

# Evaluation of anisotropic critical angle in frequency domain

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## SUMMARY

The importance of long-offset reflection data has been stressed in anisotropy analysis in recent years, because long-offset data contains more kinematic and dynamic characteristics of seismic waves and can facilitate seismic anisotropic analysis. In particular, the change of the seismic critical angle in long-offset data can be related to the anisotropy parameters of subsurface media. However, the problem in the current implementation of anisotropic critical analysis lies in the difficulty of the identification of critical offset, because the traditional diagnostic features such as the rapid increase in reflection amplitude and the separation of reflection and head waves are not sufficiently effective in practice. To solve this problem, we propose critical angle analysis in frequency domain. The method is tested on the synthetic seismograms for VTI, HTI, and orthorhombic media. The analysis in frequency domain shows that the sharp change of the reflection amplitude at the dominant frequency can give a good evaluation of the anisotropic critical angle.

## Introduction

The importance of long-offset reflection data has been highlighted in anisotropy analysis in recent years. Long-offset data contains more kinematic and dynamic characteristics of seismic waves and can facilitate seismic anisotropic analysis. P-wave non-hyperbolic moveout has been used to invert anellipticity parameter  $\eta$  for vertical transverse isotropy (VTI), and also applied to analysis for azimuthally anisotropic media, including both horizontal transverse isotropy (HTI), and orthorhombic anisotropy. On the other hand, amplitude-related dynamic features can be used through amplitude-variation-with-offset (AVO) analysis to characterize fractured reservoirs. However, conventional AVO analysis is generally performed within the pre-critical region. The reasons that wide-angle AVO analysis has not yet been common until now come from both the complexity of processing and the difficulty of interpretation in long-offset amplitude inversion. Landrø et al. (2007) suggest a more direct way to estimate anisotropy parameters and azimuthal variation by employing long-offset reflection data. But the authors also admit that the estimation of the critical offset from the picked amplitude curve in temporal domain is not straightforward, because diagnostic features, such as the rapid amplitude increase and separation of head wave, are not practical enough to detect the position of the critical offset. Sil et al. (2009) propose seismic critical-angle analysis in  $\tau$ - $p$  domain to avoid the amplitude picking process, where the critical slowness is picked through the intersection point of events of two adjacent layers in  $\tau$ - $p$  domain.

Our motivation in this paper is to find a more straightforward method to evaluate the critical offset in the presence of VTI, HTI, and orthorhombic anisotropy. The procedure is carried out in the frequency domain, and the picked results of critical angles are compared with those predicted by plane wave theory for accuracy and reliability.

## Theory

The theory on critical-angle analysis is developed by Landrø et al. (2007). Sil et al. (2009) modify the theory for the analysis in  $\tau$ - $p$  domain. Based on their theory, critical angles are related to anisotropy parameters and azimuthal angles, as below,

$$\sin \theta_c(VTI) = \frac{V_{P0,1}}{V_{P0,2} \sqrt{1 + 2\varepsilon}}, \quad (1)$$

$$\sin \theta_c(HTI) = \frac{V_{P0,1}}{V_{P0,2} \sqrt{1 + 2\varepsilon^h \cos^4 \phi}}, \quad (2)$$

$$\sin \theta_c(orth) = \frac{V_{P0,1}}{V_{P0,2} \sqrt{1 + 2\varepsilon(\phi)}}, \quad (3)$$

where  $\theta_c$  represents the critical angle for three types of anisotropic media.  $V_{p0}$  is the vertical P-wave velocity, with subscripts 1 and 2 denote upper and lower medium, respectively.  $\phi$  is the azimuthal angle;  $\varepsilon$ ,  $\varepsilon^h$ , and  $\varepsilon(\phi)$  are the Thomsen and Thomsen-style parameters for the VTI, HTI, and orthorhombic media, respectively.

Due to the interference between reflection wave and head wave generated at the critical offset, the AVO curve at near-critical and post-critical offset does not follow the reflection coefficients predicted by plane-wave theory (Landrø et al, 2007). Following the method of Skopintseva et al (2008, 2009), we will analyze the AVO response in frequency domain and try to find a straightforward way to detect critical offset.

