Pressure induced anisotropy of interconnected cracks
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Summary

The cracks in a porous matrix which is subjected to a change in the applied stress or fluid pressure will undergo a distortion related to their orientation relative to the principal directions of the applied stress. Both the crack distribution and the fluid-flow properties of the aggregate will be altered as a consequence of a change in either the applied stress or fluid pressure, resulting in a change in the effective elastic parameters of the material.

An effective medium theory, based on the method of smoothing and incorporating a transfer of fluid between connected cracks via non-compliant pores, is used to derive an expression for the effective elastic parameters of the material, to first order in crack density \( \epsilon \). This expression involves a dependence on both the applied stress and the fluid pressure, and is used to determine the effects on the anisotropy of the effective medium of the applied stress and fluid pressure.

A biaxial compressive stress is applied to an isotropic crack distribution to determine the anisotropy of the (transversely isotropic) effective medium as a function of differential pressure. As a result of competing processes, the theory predicts that there is a pressure at which the anisotropy reaches a maximum value before the properties of the effective medium decay, under increasing stress, to those of the uncracked matrix.

Introduction

The cracks in a matrix rock are, potentially, a result of a number of competing processes that dictate the resulting crack distribution. Within unstressed rock, it may be assumed that the cracks are isotropically distributed and exist as a result of either the mineralogy of the rock or thermal processes. A non-isotropic stress applied to the rock will result in preferential crack closure. The greater the alignment between the crack normal and the principal directions of the stress, the less the stress needed to close any given crack. The resulting crack distribution will be anisotropic. Further, an applied stress will, in general, result in the formation of new cracks that are likewise predominantly oriented parallel to the direction of maximum principle stress.

The presence of pores or cracks in matrix rock will influence the mechanical properties of the rock. Thus, by modeling the crack distribution to account for the effects of stress we may study the effects of the cracks on these mechanical properties, such as the velocities or the permeability. One may then attempt to invert measurements of such properties for crack or pore parameters. Clearly then, advancements in the modeling of the mechanical properties resulting from a distribution of cracks will aid the accuracy of inversion techniques.

The effects of the application of an ambient stress to a cracked medium has been studied both analytically (e. g. Pecorari, 1997) and experimentally (e. g. Wulff et al., 1999) concluding that wavespeeds increase with differential pressure, defined as the difference between the compressive stress and fluid pressure, whatever the form of the compression that the rock is subjected to. The behavior of fractures under compression has also been the subject of analytic (Bai et al., 2000) and experimental (Brown and Scholz, 1986) studies. Results are explained in terms of preferentially oriented crack closure. However, none of these models have an explicit dependence on the flow of fluid between cracks.

The method of smoothing (Hudson, 1980), an effective medium theory, gives the effective elastic parameters of a cracked material, which allows the calculation of the speeds of waves of long wavelengths. The method has recently been extended to allow for the cracks to be connected through the porosity of the matrix rock (Hudson et al., 1996; Tod, 2001) where fluid may flow between cracks which have been distorted differently by an incident wave due to differences in their orientation and aspect ratio.
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(Fig. 1). This model, up to Tod (2001), fails to account for the dependence of the crack distribution on both the applied stress field and the fluid pressure, and the fact that the distributions of crack aspect ratio and orientation are interdependent.

In this paper a model for the number density of cracks is developed that depends on both the aspect ratio and orientation of the cracks, and further depends on both the applied stress and the fluid pressure. While Sun and Goldberg (1997) categorize the dynamic process of rock deformation under a changing differential pressure into four stages, only the first and third of these are considered in the model herein; collapse of, and closure of, the original cracks. The remaining two stages are defined as: the deformation of pores into cracks and the formation of new cracks of small aspect ratio. The model developed here thus predicts that during subsequent unloading, the material relaxes to its original state. Thus, this model is purely elastic, and assumes that that the effect of stress and fluid pressure on aspect ratio and crack growth.

This model for the crack number density is used in conjunction with the dependence of the effective elastic parameters on the crack distribution (Tod, 2001) to model the relationship between the anisotropy of the effective medium, incorporating fluid-flow, and the applied stress and fluid pressure.

