Abstract
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Sensitivity of symmetry determination by decomposition.

The concept of a tensor is fundamental to elastic anisotropy. In our field we utilize the tensor to describe physical properties. As such the tensor is independent of coordinate systems, and can be represented by its components referred to a coordinate system. A vector – which is a first rank tensor – is the most familiar quantity where the components are dependent on the coordinate system. The set of components is a representation of the vector. A scalar quantity like the length of a vector is independent of the coordinate system to which the vector is referred. The length of the vector is an example of a coordinate-free quantity. The most familiar definition of a tensor is given by the rule which gives the connection between tensor components in two orthonormal coordinate systems. From mathematics a tensor in \( N \) dimensions can be defined in a formal way as a multilinear functional, mapping from the \( N \)-dimensional vectorspace to the vectorspace of real numbers. We will study the connection between the two definitions and give a physical application of the formal definition in elasticity. Further it will be indicated how tensor algebra can easily be derived from such a formal definition. The formal definition is the basis for decomposition of the elastic tensor by means of harmonic tensors. These harmonic tensors are uniquely related to spherical harmonics. Each harmonic tensor in the decomposition can be geometrically represented by vectors in three dimensional space (Maxwell multipoles).

In an arbitrary coordinate system an elastic tensor has 21 non-zero components. The exception is the isotropic tensor with a specific set of vanishing components for all coordinate systems and only two independent components. For ideal media with their specified symmetry there exist coordinate systems
where some of the components can be made zero. For experimentally observed tensors, there are no vanishing components due to deviation from ideal symmetry and inaccuracy in measurements. Thus there are 21 components for real media in any coordinate system. Given such a tensor, the natural question in elastic anisotropy is how its material symmetry can be determined from the components of the elastic tensor. How can a coordinate system which reduces the number of constants be determined, and what can be said about symmetry? Those and similar questions are the motivation for decomposition of the elastic tensor into harmonic tensors.

In an attempt to answer these questions, numerical models of real media of monoclinic, TI- and orthorhombic symmetry are shown, and the sensitivity of Maxwell multipoles analysed. These particular symmetries are considered as they are most commonly employed in practical applications. The different elastic tensors are generated using known relations for fractured and laminated media, thus restricting the study to tensor elements in a domain of common usage. The tensors are generated together with a random noise process, and a geometric visualization obtained for each separate realization. This then provides the basis for studying the discriminatory ability of the harmonic expansion, and isolating terms of specific sensitivity. The study may ultimately result in a way of constructing relationships between elastic constants which can provide an aid to the analysis of experimental data.