Accuracy and sensitivity analysis for estimating anisotropic parameters from 4C seismic data
Xiang-Yang Li*, Edinburgh Anisotropy Project, British Geological Survey, and Jianxin Yuan, PGS Inc.

Summary

In transversely isotropic media with a vertical symmetry axis (VTI), the converted-wave (C-wave) moveout over the intermediate-to-far offset range is determined by four parameters. These are: the C-wave stacking velocity $V_{C2}$, the vertical and effective velocity ratios $\gamma_0$ and $\gamma_{\text{eff}}$, and the anisotropic parameter $\chi_{\text{eff}}$. We investigate the accuracy and sensitivity of the estimation of these four parameters from 4C seismic data. One of the key steps during 4C reflection moveout inversion is the accurate determination of the converted-wave (C-wave) short-stack stacking velocity ($V_{C2}$). $V_{C2}$, as determined from conventional hyperbolic procedures, has an error margin of 3-5%, which is not sufficiently accurate for inversion of anisotropic parameters due to error propagation. To invert the anisotropic parameter $\chi_{\text{eff}}$ with error bars of 10-15%, $V_{C2}$ is required with errors less than 2%. A practical procedure is presented for this purpose by including a non-hyperbolic term in the moveout analysis. This term is determined by a background velocity ratio $\gamma$ and is relatively insensitive to the variation of $\gamma$ over the intermediate offset-ranges (offset-depth ratio $x/z$ from 1.0 to 1.5). Based on this, a robust work flow is developed for estimating all four parameters from reflection moveout.

Introduction

Parameter estimation for anisotropy becomes increasingly important for 4C seismic data processing. Four Thomsen’s (1986) parameters are associated with for P- and converted-wave (C-wave) propagation in transversely isotropic media with a vertical symmetry axis (VTI, or polar, anisotropy). The key issue is how to estimate these four parameters from reflection data and build an accurate anisotropic velocity model. The problem lies in that various parameters may be derived from different combinations of the four Thomsen’s parameters, each of which may leave an imprint in the reflection moveout, depending on how one parameterizes the moveout. For C-wave moveout, various velocities, velocity ratios, and anisotropic parameters have been used in the literature. For example, there are stacking velocities for P-, S- and C-waves, $V_{P2}$, $V_{S2}$, and $V_{C2}$, respectively, vertical and effective velocity ratios $\gamma_0$ and $\gamma_{\text{eff}}$, and anisotropic parameters $\eta$ and $\sigma$, etc. Consequently, which parameters can be recovered from the moveout data and how to retrieve them robustly has not been fully understood.

Here, we fill this gap by examining the Taylor series expansion of the C-wave moveout. We first investigate the sensitivity and error propagation during parameter inversion. We then present robust schemes for accurately determining $V_{C2}$ and all other parameters. Both synthetic and real data will be used to illustrate these schemes.

Parameterization of the moveout signature

For vertically inhomogeneous VTI media, we use the Taylor series expansion of Yuan et al. (2001),

$$t_C^2 = t_{C0}^2 + \frac{x^2}{V_{C2}^2} + \frac{A_2 x^4}{1 + A_4 x^2} + \frac{A_3 x^6}{1 + A_6 x^2} + \frac{A_4 x^8}{1 + A_8 x^2} + \frac{A_5 x^{10}}{1 + A_{10} x^2} + \frac{A_6 x^{12}}{1 + A_{12} x^2},$$

where $t_{C0}$ is the vertical two-way time, and $x$ is an anisotropic parameter. In a single layer, $\chi = (\gamma_0 - 1) \gamma_{\text{eff}}$, where $\eta$ is the anisotropic parameter defined by Alkhalifah (1997). Thus, the C-wave moveout is fully controlled by four parameters: $V_{C2}$, $\gamma_0$, $\gamma_{\text{eff}}$ and $\chi_{\text{eff}}$, which are referred to as moveout attributes. Equation (1) is accurate up to an offset-depth ratio of $x/z=2.0$ (Yuan et al., 2001). This understanding forms the basis for parameter estimation.

Sensitivity analysis

The above parameterization isolates $V_{C2}$ from the other three parameters. $V_{C2}$ acts on the quadratic term and controls the near-offset moveout, whilst the other three parameters all act on the quartic term. Thus $V_{C2}$ can be retrieved reliably, as in P-wave moveout analysis. Numerical analysis is performed over the other three parameters to understand the parameter dependencies and sensitivity. The findings are:

1. The moveout signature is insensitive to the variation in $\gamma_0$. The influence of $\gamma_0$ decreases as offset increases. When other parameters are fixed, changes in $\gamma_0$ up to 15% still have only a very small effect on the moveout, and inversion for $\gamma_0$ from C-wave reflection moveout shows poor resolution (Figures 1a and 1b).

2. Inversion for $\chi_{\text{eff}}$ by semblance analysis when other parameters are fixed also shows poor resolution (Figure 1b).
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3. In contrast, given $\gamma_0$ and $\chi_{\text{eff}}$, $V_{C2}$ and $\chi_{\text{eff}}$ can be inverted by a double-scanning procedure of semblance analysis with sufficient accuracy and resolution (Figure 1c).

