Summary

We assume a scatter point located beneath a stack of layers with vertical transverse isotropy (VTI), and derive an accurate double-square-root (DSR) C-wave diffraction equation. We demonstrate how this DSR equation can be incorporated into the Kirchhoff prestack time migration. The DSR equation is accurate for long offsets and is controlled by five parameters: P- and S-wave stacking velocities $V_{P2}$ and $V_{S2}$, vertical velocity ratio $\gamma_0$, and anisotropic parameters $\eta_{eff}$ and $\zeta_{eff}$. These parameters can be inverted from the four C-wave moveout attributes ($V_{C2}$, $\gamma_0$, $\gamma_{eff}$ and $\zeta_{eff}$) as described in Part I (Li and Yuan, 2001). The DSR equation has a similar form to its isotropic counterpart. This allows an efficient implementation of prestack time migration, and the generation of a realistic anisotropic model for depth migration. Applications to real data show that the C-wave imaging obtained by the new approach is more focused and coherent than the imaging by isotropic methods.

Introduction

Over recent years, a considerable amount of effort has been devoted to prestack migration (e.g. Sena and Toksöz, 1993, Nolte, et al, 1999). For C-wave prestack migration in the presence of anisotropy, the key issue is how to generate a reliable anisotropic velocity model. For transverse isotropy with a vertical axis of symmetry (VTI), it requires the estimation of all four Thomsen parameters to build an anisotropic velocity model, and so far there is no practical solution to this problem. Additional concern also arises with the cost-effectiveness of anisotropic time migration, since the exact migration requires calculation of the group velocity vector for each propagation direction. To address these issues, we derive an anisotropic double-square-root (DSR) equation for a scatter point located beneath a stack of VTI layers, and incorporate it into prestack migration. This is an extension of Li and Druzhinin (2000) in which a single-layer DSR equation is used and is thus restricted to homogeneous media. The new DSR equation is valid for vertically inhomogeneous anisotropic media, and is fully determined by five parameters which can be estimated from non-hyperbolic moveout analysis. We apply this method to a 4C Alba dataset from the North Sea (MacLeod et al. 1999) to evaluate the merit of anisotropic migration.

Notation

Thomsen’s (1999) notation is used. Subscripts P, S, and C denote P-, S-, and C-waves respectively. Subscript i denotes interval quantities, subscript 2 denotes root-mean-squared (rms) quantities, and subscript 0 denotes vertical, or average quantities where appropriate. $t$ stands for travel time, $V$ for velocity, and $\gamma$ for velocity ratio.

The type of anisotropy considered is transverse isotropy with a vertical symmetry axis (VTI). Anisotropic parameter $\eta$ (Alkhalifah, 1997), and two other new parameters $\zeta$ and $\chi$ are used. In a single VTI layer with Thomsen (1986) parameters $\varepsilon$ and $\delta$, they are defined as,

$$\eta = (\varepsilon - \delta)/(1 + 2\delta), \quad \zeta = (\gamma_2^2/\gamma_0^2)\eta, \quad \chi = \gamma_0\gamma_{eff}^2\eta - \zeta,$$

where $\gamma_0$, $\gamma_2$ and $\gamma_{eff}$ are vertical, stacking and effective velocity ratios, respectively.
C-wave diffraction equation for VTI

Consider a scatter point at \((x, z)\) located immediately below a stack of VTI layers (Figure 1). The C-wave diffraction curve equation can be derived as,

\[
t_C = \left( \frac{t_{c0}}{1 + \gamma_0} \right)^2 + \frac{(x + h)^2}{V_{p2}^2} - 2\eta_{eff}\Delta t^2 + \left( \frac{\gamma_0 t_{c0}}{1 + \gamma_0} \right)^2 + \frac{(x - h)^2}{V_{s2}^2} + 2\xi_{eff}\Delta t_s^2,
\]

