Abstract

In this study, we examine the accuracies of three approximations for the elastic impedance through comparisons of PP-wave reflection coefficients for three typical classes of AVO models. Three approximations for the PP-wave elastic impedance are respectively derived by Connolly (1999), Ma (2004) and Santos and Tygel (2004), in which the later two were developed with an attempt to improve the accuracy of Connolly’s approximation. However, we find that for most practical applications, i.e. incident angles less than 40°, Connolly’s equation is still the most accurate equation of all three approximations for Classes I and II AVO models, though the other two approximations pick accurately the critical angles for large incident angles, and for Classes III AVO both Ma’s and Santos and Tygel’s equations are accurate for incident angles up to at least 70°, whereas Connolly’s equation is only accurate for incident angles up to 20°. This study provides a useful guide to the choice of equations for the PP-wave elastic impedance.

Introduction

The concept of elastic impedance was introduced by Connolly in his award winning paper published in The Leading Edge in 1999 (Connolly, 1999). Since then there have been a increasing interest in this idea as it has been proven to be a useful parameter for fluid and lithology characterization. Recently, there have been several approximations aiming at improving the accuracy of the Connolly’s equation for the elastic impedance, notably the ray-path approximation of Ma (2004); Ma and Morozov (2004) and the impedance-type approximation of Santos and Tygel (2004). In this study, we shall compare the accuracy of these three approximations using models that are representative of three classes of AVO.

Approximations of elastic impedance

Inspired by the simple expression for the normal-incidence reflection coefficient across a single interface, which is the ratio of the difference and summation of acoustic impedance of lower and upper media, Connolly (1999) suggests an alternative expression for the PP reflection coefficients at oblique incidence in the following form

\[ R_{PP} (\theta) = \frac{EI_2 - EI_1}{EI_2 + EI_1}. \]

(1)

EI is termed elastic impedance and is the function of both P- and S-wave velocities as well density. The subscripts 1 and 2 denote the upper and lower media, respectively. The original expression of EI given by Connolly (1999) is denoted \( EIP \), i.e.
EI^\alpha = \alpha \cdot \sin^2(\theta) \cdot \beta \cdot \sin^2(\theta) \cdot \rho \cdot \sin^2(\theta),  \quad (2)

where \(\alpha\) is P-wave velocity, \(\beta\) is S-wave velocity, \(\rho\) is density, and \(\theta\) is the P-wave incident angle. In deriving the above equation, it is assumed that \(K\), the average of squared S- and P-wave velocity ratio of two media, is constant, i.e.

\[
K = \frac{1}{2} \left[ \left( \frac{\beta}{\alpha} \right) + \left( \frac{\beta}{\alpha} \right)^2 \right] = \text{constant}.
\]

Since the publication of equation (2) by Connolly (1999), there have been several alternative equations for the elastic impedance. Ma (2004), Ma and Morozov (2004) extend Connolly’s equation based on the ray-path reflectivity to obtain the following equation (denoted by \(EI^m\)),

\[
EI^m = \frac{\rho \alpha}{\cos(\theta)} \left[ 1 - \left( \frac{\beta}{\alpha} \right)^2 \sin^2(\theta) \right]^{\frac{1}{2}}.  \quad (3)
\]

Santos and Tygel (2004) propose the so-called reflection coefficients based on the impedance-type approximation, and their equation is given below (denoted by \(EI^n\)):

\[
EI^n = \frac{\rho \alpha}{\cos(\theta)} \exp \left[ -2 \left( 1 + \gamma \right) \left( \frac{\beta}{\alpha} \right) \sin^2(\theta) \right],  \quad (4)
\]

where \(\gamma\) is assumed to be a constant across the interface and is given by

\[
\gamma = \ln(\rho_x / \rho) / \ln(\beta / \beta_x) = \text{constant}.
\]

Note that the normalization factors in equations (2), (3) and (4) have been ignored. Also note that equations (3) and (4) were originally written as a function of ray-parameter \(p\) by their corresponding authors, who have also discussed the advantages of using ray parameters. Here we have re-written the elastic impedance in terms of P-wave incident angles for the consistent comparison with Connolly’s original equation using \(p = \sin(\theta) / \alpha\).

All the three approximations have the same assumption, i.e. they are valid for small difference in P-, S-wave velocities and density across the interface. The authors of above equations have, in their respective studies, discussed alternative simplifications of these equations under other conditions, which we shall not repeat here.