4. Furthermore, given $V_{C2}$, $\gamma_0$ and $\chi_{\text{eff}}$, $\chi_{\text{eff}}$ can be inverted by a single-scanning procedure with sufficient accuracy and resolution (Figure 2).

To sum up, although C-wave reflection moveout is controlled by four parameters, only two ($V_{C2}$ and $\chi_{\text{eff}}$) are likely to be recoverable from the moveout data with sufficient accuracy and resolution. Normally, inversion for velocity ratios directly by semblance analysis is not recommended.

![Image](https://via.placeholder.com/150)

Figure 1: Sensitivity analysis for a single VTI layer of Dog Creek shale ($V_P=1875\text{m/s}$, $V_S=826\text{m/s}$, $\varepsilon=0.225$ and $\delta=0$) at depth $z=1000\text{m}$. (a) Synthetic gather. (b) Double-scanning for $\gamma_0$ and $\chi_{\text{eff}}$ by fixing $V_{C2}$ (1540m/s) and $\chi_{\text{eff}}$ (0.187), and (c) Double-scanning for $V_{C2}$ and $\chi_{\text{eff}}$ by fixing $\gamma_0$ (2.270) and $\chi_{\text{eff}}$ (1.191). The dots indicate model results.

![Image](https://via.placeholder.com/150)

Figure 2: Sensitivity analysis for a three-layer VTI model with Dog Creek shale, as in Figure 1, Limestone shale ($V_P=3306\text{m/s}$, $V_S=1819\text{m/s}$, $\varepsilon=0.134$ and $\delta=0$), and Taylor sandstone ($V_P=3368\text{m/s}$, $V_S=1829\text{m/s}$, $\varepsilon=0.110$ and $\delta=-0.035$). Each layer is 500m thick. (a) Synthetic gather. (b) Semblance analysis for $\chi_{\text{eff}}$ given exact inputs of $V_{C2}$, $\gamma_0$ and $\chi_{\text{eff}}$. (c) The same as (b) but given 2% perturbation in $V_{C2}$, and exact inputs of $\gamma_0$ and $\chi_{\text{eff}}$. The thick line is for model values, and the thin line for picked values.

Error analysis

$\gamma_0$ and $\chi_{\text{eff}}$ have to be estimated separately before the anisotropic parameter $\chi_{\text{eff}}$ can be estimated. The following procedures have been proposed in the literature to determine these two parameters (e.g. Gaiser and Jackson, 1998; Thomsen 1999): determining $V_{C2}$ from short-spread hyperbolic moveout analysis, and $\gamma_0$ from a coarse correlation of $P$- and $C$-wave stacked sections. $\gamma_{\text{eff}}$ is then inverted from $V_{C2}$ and the $P$-wave short-spread stacking velocity $V_{P2}$, by

$$\gamma_{\text{eff}} = \frac{V_{P2}^2}{V_{C2}^2(1+\gamma_0)-V_{P2}^2}.$$  

![Image](https://via.placeholder.com/150)

Figure 3: Accuracy of hyperbolic velocity analysis for a five-layer isotropic model. (a) Synthetic gather calculated by ray tracing. (b) Hyperbolic analysis with $x/z=1.0$. (c) Hyperbolic analysis with $x/z=1.5$. The thick line indicates model values, and the thin line denotes picked values.

Error propagation is a severe problem in converted-wave parameter estimation. We have also carried out a detailed analysis to understand this process. Our findings are:

1. The magnitude of C-wave non-hyperbolic moveout increases sharply beyond near offsets, compared with that of $P$-wave. This can introduce a significant error when hyperbolic velocity analysis is employed. At offsets with $x/z=1.0$, a 3% error is observed in $V_{C2}$, even for noise-free data (Figures 3a and 3b). This error increases to about 10% at offsets with $x/z=1.5$ (Figure 3c).

2. Since the C-wave moveout is less sensitive to variations in $\gamma_0$ and $\chi_{\text{eff}}$, moveout inversion is also relatively robust to variations in $\gamma_0$ and $\chi_{\text{eff}}$. It is found that moveout inversion can tolerate 10-15% errors in $\gamma_0$, 5-10% errors in $\chi_{\text{eff}}$. A coarse correlation of the $P$- and $C$-wave stacked sections will often give $\gamma_0$ with sufficient accuracy.

3. Error propagation in equation (3), when used to determine $\chi_{\text{eff}}$, is severe. Errors in $V_{P2}$ and $V_{C2}$ will be amplified. For typical North Sea sediments with a velocity ratio of 2.5, an average 3% error in $V_{C2}$ results in more than 15% error in estimates of $\chi_{\text{eff}}$, which may invalidate the whole inversion process.

