Abstract
Recently, from the analysis of a multi-component VSP for frequency-dependent anisotropy, we find that shear-wave anisotropy as inferred from measured time delays between split shear-waves decreases systematically as frequency increases. While in general the polarizations of the fast split shear-waves remain almost constant for deep receivers, we notice systematic rotation (up to 20°) of polarizations with frequency for shallow receivers. The variation of time-delays with frequency has been successfully interpreted and modelled using a multi-scale rock physics model incorporating aligned fractures in a porous medium with randomly distributed micro-cracks. However this model fails to explain the frequency-dependent shear-wave polarizations. It is speculated that if aligned micro-cracks and aligned fractures are in different directions (i.e. the conjugate angle is not zero), low frequency would indicate the direction of fractures and high frequency would give the orientation of micro-cracks. In this study, we present some theoretical results of shear-wave propagation in media with one set of aligned micro-cracks and one set of aligned fractures, demonstrating that the shear-wave polarizations will change with frequency and the variation depends on the azimuths and angle of incidence relative to the orientations and crack densities of individual sets.

Introduction
Recently, strong observational evidence suggests that the measured seismic anisotropy as inferred from time delays of split shear-waves does actually depend on frequency (Liu et al. 2003). In our analysis of multi-component shear-wave VSPs for frequency-dependent anisotropy, we notice systematic rotation (about 20°) of shear-wave polarizations with frequency for shallow receivers (Figure 1). Changes in shear-wave polarizations have been reported by Fouch and Fischer (1998) and Liu et al. (2001), who interpreted these changes as different frequency responses of shear-waves to different alignments with different scales in the rockmass. If two scales are present, say, micro-cracks are aligned in different directions from aligned fractures, low frequency would expect to give orientation of fractures and high frequency would give direction of micro-cracks. However such a model is not yet available. The nearest analogue is the combination of two micro-crack sets, which has been studied by Liu et al. (1992). It was found that the polarization of the fast split shear-wave gives the crack-density weighted average direction between the conjugate sets. In the present study, we extend the model of Chapman (2002) to incorporate both aligned micro-cracks and aligned fractures, and this is done by adding...
additional elastic compliances due to aligned micro-cracks and fractures, respectively, using the technique of Schoenberg and Muir (1989). Numerical results show that when aligned micro-cracks and aligned fractures are in different directions, shear-wave polarization will change with frequency, and this variation, which is obvious for non-normal incidence, depends primarily on the relative orientations and densities of micro-cracks and fractures.

Modelling multi-scale aligned fractures
In this Section, we summarize the model developed by Chapman (2003) to model fractures in porous rocks with isotropic distribution of random micro-cracks. The effective elastic stiffness tensor $C$ is written as

$$C = C_0 + \Delta C = C_0 + \phi C_1 + e_c C_2 + e_f C_3,$$  

where $\phi$ is porosity of the matrix, $e_c$ is the density of micro-cracks, and $e_f$ is the density of fractures. $C_0$ is the elastic stiffness of the matrix (solid frame), $C_1$, $C_2$ and $C_3$ are the contributions of pores, cracks and fractures, respectively, and the explicit expressions are given in the Appendix of Chapman et al. (2003). $\Delta C = \phi C_1 + e_c C_2 + e_f C_3$ is additional stiffness due to pores, cracks and fractures. The size-dependent terms of elastic stiffness are linked by the relationship between two relaxation time scales, $\tau_m$ and $\tau_f$, i.e. $\tau_f = (a_f / a_m) \tau_m$, where $\tau_m$ is related to the standard micro-structural squirt flow and $\tau_f$ is related to the macro-scale flow due to the presence of fractures. $a_f$ is the fracture radius, and $a_m$ is the grain size (the size of pores and micro-cracks). Note that $a_f$ can be much larger than $a_m$. The frequency-dependent terms are governed by two terms: $(1 + i \omega \tau_f)^{-1}$, which is related to fluid flow into and out of fractures, and $(1 + i \omega \gamma \tau_{m})^{-1}$, which is related to pore-scale flow ($\gamma$ is related to the Poisson’s ratio of the matrix and the fluid compressibility). In the absence of fractures the model returns to the grain-scale squirt flow model. With the introduction of a fracture set two characteristic frequencies exist: the traditional squirt flow frequency which can be estimated from laboratory data, together with a lower characteristic frequency which depends on the size of the fractures. A consequence of this is that propagation at seismic frequencies can be very different from that predicted in the low frequency limits implying that dispersion can occur at seismic frequency, or in other words seismic frequency can no longer be safely regarded as the low frequency limit.