## Modelling studies

To test the method of critical offset identification in frequency domain, we set the model with the interfaces separating an isotropic overburden and HTI, VTI, and orthorhombic half space,

respectively. The subsurface interfaces are located at a depth of 300m and the geometry has an offset up to 2000m, corresponding to a maximum reflection angle of  $73^{\circ}$  which covers the critical offset. The isotropic overburden has the P and S wave velocities  $V_{P0,1}=2800\text{m/s}$ ,  $V_{S0,1}=1400\text{m/s}$ , and density  $\rho=2100\text{kg/m}^3$ . For the underlying VTI medium:  $V_{P0,2}=3048\text{m/s}$ ,  $V_{S0,2}=1536\text{m/s}$ . For the HTI medium:  $V_{P0,2}=3400\text{m/s}$ ,  $V_{S0,2}=1600\text{m/s}$ , with anisotropic parameters  $\varepsilon=-0.1$ ,  $\gamma=0.15$ , and  $\delta=-0.05$ ; and the orthorhombic medium:  $V_{P0,2}=2900\text{m/s}$ ,  $V_{S0,2}=1283\text{m/s}$ , with anisotropic parameters  $\varepsilon^{(1)}=0.25$ ,  $\gamma^{(1)}=0.30$ ,  $\delta^{(1)}=0.15$ , and  $\varepsilon^{(2)}=0.10$ ,  $\gamma^{(2)}=0.20$ ,  $\delta^{(2)}=-0.15$ ,  $\delta^{(3)}=-0.10$ . All underlying anisotropic media have a density of  $2300\text{kg/m}^3$ .

Synthetic seismograms are generated with pseudo-spectrum method. The explosion source is 25Hz Ricker wavelet. Figures 1, 2 and 3 show the synthetic seismograms of the vertical Z-component for P-P wave and the relevant frequency spectra. Figure 1 corresponds to the VTI case with varying  $\varepsilon$ . Figures 2 and 3 correspond to HTI and orthorhombic case with varying azimuth of  $0^{\circ}$ ,  $30^{\circ}$ ,  $60^{\circ}$  and  $90^{\circ}$ , respectively. Random noise with up to 5% of the maximum amplitude is added to the synthetics.

As shown in Figure 1, the separation of the head waves from the reflection P-P waves indicates the presence of critical angles. But this separation is not always visible, such as in the case in Figure 1a and b. Moreover, the offsets corresponding to the rapid increase in reflection amplitudes are also not clearly visible. Therefore, it is very difficult to use these plane-wave features to identify the critical offset accurately. However, after transforming the data from t-x to f-x domain as shown in Figure 1e, 1f, 1g and 1h, corresponding to the rapid increase of reflection amplitude, there is a sharp boundary defined by the contrast of colours from dark to bright in the colour displays of the spectra, and the far left boundary (edge) along the x-axis at the dominate frequency of 25 Hz indicates the critical offset. Following this way, we pick up the critical offsets from all synthetics shown in Figures 1, 2 and 3, and the results are displayed in Figure 4, where the critical angles calculated from equations (1), (2) and (3) are also plotted for comparison purpose. In Figure 4a for VTI, the errors of the picked critical offsets increase with the anisotropy parameters  $\varepsilon$  from 5% to 12%. In Figure 4b for HTI, the errors vary from 0% to 5% as azimuth changes. For orthorhombic medium in Figure 4c, the minimum errors occur at  $0^{\circ}$  and  $90^{\circ}$  azimuth, which are less than 3%, and the errors are about 7% and 6% at  $30^{\circ}$  and  $60^{\circ}$  azimuth, respectively. Here, we only concentrate on the identification of the critical offset, and their applications in the estimation of anisotropy parameters can be found in Landrø et al. (2007).

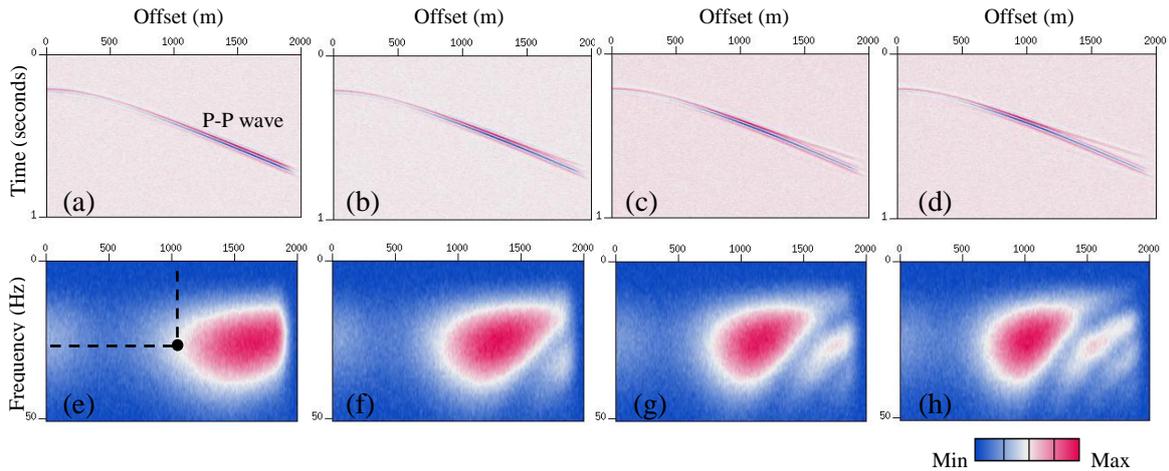
Finally, Figure 5 shows the reflection coefficients from the spectra of synthetic seismograms based on Plancherel's theorem (Skopintseva et al., 2009). The reflection coefficients are normalized by maximum values and equivalent to the RMS amplitudes. It is difficult to identify the critical offsets marked by the black arrows using the plane-wave features, such as rapid increase in reflection amplitudes.

