A simplified moveout formula for P-, S- and converted-waves in multi-layered media

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Abstract

A simplified moveout formula is derived for P-, S-, and PS converted (C-) waves in multi-layered media. This formula uses two parameters (stacking velocity and anisotropic coefficient) for each wavetype. This formula shows that only two parameters can be extracted from reflection moveout for each wave. The result from a synthetic model shows that this simplified formula has the same accuracy as the other formulae for P-, S- and C-waves. The processing of 4C datasets, especially for C-wave data, can then be simplified using this formula.

Introduction

4C seismic acquisition has become increasingly common in the oil industry due to its ability for imaging through gas clouds and lithology-fluid prediction. The moveout in 4C seismic data takes a crucial role in imaging and velocity model estimation. Different formulae have been derived for P-, S-, and C-waves with different kinds of parameterization (Alkhalifah, 1997; Cheret, et al, 2000; Li and Yuan, 2003; Thomsen, 1999, Tsvankin and Thomsen, 1994). For P- and S-waves, two formulae with two parameters are commonly used; however for C-waves, several formulae with three, four, or five parameters are used. Therefore different approaches and software need to be applied to P-, S-, and C-wave datasets. In this paper, we revisit this issue and derive a simplified moveout formula that can be applied to P-, S-, and C-waves. We examine the numerical accuracy of this simplified formula using a synthetic model, and discuss its advantages and disadvantages for data processing.

Travel-times in multi-layered anisotropic media

Consider an \( N \)-layer VTI medium with interval parameter \((V_{pi}, \eta_i, V_{si}, \zeta_i, \text{and } z_i, i = 1,2,3,...,n)\) for each layer (Figure 1). The P and S waves are reflected at the bottom of the \( N \)-th layer. The PS converted wave (C-wave) is converted at the conversion point of the \( N \)-th layer with a down-going P-leg and an up-going S-leg. Their moveouts are approximately written with effective parameters using Thomsen’s notation (1999) as (Alkhalifah, 1997, Li and Yuan 2003):

\[
t_p^2 = (2t_{p0})^2 + \frac{(2h)^2}{V_{p2}^2} - 2\eta_{eff} V_{p2}^2 (2t_{p0})^2 V_{p2}^2 + (1 + 2\eta_{eff})(2h)^2 \right];
\]

\[
t_s^2 = (2t_{s0})^2 + \frac{(2h)^2}{V_{s2}^2} - 2\zeta_{eff} V_{s2}^2 [(2t_{s0})^2 V_{s2}^2 + (2h)^2] \right];
\]

\[
t_c^2 = t_{c0}^2 + \frac{(2h)^2}{V_{c2}^2} - \frac{(\gamma_0 \gamma_{eff} - 1)^2 + 8(1 + \gamma_0) \chi_{eff}}{4t_{c0}^2 V_{c2}^4 \gamma_0 (1 + \gamma_{eff})^2} (2h)^4 \right],
\]

(1)

(2)

(3)
where $h$ is the half offset; $t_{p0} = \sum_{i=1}^{n} t_{p0i}$, $t_{s0} = \sum_{i=1}^{n} t_{s0i}$, and $t_{c0} = \sum_{i=1}^{n} t_{p0i} + \sum_{i=1}^{n} t_{s0i} - t_{p0} + t_{s0}$ are the vertical traveltimes for the P-, S- and C-wave respectively; $\gamma_0 = \frac{t_{s0}}{t_{p0}}$ is the vertical velocity ratio; $V_{p2}^2 = \frac{1}{t_{p0}} \sum_{i=1}^{n} V_{p2}^2 t_{p0i}$, $V_{s2}^2 = \frac{1}{t_{s0}} \sum_{i=1}^{n} V_{s2}^2 t_{s0i}$, and $V_{c2}^2 = \frac{1}{t_{c0}} \sum_{i=1}^{n} \left[ V_{p2i}^2 t_{p0i} + V_{s2i}^2 t_{s0i} \right] = \frac{1}{1 + \gamma_0} V_{p2}^2 + \gamma_0 V_{s2}^2$

