WAVE DECOMPOSITION FOR POROELASTIC MEDIA AND ALGORITHM FOR REFLECTION COEFFICIENTS

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INTRODUCTION

Our aim is to study wave propagation in fluid saturated poroelastic media, properly taking into account attenuation and the possible effects of the Biot wave and squirt flow which appear in such media. In this abstract we present the theory for an up-going/down-going wave decomposition and use it to derive reflection coefficients at the interface between two poroviscoelastic media in a block-matrix form suitable for use in existing software programs.

METHOD

Following Biot and Stoll we write two dynamic equations of motion and five constitutive equations for the coupled system of fluid/solid porous frame in terms of displacement of solid frame and displacement of fluid relative to the frame. We then rederive the dispersion equation by assuming that the fluid follows the solid frame with a linear proportional displacement. This gives us all the possible modes of propagation: four up-going and four down-going waves. We have written for all these modes the explicit polarisations.

In order to solve for the reflection coefficients, we follow Bourbie(1988) and introduced an eighth equation for the gradient of pressure at an interface between two fluid saturated porous media. We can write then eight equations of continuity in which we introduced the explicit polarisations. We have written an algorithm to solve for the reflection coefficients; it is written in a condensed matrix form which is suitable and efficient for numerical computations.

REVIEW OF BASIC EQUATIONS OF POROELASTICITY

Each media is characterised by its total density $\rho$, its fluid density $\rho_f$, its porosity $\phi$, an equivalent density $m = \rho_f c/\phi$ and a set of parameters introduced by Biot: $H,C$ and $M$. These Biot parameters are related to $K_s$, the bulk modulus of the free-draining porous frame, $K_f$ the bulk modulus of the solid material composing the porous frame and $K_f$ the bulk modulus of the fluid by the following relations:

$$H = \frac{(K_s - K_f)^2}{D - K_s} + K_s + 4\mu/3 \quad C = \frac{K_s(K_s - K_f)}{D - K_s} \quad M = \frac{K_s^2}{D - K_s}$$

where $D = K_s(1 + \phi(K_s/K_f - 1))$.

Considering plane waves, we can define the solid displacement as $\mathbf{u} : u_i = a_i \exp(-i\omega q(x + pz)) \exp(\omega t)$ where $q$ is the horizontal slowness of the wave and $p$ is the cotangent of the angle of the wave direction with the vertical (we will use $p$ for the P-wave, $b$ for the Biot wave, $s$ for the $S_V$ wave and $x$ for the $S_H$ wave. Stating that the fluid follows the solid frame with a proportional displacement $\mathbf{U} = \mathbf{U} \mathbf{u}$ we can write the dispersion equations by introducing the square of the slowness $X = q^2(1 + p^2)$ and $R = \phi(1 - U)$. The equations for a compressional wave and for a shear wave are then respectively:

$$\begin{bmatrix} -HX + \rho & CX - \rho_f \\ -CX + \rho_f & MX - m + \eta_0/\omega \end{bmatrix} \begin{bmatrix} 1 \\ R \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and

$$\begin{bmatrix} -\mu X + \rho & -\rho_f \\ \rho_f & -m + \eta_0/\omega \end{bmatrix} \begin{bmatrix} 1 \\ R \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For the right hand-side to be null, the matrix on the left-hand side must be singular and its determinant is null. For a compressional wave this gives us a quadratic equation for $X$ and therefore we have two values for the possible slownesses, the higher value of slowness corresponding to the wave of the second kind predicted by Biot. For a shear wave the equation on the right gives us only one solution in $X$. 
WAVE DECOMPOSITION

We decompose the propagating wave in the porous medium in four up-going waves and four downgoing waves: compressional wave (P), Biot wave (B), S(0) wave (S) and S(H) wave (Z). The polarisations of these waves are deduced from the constitutive equations.