4. In semblance analysis for $\chi_{\text{eff}}$, a 2% error in $V_{C2}$ will yield about a 10-15% error in $\chi_{\text{eff}}$ (Figure 2c). If this is considered as the minimum acceptable error margin for $\chi_{\text{eff}}$, $V_{C2}$ has to be determined with errors less than...
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2%. Thus, the conventional hyperbolic short-spread approach is not suitable for anisotropic parameter inversion.

Figure 4: Analysis of the sensitivity of C-wave moveout to the variation in the background velocity ratio $\gamma$ using equations (4) and (5). The model curve is defined by $t_{C0}=1.5$ seconds, $V_{C2}=1500$ m/s, and $\gamma=2.5$. The residual moveout is the difference between the model curve and the curves calculated using five other ratios from 2.0 to 3.0 while $V_{C2}$ and $t_{C0}$ are fixed.

\[ C0 = 1.5 \text{ seconds}, V_{C2} = 1500 \text{ m/s}, \gamma = 2.5. \]

Figure 5: Accuracy of non-hyperbolic velocity analysis for the data in Figure 3a. (a) Non-hyperbolic analysis with $x/z=1.0$, and $\gamma=2.5$; (b) the same as (a) but for $x/z=1.0$, and $\gamma=3.0$; (c) for $x/z=1.5$, and $\gamma=2.5$.

**Non-hyperbolic analysis for $V_{C2}$ over intermediate spread**

We discuss the accuracy of determining $V_{C2}$ by non-hyperbolic analysis over the intermediate-spread ($x/z$ up to 1.5). Unlike for P-waves, the magnitude of the non-hyperbolic moveout for converted-waves for spreads up to $x/z=1.0$ is no longer negligible. However, we find that the non-hyperbolic term is significant but relatively insensitive to the variation of $\gamma$ in the intermediate offsets up to $x/z=1.0$ (Figure 4). Thus the Taylor-series expansion with a non-hyperbolic correction controlled by a background $\gamma$ may be used to determine $V_{C2}$, e.g.,

\[ t_{C}^{2} = t_{C0}^{2} + \frac{x^{2}}{V_{C2}^{2}} + A_{1}x^{4} + A_{5}x^{7} + A_{3}x^{6} + A_{4}x^{8}, \]

where

\[ A_{i} = \frac{(-\gamma_{0})^{i-1} \gamma_{eff}^{i-1} \gamma_{0}}{4t_{C0}^{i} V_{C2}^{2}} \text{ and } A_{i} = \frac{A_{i} V_{C2}^{2} \gamma_{0}}{1-\gamma_{0}}. \]
Finally, with $V_{C2}$, $\gamma_0$, and $\gamma_{eff}$ all determined satisfactorily, $\chi_{eff}$ can be determined by semblance analysis over the entire offset range using equations (1) and (2). This procedure is more efficient than other double scanning methods (Yuan et al. 2001).

**Data example**

The above workflow was applied to a 4C dataset from the North Sea. Figure 7 shows the data and the results of the non-hyperbolic velocity analysis using offset ranges up to $x/z=1.5$. Figure 8a shows the result of semblance analysis for $\chi_{eff}$ over the entire offset range. For comparison, we perturbed the $V_{C2}$ with a 5% error, and scanned $\chi_{eff}$ again. Figure 8b shows the re-scanned result. There is a good resolution on $\chi_{eff}$ but the result is very sensitive to $V_{C2}$. This confirms that $\chi_{eff}$ can be determined from real data with sufficient resolution and accuracy.

**Discussion and conclusions**

The C-wave reflection moveout in anisotropic media is controlled by four parameters: $V_{C2}$, $\gamma_0$, $\gamma_{eff}$ and $\chi_{eff}$. Error propagation is a severe problem in C-wave reflection-moveout inversion. The current short-spread stacking velocity as deduced from hyperbolic moveout does not provide sufficient accuracy to yield meaningful inverted values for the anisotropic parameters. The non-hyperbolic moveout over the intermediate-offset ranges ($x/z$ from 1.0 to 1.5) is no longer negligible, but can be quantified using a background $\gamma$. Non-hyperbolic analysis with a $\gamma$ correction over the intermediate offsets can yield $V_{C2}$ with errors less than 1% for noise free data. The procedure is very robust, allowing initial guesses of $\gamma$ with up to 20% errors. It is also applicable for vertically inhomogeneous, anisotropic media. This improved accuracy makes it possible to estimate anisotropic parameters using 4C seismic data. A practical workflow is presented for this purpose. Real data results confirm the analysis.

**Acknowledgements**

We thank Leon Thomsen for various useful discussions. This work is funded by the Edinburgh Anisotropy Project (EAP) of the British Geological Survey, and had been carried out when the first author, Jerry Yuan, was a PhD student with EAP and University of Edinburgh. The work is published with the approval of the Director of BGS (NERC) and EAP sponsors: Agip, BP, Chevron, Conoco, ExxonMobil, Landmark, Norsk Hydro, PGS, Phillips, Schlumberger, Shell, Texaco, TradePartners UK, Veritas DGC.

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