where

\[
\eta_{eff} = \frac{1}{8t_{p0}V_{p2}^4} \sum_{i=1}^{n} V_{p2}^4 \Delta t_{pi0} (1 + 8\eta_0 - t_{p0}V_{p2}^4),
\]

\[
\xi_{eff} = \frac{1}{8t_{s0}V_{s2}^4} \sum_{i=1}^{n} V_{s2}^4 \Delta t_{si0} (1 - 8\gamma_0 - t_{s0}V_{s2}^4),
\]

\[
\Delta t^2 = \frac{(x + h)^4}{V_{p2}^2 V_{c0}^2 V_{p2}^2 (1 + \gamma_0)^2 + (1 + 2\eta_{eff})(x + h)^2}
\]

and

\[
\Delta t_s^2 = \frac{(x - h)^4}{V_{s2}^2 V_{c0}^2 V_{s2}^2 (1 + \gamma_0)^2 + (x - h)^2}
\]

and \(h\) is the half source-receiver offset. Although equation (2) is an approximation, it is accurate up to, at least, offset depth ratio of 2.0 (Yuan, 2000).

Kirchhoff prestack time migration

Given \(V_{p2}, V_{s2}, \gamma_0, \eta_{eff}\) and \(\xi_{eff}\), equation (2) can be utilized to perform Kirchhoff prestack time migration. Similarly to isotropic migration (Solid et al., 1997), the anisotropic Kirchhoff prestack time migration may also be implemented as a weighted summation of amplitudes along diffraction curves, that is,

\[
I(\tau, y, h) = \int W(\tau, y, b, h) \frac{\partial}{\partial t} u(\tau = t_C, y, b, h) db,
\]

where \(I\) is the imaging, \(t (=t_{c0})\) is the time depth, \(W\) is a weighting function, \(b, (=x-y)\), is the imaging point offset from the midpoint, \(u\) is the input data, and \(t_C\) is the anisotropic diffraction curve defined by equation (2).

Construction of the anisotropic velocity model

The diffraction curve equation (2) requires five parameters: \(V_{p2}, V_{s2}, \gamma_0, \eta_{eff}\) and \(\xi_{eff}\), and needs to be constructed from the four C-wave moveout attributes: \(V_{c2}, \gamma_0, \eta_{eff}\) and \(\chi_{eff}\) as described in Part I. This requires a Dix-type layer-stripping procedure. Note that \(\chi_{eff}\) is defined as,

\[
\chi_{eff} = \eta_{eff} \gamma_0^2 \chi_{eff} - \xi_{eff},
\]

and equation (3) can be re-written as,

\[
\eta_{eff} = \frac{1 + \gamma_0}{8\gamma_0 V_{c2}^2 V_{p2}^2} \sum_{i=1}^{n} V_{p2}^2 \Delta t_{ci0} \left( \frac{1 + \frac{8\gamma_0}{1 + \gamma_0} (\frac{1}{1 + \gamma_0} - \frac{8\chi_{eff}}{r_0 - 1})} {r_0 - 1 + \frac{8\chi_{eff}}{r_0 - 1}} - \frac{t_{c0}}{1 + \gamma_0} V_{p2}^4 \right),
\]

and

\[
\xi_{eff} = -\frac{(1 + \gamma_0)}{8\gamma_0 t_{c0} V_{s2}^2} \sum_{i=1}^{n} V_{s2}^4 \gamma_0 \Delta t_{ci0} \left( \frac{1 - \frac{8\chi_{eff}}{r_0 - 1}} {r_0 - 1 + \frac{8\chi_{eff}}{r_0 - 1}} - \frac{t_{s0} V_{s2}^2}{1 + \gamma_0} \right).
\]

Thus, for each input parameter set \((V_{c2}, \gamma_0, \eta_{eff}\) and \(\chi_{eff}\)) at time \(t_{c0}\), the following steps can be used to construct the anisotropic velocity model (Figure 2):