## Conclusions

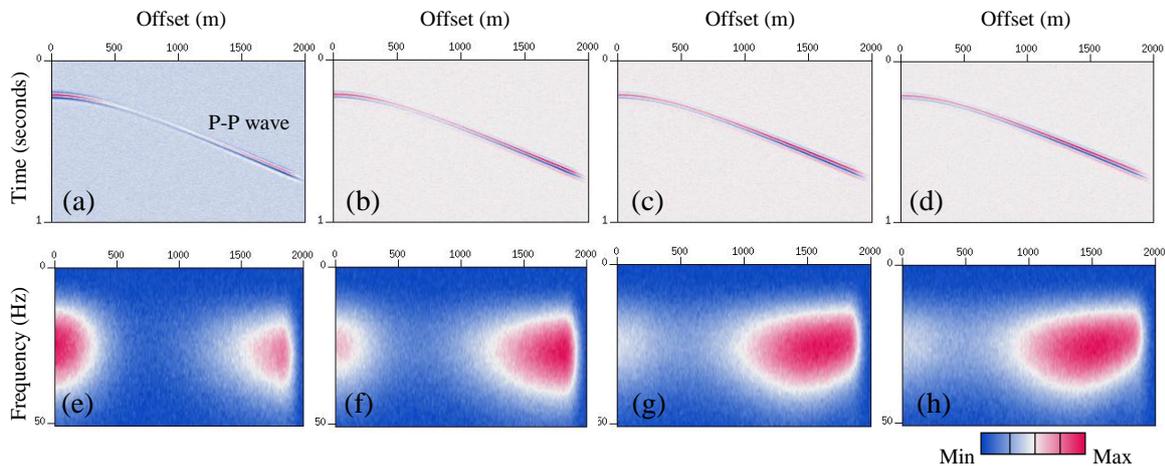
For VTI, HIT, and orthorhombic anisotropy, the Thomsen-style anisotropic parameters may be estimated from the critical offsets if the offset can be reliably identified from reflection events. This study shows that anisotropic critical offsets can be more accurately identified in the frequency domain than in the time domain. In the f-x domain along the x-axis, the offset corresponding to a sharp change of reflection amplitude at the dominant frequency is a good estimate of the critical offset. This sharp change of amplitude can be easily identified using colour displays of the amplitude spectra.

## Acknowledgements

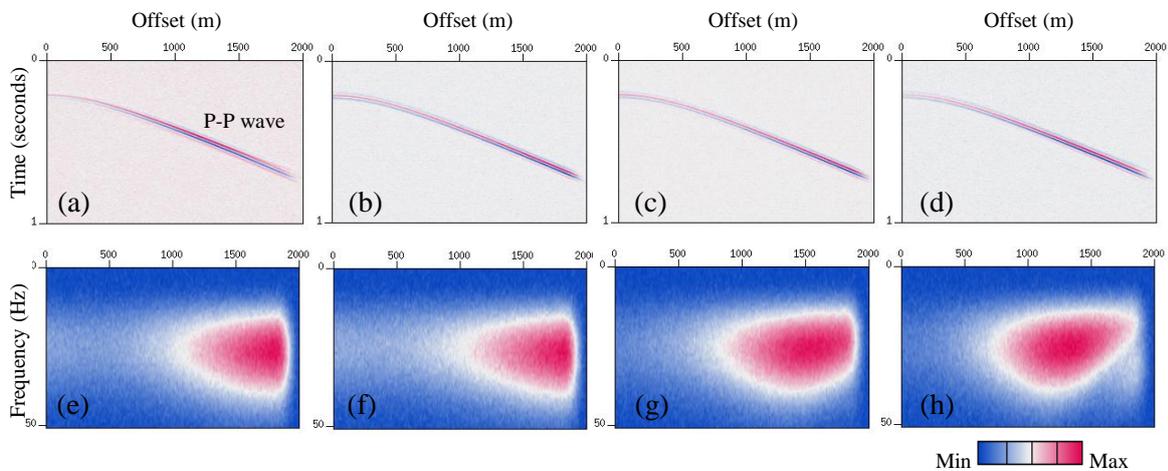
This work is supported by Edinburgh Anisotropy Project of the British Geological Survey, and is presented with the permission of the Executive Director of British Geological Survey (NERC).



