THE EFFECT ON SEISMIC ANISOTROPY OF FLUID FLOW IN CRACKED MEDIA

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Summary

Transfer of fluid between connected cracks may occur during the passage of seismic waves. Such fluid flow can be modelled using an extension of effective medium theory (Hudson et al. 1996) and is effected via non-compliant pores. The flow is governed by a parameter \( \tau \) representing the relaxation time of pressure equalization between cracks. However, if the cracks are fully aligned and have the same aspect ratio, the theory produces the unexpected result that, at low frequencies the cracks are effectively isolated and at high frequencies they are fully drained. The artificial restriction of the model to perfectly aligned cracks of identical aspect ratio is seen to be the cause of this result. By reworking the model to allow the crack orientation and aspect ratios to vary, we see that a more realistic model has the usual properties in which the cracks are isolated at high frequencies and undrained at low frequencies. Thomsen’s parameters (Thomsen 1986) and the attenuation coefficients are seen to be frequency dependent via the non-dimensional parameter \( \omega \tau \).

Introduction

Hudson et al. (1996) derived equations for the effective properties of material with aligned connected cracks by a method which ignores local crack-to-crack flow since it was assumed that, because the cracks all have the same orientation, it would be unimportant. However, if the formulae for non-aligned cracks are specialized to cracks with a single orientation, the result differs from the above in the addition of one term which becomes important at high frequencies. The incorporation of crack-to-crack flow shows that it cannot be neglected when the frequency is sufficiently high that the wavelength approaches the size of the inter-crack spacing. As well as depending on the assumption that the cracks are fully aligned, Hudson’s et al. (1996) results also rely on the fact that the cracks were assumed all to have the same aspect ratio. Relaxing either of these two assumptions leads to local fluid flow between neighbouring cracks which have been distorted in different ways by the incoming wave because of their different orientations, aspect ratios, or both. By analyzing the effect of small variations in alignment and aspect ratio we find that the behaviour of the material is that of undrained cracks at low frequencies, in accordance with physical expectation. We present numerical results demonstrating the behaviour of cracked solids at all frequencies when the cracks are nearly aligned and when they all have nearly the same aspect ratio.
Modelling the Fluid Flow

The model of connected cracks proposed by Hudson et al. (1996), for the transfer of fluid between cracks by non-compliant pores (seismically transparent pathways), see Figs. (1a) and (1b), makes the assumptions that the distortion of the pores is negligible compared with that of the cracks during the passage of a wave and that the pore porosity is low, so that we neglect compression of the fluid in the pores. The same set of governing equations is assumed as in Hudson et al. (1996), where the population of cracks is divided into families of parallel cracks with identical aspect ratio and identical radius, labelled by $n = 1, 2, \ldots$. A first order expression for the porosity, $\phi_n$, of the $n$th set of cracks, is then given by:

$$
\phi_n = \phi_n^0 + \phi_n^1 : (\sigma^0 + p_f^0 \mathbf{I}) - \frac{\phi_n^0 p_f^0}{\kappa},
$$

(1)

where $\kappa = \lambda + 2\mu/3$ is the bulk modulus of the (porous and isotropic) matrix material ($\lambda$ and $\mu$ the Lamé constants), $\phi_n^0$ and $\phi_n^1$ are the stress-free porosity and the first order dependence on stress, respectively, of the $n$th set of cracks; $\sigma^0$ is the imposed static stress field and $p_f^0$ the fluid pressure in the $n$th set of cracks. The flow is governed by a local expression:

$$
\frac{\partial}{\partial t} (\rho_f^0 \phi_n) = -\frac{\phi_n^0 \rho_0}{\kappa_f} (p_f^0 - p_f)
$$

(2)

and a global flow rate derived from the conservation of mass and D’Arcy’s law:

$$
\frac{\partial}{\partial t} \left( \sum_n \rho_f^0 \phi_n \right) = \nabla \cdot \left( \frac{\rho_f}{\eta_f} \mathbf{K}' \cdot \nabla p_f \right),
$$