A fluid-flow model

We follow the model of connected cracks proposed by Hudson et al., (1996) and extended by Tod (2001) for the transfer of fluid between cracks by non-compliant pores. The model is based on a local flow governed by diffusion

$$\frac{\partial}{\partial t} \left( \rho_f^* \phi_n \right) = -\frac{\phi_n^0 \rho_f}{\kappa_f \tau} \left( p_f^0 - p_f \right)$$

(1)

and a global flow resulting from D’Arcy’s law

$$\frac{\partial}{\partial t} \left( \sum_n \rho_f^* \phi_n \right) = \nabla \cdot \left( \frac{\kappa^*}{\eta_f} \nabla p_f + \nabla \phi_n \right),$$

(2)

where $\phi_n$, $\phi_n^0$, $\rho_f^0$ and $p_f^0$ are the porosity, stress-free porosity, fluid density and fluid pressure respectively, in the $n$th set of cracks, where the cracks have been divided into families of parallel cracks with identical aspect ratio. $\rho_f$ is the unstressed density, $\kappa_f$ the bulk modulus of the fluid, $\rho_f$ the average fluid density, $p_f$ the average (local) pressure in the fluid, $\eta_f$ the fluid viscosity, $\kappa^*$ the permeability tensor of the matrix and $\tau$ a relaxation parameter.

The effect of stress and fluid pressure on aspect ratio

A change in the stress and/or pressure on a cracked material will result in a distortion of the cracks, that will alter the effective elastic parameters. Hudson (2000) derives an expression for the change in crack aspect ratio $\delta a$ due to a change in applied stress field $\delta \sigma$ and fluid pressure $\delta p_f$ on a crack whose normal lies in the direction $n$:

$$\delta a = \frac{2(1-\nu)}{\pi \mu} (\delta \sigma_{ij} n_i n_j + \delta p_f),$$

(3)

where $\nu$ is Poisson’s ratio for the matrix material. The sign convention is such that a positive $\delta \sigma$ or $\delta p_f$ will cause the cracks to open.

Attention is restricted to biaxial compression;

$$\sigma_{ij} = -\sigma (\delta i \delta j_n + \delta j \delta i_n)$$

(4)

We then have that the differential pressure is given by

$$p^{diff} = - (\| \sigma_{ij} \| + p_f) = \sigma - p_f.$$

(5)

Crack density

The total crack density $e$ is given by

$$e(\sigma, p_f) = e_0 \int \Sigma \, d\phi^*,$$

(6)

where $e_0$ is the crack density in the absence of an applied stress or fluid pressure and

$$dn^* = n^*(\alpha; \sigma, p_f, \theta) \, d\alpha \sin \theta d\theta$$

(7)

is the probability density of cracks in an elemental area $d\alpha \sin \theta d\theta$ of the region $\Sigma$, such that

$$\int_\Sigma n^*(\alpha; 0, 0, \theta) \, d\alpha \sin \theta d\theta = 1.$$

(8)

$\Sigma$ is defined as the region $0 \leq \alpha \leq \infty$ and $0 \leq \theta \leq \pi / 2$. The angular dependence is defined such that $\theta = 0$ corresponds to the $x_3$-direction.

We make the assumption that in the absence of any applied stress or fluid pressure the distribution of cracks is independent of the crack normal orientation;

$$n^*(\alpha; 0, 0, \theta) = n^0(\alpha).$$

(9)

Using eq. (3) we may then write

$$n^*(\alpha; \sigma, p_f, \theta) = n^0(\alpha - f(\sigma, p_f, \theta))$$

(10)

for

$$\alpha \geq \max \{ f(\sigma, p_f, \theta), 0 \}$$

(11)

where

$$f(\sigma, p_f, \theta) = - \frac{2(1-\nu)}{\pi \mu} (\sigma \sin^2 \theta - p_f).$$

(12)
As Tod (2001), we approximate the initial crack aspect ratio distribution, \( n_0(\alpha) \), with a Gamma \( \Gamma(1/\delta^2, 1/\delta^2) \) distribution with \( c_0 = 0.00837 \) and \( \delta = 0.703 \), to give an approximation to the measurements of Hay et al. (1988).

\[
n_0(\alpha) = \frac{(c_0\delta)^{1/\delta^2}}{2\pi \Gamma(1/\delta^2)} \exp(-\alpha / c_0\delta^2),
\]

where \( \Gamma(x) \) is the Gamma function.