Following Schoenberg and Muir (1989), when there exist two sets of aligned micro-cracks and fractures, the effective fracture compliance can be simply written as the sum of effective elastic compliances of individual cracks and fractures, i.e.

$$C = S^{-1} = (S^0 + \Delta S^1 + \Delta S^2)^{-1},$$

where effective compliance $S$ is reciprocal of effective stiffness $C$. $\Delta S^i$ is additional compliance due to $i$th set of cracks (or fractures), which can be calculated using Chapman’s (2003) model.

Results
We consider various combinations of aligned cracks and fractures with different crack densities. Figure 2 shows the equal-area projection of the three-dimensional pattern of the behaviour of shear-waves propagating through rock with two fracture sets with crack densities of $e_c=0.02$ and $e_f=0.02$ striking at east (Y-direction, north is in X-direction) and N60°E, respectively. The cracks and fractures are aligned vertically. We consider two different frequencies, 10Hz and 100Hz. Figure 3 shows the similar plots to Figure 2, except that the fracture size is increased to 5m. From Figure 2, we can see that when two micro-crack sets are combined, shear-wave polarizations do not vary with frequency, and the mean polarization for near-normal incidence (inner circle in the equal-area projection) is exactly 30° between two crack sets. In general, if the crack densities of the two sets are not equal, the mean polarization will be crack density-weighted average, which have been systematically studied by Liu et al. (1993). However, when we increase the size of one set of fractures (Figure 3), frequency-dependent polarizations can be clearly observed, particularly, for non-normal incidence. Comparing Figures 2 and 3, we may immediately conclude that there is a strong dependence of shear-wave polarizations with frequency and fracture sizes.
Figure 2. Equal-area projections out to 90° of the horizontal polarizations of the faster split shear-waves. The crack density \(e_c\) and fracture density \(e_f\) are both equal to 0.02, and micro-cracks and fractures have the same size (1mm). The conjugate angle is 60°.

Figure 3. Equal-area projections out to 90° of the horizontal polarizations of the faster split shear-waves. The crack density \(e_c\) and fracture density \(e_f\) are both equal to 0.02, and the radius of micro-cracks is 1mm and the size of fractures is 5m. The conjugate angle is 60°.

Figure 4 shows the similar plots to Figure 3, except the fractures are rotated 20° from vertical to the south (-X direction) direction. Shear-wave polarizations no longer have a perfect alignment within the shear-wave window (inner circle marked incidence angle of 35° in Figures 2, 3 and 4). For low frequency (10Hz), the polarizations are in general aligned in the direction of 30° (or crack-density weighted average direction), which is expected. Whereas for high frequency (100Hz), a complicated pattern can be seen, which is characterized by a systematical rotation of shear-wave polarizations with azimuths and also with incident angles. Interestingly, in some directions, we may recover the fracture orientation, and in other directions, we can identify the crack orientations, which depends primarily on the angle of incidence and azimuths relative the fracture and crack orientations.

Summary
We have presented theoretical results demonstrating the dependence of shear-wave polarizations on frequency using the model of Chapman (2003), which have been extended to incorporate aligned
micro-cracks and fractures. If the sizes of both micro-cracks and fractures are small, by definition, the effective crack density is the sum of the crack densities of the individual crack densities and the effective mean shear-wave polarization is the crack-density-weighted mean of the individual crack orientation. When the fracture size is significantly larger than the size of micro-cracks, the shear-wave polarization will depend on frequency and also on the angle of incidence and azimuth relative to the crack orientations of individual sets. Our results can explain observed variation of shear-wave anisotropy with frequency seen in Figure 1 (from analysis of a multi-component shear-wave VSP). Our study may provide a mean to use anisotropy measurements to estimate the fracture sizes and orientations of aligned micro-cracks and aligned fractures, which are ultimately needed in reservoir simulation. The speculation of low frequency indicating direction of fractures and high frequency giving orientation of micro-cracks should be used with care as the variation of shear-wave polarizations with frequency also depends on azimuths and angle of incidence relative to the orientations and crack densities of micro-cracks and fractures.

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References