Summary

Seismic velocity is important to migration of seismic data, interpretation of lithology and lithofacies as well as prediction of reservoir. The information of shear wave velocity is required to reduce the uncertainty for discriminating lithology, identifying fluid type in porous material and calculating gas saturation in reservoir prediction. Based on Zoeppritz equations, a numeral and scanning method was proposed in this paper. Shear wave velocity can be calculated with prestack converted wave data. The effects were demonstrated by inversion of theoretical and real seismic data.
Introduction

Seismic wave propagation velocity varies with rock component, fluid type and gas saturation in porous reservoir. Velocity is not only significant parameter to describe and characterize reservoir, but also important basis for reservoir prediction, such as fluid type, gas saturation, porosity. The information of shear wave velocity is required to reduce the uncertainty for discriminating lithology, identifying fluid type and calculating gas saturation in porous material only by P wave velocity.

Zoeppritz (1919) derived reflection and transmission coefficient equations when P-wave impact on interface between two infinite solid spaces. The equations can be expressed as follow:

\[
\begin{bmatrix}
\cos \alpha_i & -\sin \alpha_i \\
-\sin \alpha_i & \cos \alpha_i \\
\sin 2\alpha_i/v_{pl} & \cos 2\beta_i \\
\cos 2\beta_i & -\sin 2\beta_i/v_{pl}
\end{bmatrix}
\begin{bmatrix}
\rho_1 v_{p1}^2 & \rho_2 v_{p2}^2 \\
\rho_1 v_{s1}^2 & \rho_2 v_{s2}^2
\end{bmatrix}
= \begin{bmatrix}
R_{PP} \\
R_{PS} \\
T_{PP} \\
T_{PS}
\end{bmatrix}
= \begin{bmatrix}
-\sin \alpha_i \\
\cos \alpha_i \\
\sin 2\alpha_i \\
-\cos 2\beta_i
\end{bmatrix}
\]

(1)

Aki and Richards (1980) by simplifying Zoeppritz equations derived converted wave reflection coefficient expression as follow:

\[
R_p(\theta, \varphi) = -\frac{v_p}{2v_r} \tan \varphi \left[(1 - \frac{2v_s^2}{v_p^2} \sin^2 \theta + \frac{2v_s}{v_p} \cos \theta \cos \varphi) \frac{\Delta \rho}{\rho} - \frac{4v_s^2}{v_p^2} \sin^2 \theta - \frac{4v_s}{v_p} \cos \theta \cos \varphi \frac{\Delta v_r}{v_r}\right]
\]

(2)

The representative formula is that converted wave reflection coefficient is expressed as the function of impedance:

\[
R_p(\theta, \varphi) = -\frac{v_p}{2v_r} \tan \varphi \left[(1 + 2\sin^2 \theta - \frac{2v_s}{v_p} \cos \theta \cos \varphi) \frac{\Delta \rho}{\rho} - (4\sin^2 \theta - \frac{4v_s}{v_p} \cos \theta \cos \varphi) \frac{\Delta J}{J}\right]
\]

(3)

Cosine’s odd power of incident angle is

\[
R_{pp} = \left(-\frac{1}{2}(1 + \frac{2v_s}{v_p}) \frac{\Delta \rho}{\rho} + \frac{2v_s}{v_p} \frac{\Delta v_r}{v_r}\right) \sin \theta + \left\{\frac{1}{2} \frac{v_p}{v_r} + \frac{3}{2} \frac{v_p}{v_r} \frac{\Delta \rho}{\rho} + 2 \frac{v_s}{v_p} + (\frac{1}{2} + \frac{2v_s}{v_p}) \frac{\Delta v_r}{v_r}\right\} \sin^3 \theta
\]

(4)

At present, poststack inversion method for S wave interval velocity is almost based on typical approximation (Equation 1) or corresponding simplified formulae (Equation 2, 3 or 4). All of those approximations have good precision on the condition that variance rate of elastic parameters is smaller and incident angle is less than moderate angle (Antonio and John, 2001), so the precision of S wave interval velocity is limited no matter what the improvement of resolving algorithm. In this paper, we seek to method for obtaining numerical resolution from Zoeppritz equations (Equation 1), and acquiring S wave interval velocity on multi-wave seismic data by velocity scanning method, then calculating velocity ratio and Poison ratio.

The inversion method

Zoeppritz equations are complex function of P wave velocity, S wave velocity and density of layer above and below interface, reflected and transmitted angle of P wave and S wave. Compared with some other simplified equations, more accurate S wave interval velocity can be obtained from velocity scanning method by resolving equation (1) directly.

When P wave interval velocity and density are available, we trace propagation path according to each S wave velocity with given scanning range and step. Secondly, collect theoretical common converted point gather \(R_{PSV}\), and then make theoretical reflect record and real data S the best square approach.

\[
\min_{\nu} \sum_{i=1}^{N} [K \times R_{PSV}, - S_i]^2
\]

(5)

Where \(K\) is scale coefficient of amplitude between theoretical reflection coefficient and real data, \(N\) is number of trace in common converted point gather. The value that minimizes the
least square approach function is the S wave interval velocity we seek. Thus ratio of P to S velocity and Poison ratio can be attained.