The resulting crack density is given by

\[
e(\sigma, p_f) = e_0 \int_0^{\pi/2} \frac{\Gamma(1/\delta^2, g(\sigma, p_f, \theta))}{\Gamma(1/\delta^2)} \sin\theta d\theta,
\]

where

\[
g(\sigma, p_f, \theta) = \max \{ -f(\sigma, p_f, \theta) / c_0\delta^2, 0 \}
\]

and \( \Gamma(x, y) \) is the incomplete Gamma function such that \( \Gamma(x, 0) = \Gamma(x) \).

**Effective elastic parameters**

This model is used by Tod (2001) to derive an expression, in frequency - wavenumber space, for the effective elastic parameters of a cracked material to first order in the crack density \( \epsilon \):

\[
c_{ip,jq} = c_{ip,jq}^0 + \epsilon c_{ip,jq}^1 + O(\epsilon^2),
\]

where \( c^0 \) is the elastic tensor for the, assumed isotropic, porous matrix material,

\[
c_{ip,jq}^0 = \lambda \delta_{ij} \delta_{jq} + \mu (\delta_{iq} \delta_{jp} + \delta_{jq} \delta_{ip}),
\]

\( \lambda \) and \( \mu \) are the Lamé constants of the material; \( c^1 \) accounts for scattering off individual cracks.

The expression derived for the first order correction to the effective elastic parameters by Tod (2001) may be given in terms of a continuous distribution of crack aspect ratio and orientation as

\[
\epsilon c_{ip,jq} = \frac{\epsilon_0}{\mu} c_{ip,jq}^0 \int_{\Sigma} \hat{T}_{\lambda \nu} \omega_d \omega^c \left( \frac{\omega \cdot \hat{n}^c}{\omega \cdot \hat{n}} \right),
\]

where \( \hat{T}_{\lambda \nu} \omega_d \omega^c \) is a function of crack aspect ratio and orientation, and also depends upon frequency and material properties of the matrix and fluid.

**Results**

When the differential pressure \( p_f \) (eq. 5) is negative, that is \( \sigma / p_f \leq 1 \), there is no crack closure, thus no change in crack density.

Once \( \sigma / p_f \geq 1 \), preferentially oriented crack closure occurs and the crack density decreases with increasing compressive stress (Fig. 2). At large stresses only those cracks with normals lying in, or close to, the \( x_2 \)-direction remain open. The presence of a non-zero pressure within the fluid medium increases the crack density at any given value of the compressive stress, as it hinders the process of crack closure (Fig. 2).

For waves propagating in a direction parallel to the symmetry axis, there is no shear wave splitting, only two wavespeeds. Both of these wavespeeds are seen to increase with differential pressure (Fig. 4), approaching the matrix wavespeeds at large pressures. The shear wavespeed shows a greater proportional change with pressure than the compressional wavespeed does, indicating that shear waves are more sensitive to pressure changes than compressional ones.

**Discussion**

The model proposed by Hudson et al. (1996) for the transfer of fluid between connected cracks via non-compliant pores has been extended to allow for a continuous distribution of values of both crack orientation and aspect ratio, that depend upon each other via the applied stress and fluid pressure. This more realistic model has the expected properties that at high frequencies the cracks behave as if isolated, while at low frequencies they behave as if undrained; in addition, the relationship between the expressions for undrained and isolated conditions agrees with Brown and Korriga (1975). The pressure effects are included via a mechanism that allows for the collapse and closure of the cracks present in an unstressed state, and does not allow for the possible growth of existing cracks, the deformation of pores into cracks, or the formation of...
of new cracks. This leads to a description that is purely elastic under loading and unloading, rather than exhibiting hysteresis. Further, stress history is not accounted for in the model, by assuming that in the unstressed state the crack distribution is isotropic. We may remedy this by including an initial crack distribution that is anisotropic. This would have the further advantage of increasing the magnitude of the anisotropy parameters.

While the effect of an increase in the differential pressure will increase the polarization of the distribution of remaining cracks, which will serve to increase the anisotropy, the total crack density will decrease, thus decreasing the anisotropy. These two competing processes explain the rise in the anisotropy parameters with differential pressure (Fig. 3) to a maximum absolute value, before a decay towards zero as the differential pressure increases yet further, as the properties of the effective material approach those of the isotropic matrix.

This model of an effective medium theory may be combined with a model of a fault (Tod and Hudson, 2001) to produce synthetic seismograms for the scattering from a fault, that may be compared with well-log or VSP data.

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**References**


