Abstract

In this study, we investigate the theoretical azimuthal variations of PP and PS AVO attributes in anisotropic media. In addition to three conventional P-wave AVO attributes (intercept $A$, gradient $B$ and curvature $C$), we introduce two PS AVO attributes $\hat{A}$ and $\hat{B}$ (which control near and far offset response of PS-waves, respectively). Previous studies of these parameters in isotropic media have suggested that they and their cross-plots are good fluid and lithology indicators. However, there is no equivalent study of these AVO attributes for anisotropic media, except the simple, but inaccurate analytic Rüger’s equation for PP-waves. Here we perform a systematic study of these AVO attributes in anisotropic media to examine their azimuthal dependence and we find that all these attributes ($B$ and $C$ for PP-waves; and $\hat{A}$ and $\hat{B}$ for PS-waves) vary approximately with $\cos(2\phi)$ ($\phi$ is azimuth angle). Finally, we propose a technique to extract these azimuth-dependent AVO attributes from seismic data. The main contribution from this paper is to draw attention to the azimuthal variations of these AVO attributes and their potential applications for characterizing fractured reservoirs.

Introduction

The common problem in AVO analysis is its ambiguous nature as many phenomena lead to similar AVO anomalies. For hydrocarbon exploration, one specific application of the AVO analysis is to predict lithology and to discriminate fluid effects. Despite that there have been many successful examples of applying PP-wave AVO analysis, the above problem remains and the ambiguity cannot be easily resolved with short-offset PP-waves only. As new types of data (long offset 3D P-wave data and PS-wave data from OBC) are now available, it has recently been suggested that combined analysis of PP and PS AVO should provide a much better way of predicting lithology and fluid discrimination. In this paper, we extend the work of Hansen et al. (2000) to anisotropic media and suggest that combined azimuthal analysis of the PP and PS AVO attributes should aid interpretations in fracture characterization. This is achieved through a systematic study of these AVO attributes in anisotropic media to examine their azimuthal dependence.

AVO attributes of PP- and PS-waves

In contrast to isotropic wave propagation, analytic expressions for PP and PS reflection coefficients are not generally available for anisotropic media except for VTI media and in symmetry planes of HTI media (e.g. Rüger 1998). Following Hansen et al. (2000), instead of trying to derive equations similar to those of Shuey (isotropic) or Rüger (anisotropic), which assume small offset (incident angles) and weak contrasts in elastic properties across a single
interface, we obtain the following approximate equations using the Taylor series expansion around \(\sin(\theta)\) (\(\theta\) is the P-wave incident angle). For PP-waves, we have

\[
R_{\sigma}(\theta, \phi) = A + B(\theta, \phi)\sin'(\theta) + C(\theta, \phi)\sin'(\theta),
\]

and for PS-waves, we have

\[
R_{\sigma}(\theta, \phi) = \hat{A}(\theta, \phi)\sin(\theta) + \hat{B}(\theta, \phi)\sin'(\theta).
\]

Coefficients \(A, B\) and \(C\) are PP-wave AVO attributes. As in isotropic media, \(A\) is called AVO intercept and does not depend on the incident angle \(\theta\) and azimuth angle \(\phi\), \(B\) is AVO gradient and \(C\) is AVO curvature. Similarly, \(\hat{A}\) and \(\hat{B}\) are PS-wave AVO attributes with \(\hat{A}\) controlling near to middle offset response and \(\hat{B}\) dominating the far offset response. These coefficients can be obtained through non-linear fits to the exact solutions (using numerical calculations). As a result, their validity is not restricted to small offsets (incident angles) or weak contrasts in elastic properties. Note that for small incident angles (near to middle offsets), the third term in equation (1) can be ignored as in most standard AVO processing (in which case Shuey and Rüger’s approximations are recovered). The second term in equation (2) for PS-waves may also be ignored for small incident angles.

In VTI media, all these parameters \((B, C, \hat{A}, \hat{B})\) are invariant with azimuthal angle \(\phi\). However, in HTI media, e.g. a medium with one set of vertically aligned fractures, Rüger (1998) shows that the PP-wave AVO gradient \(B\) varies approximately with \(\cos(2\phi)\). Here, we consider two typical shale/sand interface models (Classes I and III, with elastic parameters taken from Sayers and Rickett, 1997). Vertical fractures (filled with gas) are inserted in the sand formation using the equivalent medium theory of Liu et al. (2000) with the fracture normal at azimuth \(\phi=0\), and the fracture density of 10\%. We fit the variations of AVO attributes \((B, C, \hat{A}, \hat{B})\) for incident angles up to 40\(^\circ\) (but as we have said, it is not restricted to small incident angles). Figures 1 and 3 show the variations of PP-wave AVO attributes \(B\) and \(C\) with azimuths for the two typical AVO models. Similarly, Figures 2 and 4 shows the variations of PS-wave attributes \(\hat{A}\) and \(\hat{B}\) with azimuths. From these figures, we observe that all these parameters \((B, C, \hat{A}, \hat{B})\) follow a \(\cos(2\phi)\) variation. We also find that their variations are at the same order of magnitude for both classes of AVO models and the magnitudes are related to the fracture density, i.e. the higher the fracture density the larger the azimuthal variations will be. It is interesting to note that fracture normal directions are coincident with troughs or peaks in the variations of \(B, C, \hat{A}, \hat{B}\), and whether it is associated with troughs or peaks will depend on the AVO type. Note that the variations of \(B\) and \(C\) in Figures 1 and 3 are opposite in sign; and similarly \(\hat{A}\) and \(\hat{B}\) also have opposite troughs and peaks (Figures 2 and 4). This is certainly good news in terms of the simplicity in variations in these attributes. Since we may potentially extract these parameters from seismic data (see below) and plot their variations with azimuths, fracture orientation and relative density information may thus be obtained.