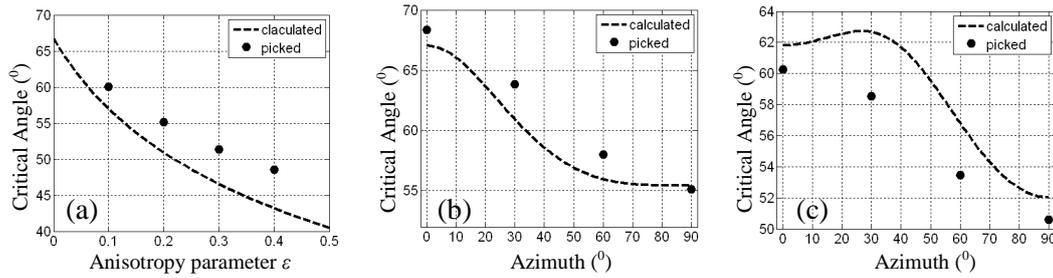
**Figure 1:** Synthetic seismograms and relevant frequency spectrums for models with underlying VTI media. (a), (b), (c) and (d) correspond to VTI media with anisotropy parameter  $\varepsilon=0.1, 0.2, 0.3$  and  $0.4$  respectively. (e), (f), (g) and (h) are spectra corresponding to (a), (b), (c) and (d). The black dot in (e) shows the criteria for picking up the critical offset.



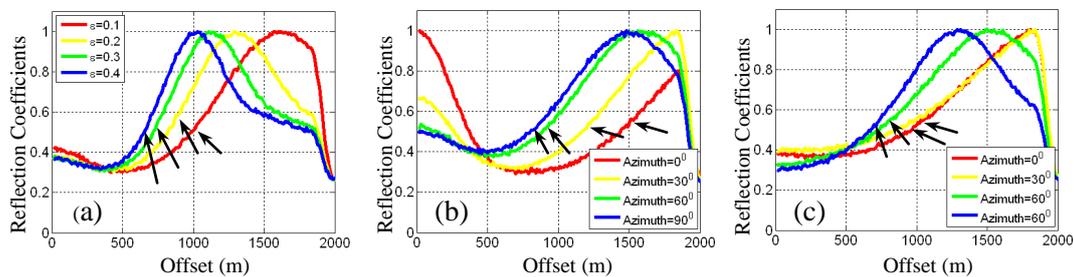
**Figure 2:** Synthetic seismograms and relevant frequency spectrums for models with underlying HTI media. (a), (b), (c) and (d) correspond to HTI media with azimuth  $\varphi=0^\circ, 30^\circ, 60^\circ$  and  $90^\circ$ , respectively. (e), (f), (g) and (h) are spectra corresponding to (a), (b), (c) and (d).



**Figure 3:** Synthetic seismograms and relevant frequency spectrums for models with underlying orthorhombic media. (a), (b), (c) and (d) correspond to orthorhombic media with azimuth  $\varphi=0^\circ, 30^\circ, 60^\circ$  and  $90^\circ$ , respectively. (e), (f), (g) and (h) are spectra corresponding to (a), (b), (c) and (d).



**Figure 4:** Comparison of the picked critical offset with the calculated offset using equations (1), (2) and (3). The picked critical offsets are denoted by black solid circles, and the calculated critical offsets are illustrated by dashed lines. (a) Variation of critical offset with anisotropy parameter  $\epsilon$  of VTI media. (b) Variation of critical offset with azimuth for the model with underlying HTI. (c) Variation of critical offset with azimuth for the model with underlying orthorhombic media.



**Figure 5:** AVO responses extracted from the synthetics shown in Figures 1, 2, and 3, corresponding to (a) VTI medium with varied anisotropy parameter  $\epsilon$  (b) HTI and (c) Orthorhombic with different azimuth. Black arrows show the corresponding critical offsets.

## References

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