During the process of S wave velocity scanning, P wave velocity and density are considered as being known. For example, P wave velocity is provided with poststack or prestack inversion of P wave data and density is provided with log data or prestack inversion of P wave data.

**Numerical simulation**

S wave velocity errors inverted with velocity scanning method applied to reflection coefficient of each layer are less than 1 m·s\(^{-1}\) as listed in Table 1 (when scan step is 1 m·s\(^{-1}\)), which indicates that this method is a high resolution inversion algorithm without composite wave.

Real seismic data contains composite wave, so in order to validate the practicability of the method, inversion results are analyzed and compared with real sonic and S wave logging data in some area. The measured depth of logging data ranges are from 1520 to 4531 m and sample is 0.125 m. For easy synthesis and inversion, 10 layers are interpolated linearly from surface to measured original depth (1520 m) and original logging data are resample at 50 m interval. Figure 2 is velocity model (Figure 1a) and prestack PSV wave gather after NMO (Figure 1b).

![Figure 1 Velocity model (a) and Prestack converted wave gather after NMO (b)](image)

![Figure 2 Inversion of shear wave interval velocity(a) Comparison of real logging shear interval velocity and inversion velocity; (b) Relative errors](image)

S wave interval velocity is achieved using velocity scanning method when P wave interval velocity and reflect time of horizon in P and PSV wave profiles are inputted. Figure 3a is comparison of real logging S wave interval velocity and inversion velocity. Where black solid line denotes real velocity in model, dash line denotes velocity by inversion. It can be seen from Figure 2a that inversion velocity consists with model velocity, but errors increase with depth. The errors (Figure 2b) are not more than 25 m·s\(^{-1}\) in general, just only a few points exceed 50 m/s. Relative errors \(\frac{v_s^{\text{Inversion}} - v_s^{\text{Real}}}{v_s^{\text{Real}}}\) increase with depth. A majority of result is restricted in 1%, and a few samples excess 2%. The maximal relative errors are still
less than 6%. The results of model data represent that S wave velocity inverted with prestack PSV wave gather is reliable and the velocity scanning method is feasible.

**Inversion of real data**

This paper applies the velocity scanning method to the process of multi-component seismic data and inverts S wave interval velocity. Layer calibration is made by synthetic seismogram of PP and PSV wave according to the character of stacked section or referring to geological interpretative result of PP and PSV wave data in study area.

Figure 3 is P wave interval velocity section inverted with poststack log constrained inversion. Figure 4 is S wave interval velocity sections inverted with velocity scanning method on common converted point gather after NMO. Initial S wave velocity model is computed from reflection time according to the same horizon on two sections. Figure 5 is velocity ratio section and Figure 6 is Poison ratio section.

The research of multi-component seismic attribution indicates that abnormal amplitude ratio of P to S wave consists with abnormal Poison ratio. In general, abnormal amplitude ratio and abnormal poison ratio all indicate change of lithology or physical property, but variation of shale content in sand gravel and fracture density of igneous rock also lead to abnormal multi-component seismic attribute. Abnormal Poison ratio indicates the exist of oil and gas in general, but high porosity, high crushed reservoir and low shear modulus also lead to low S wave velocity and high Poison ratio that indicate high quality reservoir. According to the analysis of gas well and reservoir lithology, abnormal multi-component seismic attribute indicates high quality reservoir.

**Conclusions**

Velocity scanning method for resolving Zoeppritz equations is more accurate algorithm to invert S wave velocity. There are several advantages as follows:

1) Based on the exact resolution of reflection coefficient equation, inversion error decreases compared with that of approximations.

2) Prestack PSV wave data reserves more information of reservoir and is useful to obtain relative accurate S wave interval velocity. In addition, velocity ratio and Poison ratio can be got combined with inversion result of P wave data.

Application of velocity scanning method is on the assumption that P wave velocity is available, so precision of P wave interval velocity affects directly that of P wave. It is key important to select a good method for inverting P wave interval velocity. Next step is to generalize this method to simultaneous inversion of P wave and S wave data and research high precision inversion method for simultaneous inversion of P and S wave interval velocity.

![Figure 3 P wave interval velocity section gained from poststack log constrained inversion (velocity unit: m·s⁻¹)](image-url)
Figure 4  S wave velocity section inverted with velocity scanning method (velocity : m⋅s\(^{-1}\))

Figure 5  Velocity ratio of P to S wave

Figure 6  Poison ratio section

Reference


Zoeppritz, K., 1919, Erdbebenwellen VIII B, On the reflection and penetration of seismic waves through unstable layers: Goettinger Nachrichten., 1, 66-84

**Foundation item:** Chinese National 973 Key Basic Research Development Program (No. 2005CB422104) and SINOPEC’s Scientific and Technological Development Program (No. P05063)