1) Invert \(V_{p2}\) and \(V_{s2}\) from \(V_{c2}, \gamma_0\), and \(\eta_{eff}\) by

\[
V_{p2}^2 = V_{c2}^2 \frac{\gamma_0 (1 + \gamma_0)}{1 + \eta_{eff}} \quad \text{and} \quad V_{s2}^2 = V_{c2}^2 \frac{(1 + \gamma_0)}{\eta_{eff}(1 + \gamma_0)}.
\]

2) Convert \(V_{p2}\) and \(V_{s2}\) and the average ratio \(\gamma_0\) to interval parameters \(V_{p2i}, V_{s2i}\) and \(\gamma_0i\).

3) Use equations (6) to build a Dix-type layer-stripping procedure to determine \(\chi_i\).

4) Calculate the effective parameters \(\eta_{eff}\) and \(\xi_{eff}\) using equation (7).

Updating the anisotropic velocity model

It is a common practice to update the model during migration. Updating is achieved by analyzing the residual moveout in the common imaging point (CIP) gathers. Similarly to moveout analysis, as described in Part I, we find that model updating should be restricted to \(V_{c2}\) and \(\chi_{eff}\). Updating the model can be very
time consuming, since each updating requires re-construction of the velocity model by the layer-stripping procedure in Figure 2. In practice, a two-step procedure may be used. We first update $V_{C2}$ by analyzing the residual moveout in near offset traces without considering anisotropy, and thus make no changes to $\chi_{\text{eff}}$. Once $V_{C2}$ is satisfactorily determined, we use the far-offset traces to update $\chi_{\text{eff}}$. Fortunately, we find that one iteration is usually sufficient.

A 4C data example

We test the method using the Alba 4C dataset (MacLeod et al., 1999). A 2D line is extracted for this purpose. Moveout analysis, as described in Part I, is applied to the data to determine $V_{C2}$, $\gamma_0$, $\chi_{\text{eff}}$ and $\chi_{\text{eff}}$. These parameters are then used to construct the velocity model ($V_{P2}$, $V_{S2}$, $\gamma_0$, $\eta_{\text{eff}}$ and $\zeta_{\text{eff}}$) following the procedures in Figure 2. Residual moveout in CIP gathers are used to update the velocity model. Figure 3 shows an example of this process. The final migrated imaging by anisotropic migration shows considerable improvement in focusing the faults and small structures, compared with the isotropic migrated imaging (Figure 4).

Conclusions

We have extended the single-layer DSR equation to vertically inhomogeneous, anisotropic media and presented an accurate and efficient approach for anisotropic prestack time migration. The initial velocity model is constructed from reflection moveout analysis with parameters $V_{C2}$, $\gamma_0$, $\chi_{\text{eff}}$ and $\chi_{\text{eff}}$. These parameters are then converted to $V_{P2}$, $V_{S2}$, $\gamma_0$, $\eta_{\text{eff}}$ and $\zeta_{\text{eff}}$ for the DSR equation using a Dix-type scheme. Migration velocity updating is possible, but it should be restricted to $V_{C2}$ and $\chi_{\text{eff}}$ only. Updating of $V_{C2}$ is achieved by aligning the near-offset events, and updating of $\chi_{\text{eff}}$ by aligning the far-offset events in the CIP gather. Application to real data verifies the approach and shows significant improvement in imaging quality. The model obtained from this migration scheme may then be used for pre-stack depth migration.

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References


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Figure 1. C-wave scattering in layered VTI media.

Figure 2. Building the anisotropic velocity model.

Figure 3. Updating the migration velocity model using CIP gathers. (a) A CIP gather with initial migration parameters. (b) and (c) are the gather but after updating $V_{C2}$ and $\chi_{eff}$, respectively.

Figure 4. Migrated C-wave images of the Alba data: (a) isotropic and (b) anisotropic prestack time migration.