are the squares of the stacking velocities for each wave; $\gamma_{eff} = [1 + \gamma_0] V_{c2}^2 - 1$ is the effect velocity ratio; $\eta_{eff} = \frac{1}{8t_{p0} V_{p2}^4} \sum_{i=1}^{n} V_{p2i}^4 (1 + 8\eta_i) \Delta t_{p0i} - t_{p0} V_{p2}^4$, $\zeta_{eff} = \frac{1}{8t_{s0} V_{s2}^4} \sum_{i=1}^{n} V_{s2i}^4 (1 - 8\zeta_i) \Delta t_{s0i} - t_{s0} V_{s2}^4$, and $\chi_{eff} = \gamma_0 \eta_{eff} + \zeta_{eff}$ are the anisotropic coefficients for each wavetype respectively. Note that we change the sign of $\zeta_{eff}$ for convenience.

From the above formulae, one can see that the P- and S-moveout equations have different formats although both of them are controlled by two-parameters ($V_{p2}$ and $\eta_{eff}$, or $V_{s2}$ and $\zeta_{eff}$, respectively). However, the C-wave moveout is controlled by four parameters ($V_{c2}$, $\gamma_{eff}$, $\gamma_0$, and $\zeta_{eff}$), which cannot all be estimated from the C-wave moveout alone. Therefore, which parameters can be estimated from the C-wave moveout? Can we reduce the number of parameters in the C-wave moveout equation to two, as in the P- and S-wave case? These are interesting questions worth revisiting. For this purpose, we derive a simplified moveout formula by considering the C-wave is equivalent to a P-wave leg plus the mirrored S-wave leg (Figure 1). P- and S-waves also can be considered as the down-going leg plus the mirrored up-going legs. For the P- and S-waves, all parameters are defined as before. However, for the C-wave, an anisotropy parameter can also be defined according to the raypath geometry in Figure 1, as:

$$\kappa_{eff} = \frac{1}{8t_{c0} V_{c2}^4} \left[ \sum_{i=1}^{n} V_{p2i}^4 (1 + 8\eta_i) \Delta t_{p0i} + \sum_{i=1}^{n} V_{s2i}^4 (1 - 8\zeta_i) \Delta t_{s0i} - t_{c0} V_{c2}^4 \right] = \frac{(\gamma_0 \gamma_{eff} - 1)^2 + 8 \chi_{eff} (1 + \gamma_0)}{8 \gamma_0 (1 + \gamma_{eff})^2},$$

which is analogous to the definitions of and $\eta_{eff}$ and $\zeta_{eff}$. It is interesting to note that this $\kappa_{eff}$ is the same as that obtained by Cheret et al (2000), and is a combination of $\gamma_0$, $\gamma_{eff}$, and $\chi_{eff}$. Therefore, for each wavetype of P-, S-, and C-waves, the definitions of stacking velocities and anisotropy parameters all have the same format. Accordingly, the moveout equation of P-, S-, and C-waves can be written in the following generic format as:

$$t^2 = t_{0}^2 + \frac{(2h)^2}{V_{2}^2} - 2an i \frac{(2h)^4}{V_{2}^2 [t_{0}^2 V_{2}^2 + m \cdot (2h)^2]}.$$

In the above formula, $V_{2}$ is the stacking velocity and $ani$ is the anisotropy coefficient for P-, S-, or C-wave, respectively. Parameter $m$ is an empirical parameter related to the variation of moveout in far offsets. Noting equations (1), (2) and (3), one can see that $m = 1 + 2\eta_{eff}$ for the P-wave, $m=1$ for the S-wave, and $m = -2\kappa_{eff} (1 + \gamma_0) (1 - \gamma_0 \gamma_{eff}^2 + 2 \chi_{eff})$ for the C-wave, respectively. Since the
definition of $V_2$ and $ani$ have the same format for P-, S-, and C-waves, an intriguing question is, can we find the same empirical expression of $m$ for all the three-wave types?