(3)

where $\rho_f^0$ is the fluid density in the $n$th set of cracks, $\rho_0$ the unstressed density, $\kappa_f$ the bulk modulus of the fluid, $p_f$ the average (local) pressure in the fluid and $\tau$ a relaxation parameter. $\mathbf{K}'$ is the permeability tensor of the matrix (including cracks), $\rho_f$ is the average fluid density and $\eta_f$ the fluid viscosity. We gain, to first order in $p_f/\kappa_f$ and $\phi_n^1: \sigma^0 / \phi^0$, where $\phi^0$ is the average stress-free porosity of the cracks, an expression for the relative change in porosity:

$$
\frac{\phi_n - \phi_n^0}{\phi_n^0} = \frac{3}{4\pi \mu \alpha_n} N^0_0 \sigma_{ij},
$$

(4)
thus defining \( \{N_{ij}\} \), the crack opening parameters for the \( n \)th set of cracks, where \( \alpha_n \) is the aspect ratio of the \( n \)th set of cracks. These parameters may be substituted directly into an expression given by Hudson et al. (1996) to determine the first order perturbation to the elastic stiffnesses of the background matrix material, resulting from the assumed fluid flow mechanism. Although the theory is derived from a discrete perspective, the continuous limit is henceforth assumed by introducing distribution functions for the aspect ratio and orientation of the cracks, leading to an expression for the elastic stiffnesses of the form:

\[
c_{ipjq} = c_{ipjq}^0 - \frac{\varepsilon}{\mu} c_{kp}^0 c_{usjq}^0 \hat{T}_{kr}^{krus} + \mathcal{O}(\varepsilon^2),
\]

where \( \varepsilon \) is the crack density, and the exact form of \( \hat{T}_{krus} \) depends upon the distribution functions and the non-dimensional parameters \( \omega \tau \), \( P^m \) and \( P^k \), where

\[
\begin{align*}
P^m &= \frac{\eta f}{\mu \sigma_0 \tau} \quad \\
P^k &= \frac{3\kappa f_k p_k K_{pq} q_{pq}}{4\pi \varepsilon \alpha_0 \sigma_0 v^2 \tau \eta f}
\end{align*}
\]

\( \alpha_0 \) is the average aspect ratio, \( v \) a zeroth order approximation to the wavespeed and \( \hat{k} \) the (normalized) wavenumber vector.

**Results**

The effects upon the anisotropy of the resulting effective medium are considered separately for the cases in which only one of the crack aspect ratio or orientation is allowed to vary, while the other variable remains fixed. For a model with variable aspect ratio, we choose a distribution function that closely matches the experimental results presented by Hay et al. (1988), and consider the change in Thomsen’s parameters and the attenuation coefficients with non-dimensional frequency, for a variety of values of \( P^m \) and \( P^k \). We predict a substantial variation in Thomsen’s parameters with frequency; a typical example of the \( P \)-wave anisotropy is shown in Fig. (2). A second parameter is seen to affect the frequency range within which the anisotropy parameter makes a rapid transition between maximum and minimum values. These transition regions correspond to peaks in the related attenuation coefficients, and these would be verified in laboratory experiments.

**Conclusions**

The model proposed by Hudson et al. (1996) for the transfer of fluid between connected cracks via non-compliant pores has been extended to allow for a continuous distribution of values of both crack orientation and aspect ratio. This more realistic model has the expected properties that at high frequencies the cracks behave as if isolated, while at low frequencies they behave as if undrained; in addition, the relationship between the expressions for undrained and isolated conditions agrees with Brown and Korringle (1975). In the fully aligned limit they behave as if isolated at both high and low frequencies. We examine separately the cases when the aspect ratio and crack orientation vary, while keeping the other fixed, and study the frequency dependence of Thomsen’s parameters and the attenuation coefficients. We believe that it is possible to both identify, and differentiate between, the effects of the crack density and the variation of either the aspect ratio or orientation. Critically however, there is a dependence upon the undetermined parameter \( \tau \) corresponding to the relaxation time of pressure equalization between cracks. Estimates of this parameter have
Figure 2: Thomsen’s parameter $\varepsilon_T$ as a function of non-dimensional frequency $\omega_T$, for $P^k = 10$ (solid line), $10^2$ (long dashes), $10^3$ (medium dashes), $10^4$ (short dashes), $10^5$ (long dash - dot), $10^6$ (short dash - dot) and $10^7$ (double dashes).

been made (Hudson et al. 1996; O’Connell and Budiansky 1977), suggesting that the range of frequencies where a rapid change in the anisotropy parameters is predicted, lies within the seismic range.

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References