### A strategy to extract AVO attributes \(A, B, C, \hat{A}\) and \(\hat{B}\) from seismic data

Here we follow closely the work of Hansen et al. (2000). A simple approach to obtain \(A, B,\) and \(C\) from PP-wave data is to use the reflection coefficients obtained at three angles \((\theta_1, \theta_2, \theta_3)\) for a given azimuth \(\phi\) and set a linear system of three equations. This yields:

<table>
<thead>
<tr>
<th>Model 1 (Class I)</th>
<th>(\alpha) (km/s)</th>
<th>(\beta) (km/s)</th>
<th>(\rho) (g/cm(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shale</td>
<td>3.30</td>
<td>1.70</td>
<td>2.35</td>
</tr>
<tr>
<td>Sand</td>
<td>4.20</td>
<td>2.70</td>
<td>2.49</td>
</tr>
<tr>
<td>Model 2 (Class III)</td>
<td>Shale</td>
<td>2.73</td>
<td>1.24</td>
</tr>
<tr>
<td>Sand</td>
<td>2.02</td>
<td>1.23</td>
<td>2.13</td>
</tr>
</tbody>
</table>
\[ R_{pp}(\theta, \phi) = A + B(\theta, \phi) \sin^2(\theta) + C(\theta, \phi) \sin(\theta), \]
\[ R_{np}(\theta, \phi) = A + B(\theta, \phi) \sin(\theta) + C(\theta, \phi) \sin^2(\theta), \]
\[ R_{pp}(\theta, \phi) = A + B(\theta, \phi) \sin^2(\theta) + C(\theta, \phi) \sin(\theta). \]  

(3)

To obtain \( \hat{A} \) and \( \hat{B} \), we likewise have a system of two equations
\[ R_{pp}(\theta, \phi) = \hat{A}(\theta, \phi) \sin(\theta) + \hat{B}(\theta, \phi) \sin^2(\theta), \]
\[ R_{np}(\theta, \phi) = \hat{A}(\theta, \phi) \sin(\theta) + \hat{B}(\theta, \phi) \sin(\theta). \]  

(4)

Solving equations (3) and (4), \( A, B, C, \hat{A} \) and \( \hat{B} \) are obtained
\[
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix} = \begin{bmatrix}
1 & \sin^2 \theta_1 & \sin^2 \theta_2 \\
1 & \sin^2 \theta_1 & \sin^2 \theta_2 \\
1 & \sin \theta_1 & \sin \theta_2
\end{bmatrix}^{-1} \begin{bmatrix}
R_{pp}(\theta_1) \\
R_{np}(\theta_1) \\
R_{pp}(\theta_2)
\end{bmatrix},
\]
and
\[
\begin{bmatrix}
\hat{A} \\
\hat{B}
\end{bmatrix} = \begin{bmatrix}
\sin \theta_1 & \sin^2 \theta_1 \\
\sin \theta_2 & \sin^2 \theta_2
\end{bmatrix}^{-1} \begin{bmatrix}
R_{pp}(\theta_1) \\
R_{np}(\theta_1) \\
R_{pp}(\theta_2)
\end{bmatrix}.
\]

(5)

This is repeated for all azimuths. The method is called weighted common azimuth angle stack (the matrices in front of equations 5 and 6 are angle weights). Alternatively, \( A, B, C, \hat{A} \) and \( \hat{B} \) can be obtained using the angle stack method described by Hendrickson (1999), i.e. dividing PP data into three small angle ranges (near, middle and far offsets) to obtain \( A, B \) and \( C \), and PS data into two small angle ranges (near and far) to obtain \( \hat{A} \) and \( \hat{B} \). Once AVO attributes \( A, B, C, \hat{A} \) and \( \hat{B} \) are extracted from seismic data, we may use their azimuthal variations to extract fracture information. We can also apply the standard methods to manipulate them, e.g. cross-plots of \( A-B, \ A-C, \ A-\hat{B}, \) etc. as Hansen et al. (2000) and others have demonstrated that the cross-plots of these parameters in isotropic media are indicative of fluids and lithology (through \( Vp/Vs \) ratio).

Summary

We have systematically investigated the azimuthal variations of PP and PS AVO attributes in fractured media. We suggest to combine conventional P-wave AVO attributes (intercept \( A \), gradient \( B \) and curvature \( C \)) with two newly introduced AVO attributes \( \hat{A} \) and \( \hat{B} \) (which control near and far offset response of PS-waves, respectively). We demonstrate through a systematic study of these AVO attributes that all these attributes (\( B \) and \( C \) for PP-waves; and \( \hat{A} \) and \( \hat{B} \) for PS-waves) vary approximately with a \( \cos(2\phi) \) function (\( \phi \) is azimuth). The simplicity in the azimuthal dependence of these attributes in fractured media implies that fracture orientation and relative density information may thus be obtained if we can extract these parameters from seismic data using the strategy suggested in this paper.

Acknowledgements

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References
Hansen, T.M., Foster, D. and Hornby, B., 2000, AVO attributes of compressional and converted shear-waves, presented at the SEG spring symposium 2000, Houston, TX.

Figure 1. Variation of PP-wave AVO attributes (gradient $B$ and curvature $C$) with azimuth for Model 1 (class I AVO).

Figure 2. Variation of PS-wave AVO attributes ($\hat{A}$ and $\hat{B}$) with azimuth for model 1 (class I AVO).

Figure 3. Variation of PP-wave AVO attributes (gradient $B$ and curvature $C$) with azimuth for Model 2 (class III AVO).

Figure 4. Variation of PS-wave AVO attributes ($\hat{A}$ and $\hat{B}$) with azimuth for Model 2 (class III AVO).