**Comparison of accuracies**

In this section, we examine the accuracy of the three approximations by comparing the PP-wave reflection coefficients for three typical classes of AVO given in Table 1.

<table>
<thead>
<tr>
<th>Class</th>
<th>Shale (km/s)</th>
<th>Sand (km/s)</th>
<th>(\rho) (g/cm(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class I</td>
<td>3.30</td>
<td>1.70</td>
<td>2.35</td>
</tr>
<tr>
<td></td>
<td>4.20</td>
<td>2.70</td>
<td>2.49</td>
</tr>
<tr>
<td>Class II</td>
<td>2.96</td>
<td>1.38</td>
<td>2.43</td>
</tr>
<tr>
<td></td>
<td>3.49</td>
<td>2.29</td>
<td>2.14</td>
</tr>
<tr>
<td>Class III</td>
<td>2.73</td>
<td>1.24</td>
<td>2.35</td>
</tr>
<tr>
<td></td>
<td>2.02</td>
<td>1.23</td>
<td>2.13</td>
</tr>
</tbody>
</table>

According to the standard classification of AVO types, Class I gas sands have higher acoustic impedance for normal incidence than the encasing shale with relatively large positive values of \(R_0\). Class II gas sands have nearly the same acoustic impedance as the shale and are characterized by relatively small values of \(R_0\). Class III sands have lower impedance than the shale with relatively large negative values of \(R_0\). The parameters given in Table 1 are taken from Sayers and Rickett (1997) and used here to compare the accuracies of three approximations of elastic impedance.
We consider reflection angles from $0^\circ$ to $70^\circ$, which include, of course, both pre- and post-critical reflections. The results are displayed in Figures 1 and 2 for the incident angles of up to $70^\circ$ and $40^\circ$, respectively.

The exact results are computed using the analytic equation of Aki and Richards (1980). For Class I AVO model, all three approximations give excellent agreement for incident angles up to $15^\circ$ incident angles. Connolly’s equation (denoted CP in Figures 1 and 2) and Santos and Tygel’s equation (denoted ST in Figures 1 and 2) give acceptable values up to incident angles of $40^\circ$, whereas Ma’s approximation (denoted MA) is only valid for incident angles up to $10^\circ$. Similar conclusions can be drawn for the Class II AVO model. For Class III AVO model, Ma’s equation and Santos and Tygel’s equation are surprisingly valid for all angles up to at least $70^\circ$, and in contrast Connolly’s equation is only valid for incident angles up to about $20^\circ$ to $20^\circ$. Both Ma’s equation and Santos and Tygel equation pick up nicely the critical angles for both Classes I and II AVO models, and Connolly’s equation fail to do so. In practice, for most AVO studies we normally restrict ourselves to the offset/incident angles less more than $40^\circ$, in which case, Connolly’s equation is the sufficiently accurate for Classes I and II AVO models (Figure 2, blue line). We expect from Figure 2 that in terms of AVO attributes, both AVO intercept $A$ and $B$ from Connolly’s equations will be sufficiently accurate. This is in contrast to the results given in Santos and Tygel (2004). For class III AVO, Ma’s and Santos and Tygel’s equations should be used.

Summary

In this study, we have compared the accuracy of three approximations for the elastic impedance using three typical classes of AVO models. We have also performed a systematic study for other models including those studied by the authors of the three approximations and obtain the following conclusion. For most practical applications, i.e. incident angles less than $40^\circ$, Connolly’s equation is still the most accurate equation of all three approximations for Classes I and II AVO models, and for Classes III AVO both Ma’s and Santos and Tygel’s equations are accurate for incident angles up to large incident angles (at least $70^\circ$), whereas Connolly’s equation is only accurate for incident angles up to $20^\circ$. This study provides a useful reference as to the choice of equations for the PP-wave elastic impedance.

Acknowledgements

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References

Figure 1. Comparison of PP wave reflection coefficients computed using different approximations described in the text with the exact solution. From top to bottom, we consider three different classes of AVO response (up to 70° incident angles). CP is Connolly’s approximation; ST is Santos and Tygel approximation and MA is Ma’s approximation described in the text.

Figure 2. Comparison of PP wave reflection coefficients computed using different approximations described in the text with the exact solution. From top to bottom, we consider three different classes of AVO response (up to 40° incident angles). CP is Connolly’s approximation; ST is Santos and Tygel approximation and MA is Ma’s approximation described in the text.