<table>
<thead>
<tr>
<th>Type</th>
<th>$P \downarrow$</th>
<th>$P \uparrow$</th>
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<tbody>
<tr>
<td>Displacement</td>
<td>$\begin{pmatrix} 1 \ 0 \ p \end{pmatrix}$</td>
<td>$\begin{pmatrix} 0 \ 1 \end{pmatrix}$</td>
</tr>
<tr>
<td>Traction/($-i\omega q$)</td>
<td>$\begin{pmatrix} 2\mu p \ 0 \ (1 + p^2)(H - R_P C) - 2\mu \end{pmatrix}$</td>
<td>$\begin{pmatrix} 2\mu p \ 0 \ -(1 + p^2)(H - R_P C) + 2\mu \end{pmatrix}$</td>
</tr>
<tr>
<td>Filtration velocity</td>
<td>$R_P p$</td>
<td>$R_P p$</td>
</tr>
<tr>
<td>Fluid pressure</td>
<td>$(-i\omega q)(MR_P - C)(1 + p^2)$</td>
<td>$(-i\omega q)(MR_P - C)(1 + p^2)$</td>
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<tr>
<th>Type</th>
<th>$B \downarrow$</th>
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<tbody>
<tr>
<td>Displacement</td>
<td>$\begin{pmatrix} 1 \ 0 \ b \end{pmatrix}$</td>
<td>$\begin{pmatrix} 0 \ 1 \end{pmatrix}$</td>
</tr>
<tr>
<td>Traction/($-i\omega q$)</td>
<td>$\begin{pmatrix} 2\mu b \ 0 \ (1 + b^2)(H - R_B C) - 2\mu \end{pmatrix}$</td>
<td>$\begin{pmatrix} 2\mu b \ 0 \ -(1 + b^2)(H - R_B C) + 2\mu \end{pmatrix}$</td>
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<tr>
<th>Type</th>
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</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>$\begin{pmatrix} -s \ 0 \ 1 \end{pmatrix}$</td>
<td>$\begin{pmatrix} s \ 0 \end{pmatrix}$</td>
</tr>
<tr>
<td>Traction/($-i\omega q$)</td>
<td>$\begin{pmatrix} \mu(1 - s^2) \ 0 \ 2\mu s \end{pmatrix}$</td>
<td>$\begin{pmatrix} \mu(1 - s^2) \ 0 \ -2\mu s \end{pmatrix}$</td>
</tr>
<tr>
<td>Filtration velocity</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fluid pressure</td>
<td>0</td>
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<td>$\begin{pmatrix} 0 \ 1 \end{pmatrix}$</td>
</tr>
<tr>
<td>Traction/($-i\omega q$)</td>
<td>$\begin{pmatrix} \mu z \ 0 \end{pmatrix}$</td>
<td>$\begin{pmatrix} -\mu z \ 0 \end{pmatrix}$</td>
</tr>
<tr>
<td>Filtration velocity</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fluid pressure</td>
<td>0</td>
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REFLECTION MATRIX

We obtain it by writing the boundary conditions at the interface between two poroelastic media—continuity of displacement, traction, filtration velocity and fluid pressure. However, if the fluid cannot freely cross the interface we have a discontinuity of the fluid pressure $P_f$ proportional to the gradient of filtration velocity $\xi$ by a factor $1/\kappa_s$ and the last boundary condition must be rewritten:

$$
\sum_{\alpha=1}^{4} P_f^{(\alpha)} = \sum_{\beta=1}^{4} \left( P_f^{(\beta)} - \frac{\xi^{(\beta)}}{\kappa_s} \right) \text{ or } \sum_{\alpha=1}^{4} \left( P_f^{(\alpha)} - \frac{\xi^{(\alpha)}}{2\kappa_s} \right) = \sum_{\beta=1}^{4} \left( P_f^{(\beta)} - \frac{\xi^{(\beta)}}{2\kappa_s} \right)
$$

where the index $\alpha = 1..4$ refers respectively to the P, Biot, $S_v$ and $S_h$ wave in the first medium and $\beta = 1..4$ refers to them in the second medium.