Numerical analysis shows that $m$ may be set to a value between 0.5 and 2.0, and the following empirical relationships may be used to estimate $m$: $m = 1 + 2\eta_{eff}$ for P-waves; $m = 1 + 2\zeta_{eff}$ for S-waves and $m = 4\kappa_{eff}$ for C-waves, respectively. In this way, we have obtained a simplified equation for calculating the reflection moveout for the P-, S- and C-waves. Note that for the P- and S-waves, the expression $m = 1 + 2ani$ is a good empirical approximation, but not for the C-waves. Therefore, it is not possible to define $m$ with same format for all three wavetypes.

**Numerical analysis**

To validate the above analyses, we consider a three-layer model (Table 1). Table 2 lists the effective parameters calculated from the model. To examine the accuracy of equation (5), we calculate the moveout of P-, S-, and C-waves using equation (5) with $m = 0.5$ for all waves, $m = 1 + 2ani$ for the P- and S-waves, and $m = 4ani$ for the C-waves. We compare these values with the results obtained from equations (1) (2) and (3), as well as the results from ray tracing, as shown in Figures 2, 3 and 4 for the P-, S- and C-waves, respectively. The simplified formulae with $m = 1 + 2ani$ for the P- and S-waves, and $m = 4ani$ for the C-waves give the same accuracy as other formulae for the model in Table 1.

**Discussion and Conclusions**

The simplified formula in equation (5) indicates that only two parameters ($V_2$ and $ani$) can be recovered from the reflection moveout. As a result, the velocity ratios $\gamma_0$ and $\gamma_{eff}$ cannot be recovered from the C-wave moveout alone. Note that the anisotropic parameter $\kappa_{eff}$ is a combination of $\gamma_0$, $\gamma_{eff}$ and $\chi_{eff}$, and their effects are coupled in the C-wave reflection moveout. In practice, a joint analysis of P- and C-waves is used to estimate $\gamma_0$ based on event correlation, and $\gamma_{eff}$ may be calculated from $V_{e2}$, $V_{p2}$ and $\gamma_0$. Therefore equation (5) can replace equations (1), (2), and (3) for moveout analysis with $m = 1 + 2ani$ for the P- and S-waves, and $m = 4ani$ for the C-waves. The processing of 4C data set can be simplified by using this formula.

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**References**


Table 2. Effective parameters in the three-layer model in Table 1.

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<th>$t_0^p$ (s)</th>
<th>$V_{p2}$ (m/s)</th>
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<th>$V_{s2}$ (m/s)</th>
<th>$\zeta_{eff}$</th>
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<th>$V_{c2}$ (m/s)</th>
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Figure 2. Accuracy of P-wave moveout approximations for (a) the second and (b) the third reflector in Table 1. The residual moveout is calculated as $\Delta t = t - t^0$. $t^0$ is calculated by ray-tracing. $t$ is calculated using equation 1 (long-dashed curve), equation 2 (medium-dashed curve) and equation 5 with $m = 0.5$ (solid curve).

Figure 3. Accuracy of S-wave moveout approximations for (a) the second and (b) the third reflector in Table 1. Notation as in Figure 2.

Figure 4. Accuracy of C-wave moveout approximations for (a) the first, (b) the second and (c) the third reflectors in Table 1. The residual moveout is calculated as $\Delta t = t - t^0$. $t^0$ is calculated by ray-tracing. $t$ is calculated using equation 1 (long-dashed curve), equation 2 (medium-dashed curve), equation 3 (short-dashed curve), equation 5 with $m = 0.5$ (solid curve), and $m = 4\kappa_{eff}$ (dotted curve).