient accuracy and resolution. Normally, inversion for velocity ratios directly by semblance analysis is not recommended.

**Error propagation**

$\gamma_0$ and $\gamma_{eff}$ have to be estimated separately before the anisotropic parameter $\chi_{eff}$ can be estimated. The following procedures have been proposed in the literature to determine these two parameters (e.g. Gaiser and Jackson, 1998; Thomsen 1999): determining $V_{C2}$ from short-spread hyperbolic moveout analysis, and $\gamma_0$ from a coarse correlation of $P$- and $C$-wave stacked sections. $\gamma_{eff}$ is then inverted from $V_{C2}$ and the $P$-wave short-spread stacking velocity $V_{P2}$, by

$$\gamma_{eff} = \frac{V_{P2}^2}{V_{C2}^2(1+\gamma_0)} - V_{P2}^2$$

(3)

Error propagation is a severe problem in converted-wave parameter estimation. We have also carried out a detailed analysis to understand this process. Our findings are:

1) Double-scanning semblance analysis for $V_{C2}$ and $\chi_{eff}$ is very robust, and allows 10-15% errors in $\gamma_0$. Also when $\gamma_0$ is used as an input to equation (3), any errors in $\gamma_0$ will not be amplified, but attenuated. Thus, a coarse correlation of the $P$- and $C$-wave stacked sections will give $\gamma_0$ with sufficient accuracy.

2) $\gamma_{eff}$ has a stronger influence than $\gamma_0$ in the quartic term and the error of margin is hence small. The double-scanning procedure may allow 5-10% error in $\gamma_{eff}$. However, error propagation in equation (3) is severe. Errors in $V_{P2}$ and $V_{C2}$ will all be amplified. For typical North Sea sediments with velocity ratio of 2.5, an average 3% error in $V_{C2}$ results in more than 15% error in $\gamma_{eff}$ estimation, which will invalidate the whole inversion process.

3) Because of error propagation, $V_{C2}$ has to be determined to within 2% error. Thus conventional hyperbolic analysis may not be sufficiently accurate for anisotropy parameter inversion.

**Single-scanning semblance analysis for $V_C$ and $\chi_{eff}$**

SCANNING FOR $V_{C2}$. $V_{C2}$ is determined by non-hyperbolic analysis over the middle-offsets. We find that the magnitude of the non-hyperbolic moveout is relatively insensitive to the variation of $\gamma$ in middle
offsets up to \( x/z = 1.0 \). Thus the Taylor-series equation in a single-layered medium may be used to determine \( V_{C2} \):

\[
 t^2 = t_0^2 + \frac{x^2}{V_{C2}^2} + \frac{A_2 x^4}{1 + A_3 x^2}, \quad A_4 = \frac{-\left(\gamma_0 - 1\right)^2}{4 \gamma_0^2 V_{C2}^2}, \quad A_5 = \frac{-\gamma_0}{1 - \gamma_0}. \tag{4}
\]

During the above non-hyperbolic velocity analysis, \( \gamma_0 \) is fixed as a background parameter. This non-hyperbolic velocity analysis with a constant \( \gamma \) is very robust and accurate. \( V_{C2} \) can be determined within an error of less than 1% (Figure 2c). The analysis allows for 15-20% error in the background \( \gamma \) (Figure 2d). This procedure also holds for vertical inhomogeneous, anisotropic media (Figures 3a, 3b and 3c).

**SCANNING FOR \( \chi_{\text{eff}} \).** Once \( V_C \) is accurately determined, \( \chi_{\text{eff}} \) can be determined by semblance analysis over the entire offset range (up to \( x/z = 2.0 \)) as function of \( C \)-wave two-way time (\( t_{C0} \)) (Figure 3d). This procedure is more efficient, but is more sensitive to errors in \( V_{C2} \).

**REAL DATA EXAMPLE.** The above analysis is applied to a 4C dataset from the North Sea. We test the single-scanning procedure on real data. Figure 4 confirms that \( \chi_{\text{eff}} \) can be determined from real data with sufficient resolution and accuracy.

**Discussion and conclusions**

\( C \)-wave reflection moveout is controlled by four parameters over the middle-to-long offset range. Detailed numerical analysis of the parameter dependencies and error propagation shows that only \( V_C \) and \( \chi_{\text{eff}} \) are recoverable from the \( C \)-wave moveout data. This may be achieved by either a double- or single-scanning semblance analysis. The double-scanning procedure is more robust and relatively insensitive to errors in the velocity ratios, but is more expensive to implement. In contrast, the single scanning procedure is more sensitive to errors in \( V_C \), and the error in \( V_C \) has to be less than 2%, but it is much easier to implement.

Accurate determination of \( V_C \) may be achieved by using non-hyperbolic moveout analysis based on the single-layer equation that contains a constant \( \gamma \). The non-hyperbolic moveout term is relatively insensitive to the variation of \( \gamma \) over the middle-offset range (up to \( x/z = 1.0 \)). This procedure is also applicable to vertically inhomogeneous, anisotropic media. Once \( V_C \) is accurately determined, the non-hyperbolic moveout over the ranges of \( x/z : 1.5-2.0 \) can then be used to determine the anisotropic parameter \( \chi_{\text{eff}} \). Real data results confirm the analysis.

**Acknowledgements**

This work is funded by the Edinburgh Anisotropy Project (EAP) of the British Geological Survey, and is published with the approval of the Director of the British Geological Survey (NERC) and the EAP sponsors.
References