The reflection matrix is then solution of the system:

$$
\begin{pmatrix}
P_1 & -P_2 \\
Q_1^+ & Q_2^+ \end{pmatrix} \begin{pmatrix}
R_{du} & T_{du} \\
R_{ud} & R_{ud} \end{pmatrix} = \begin{pmatrix}
P_1 & P_2 \\
Q_1^+ & Q_2^+ \end{pmatrix}
$$

with $P = \begin{pmatrix}
p & b & 0 & 0 \\
m & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \end{pmatrix}$

and $Q^+ = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$

which can be written in the following form similar to the elastic case (Graebner):

$$
\begin{align*}
R_{du} &= P_1^{-1}(Q_1^+ P_1^{-1} + Q_2^{-1} P_2^{-1})^{-1}(Q_1^+ P_1^{-1} - Q_2^{-1} P_2^{-1})P_1 \\
T_{du} &= 2P_1^{-1}(Q_1^+ P_1^{-1} + Q_2^{-1} P_2^{-1})^{-1}P_1 \\
R_{ud} &= P_2^{-1}(Q_1^+ P_1^{-1} + Q_2^{-1} P_2^{-1})^{-1}(Q_2^+ P_2^{-1} - Q_1^+ P_1^{-1})P_2 \\
T_{ud} &= 2P_2^{-1}(Q_1^+ P_1^{-1} + Q_2^{-1} P_2^{-1})^{-1}P_2
\end{align*}
$$

or in terms of $Q$, the matrix when the fluid can freely cross the interface ($1/\kappa_s = 0$):

$$
\begin{align*}
R_{du} &= P_1^{-1}(Q_1 P_1^{-1} + Q_2 P_2^{-1})^{-1}(Q_1 P_1^{-1} - Q_2 P_2^{-1})P_1 \\
T_{du} &= 2P_1^{-1}(Q_1 P_1^{-1} + Q_2 P_2^{-1})^{-1}P_1 \\
R_{ud} &= P_2^{-1}(Q_1 P_1^{-1} + Q_2 P_2^{-1})^{-1}(Q_2 P_2^{-1} - Q_1 P_1^{-1})P_2 \\
T_{ud} &= 2P_2^{-1}(Q_1 P_1^{-1} + Q_2 P_2^{-1})^{-1}P_2
\end{align*}
$$

with $\Delta = \frac{1}{\kappa_s} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$

CONCLUSION

The study of reflection-transmission-conversion (RTC) coefficients in fluid-saturated porous media is of utmost importance for the characterisation of hydrocarbon reservoirs. We have considered here the propagation of seismic waves in fluid-saturated poro-viscoelastic media taking properly into account attenuation and possible effects of the fluid-solid interaction (e.g., Biot-wave) which appears in such media.

We have derived Biot equations in poroelastic media and added Bourbie’s equation for pressure discontinuity at a porous/porous interface. We have first obtained an explicit wave decomposition in up and down-going waves and then developed an algorithm to compute RTC coefficients of plane waves at all frequencies and all angles of incidence. It encompasses both viscoelastic and poroelastic cases and considers a variable surface permeability at the interface.

Using numerical examples (some of them shown next page) we have shown that our algorithm is consistent with previously published results for asymptotic cases. Now that the theory has been developed and algorithms designed in a matrix-form suitable for numerical computations, a range of possible applications and extensions exist such as the study of multilayered porous media and the extraction of petrophysical parameters from seismic reflection data.
ACKNOWLEDGMENTS

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REFERENCES

1) Bourbié T., Coussy O. and Zinszner B. Acoustique des milieux poreux. Editions Technip 1986

NUMERICAL RESULTS

Angle dependence of some reflection coefficients for poroelastic media at low frequency: