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Frequency dependent AVO inversion using smoothed pseudo Wigner-Ville distribution

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Summary

Fluid-saturated rocks are generally expected to have frequency-dependent velocities, and it is attractive to try to use this property to discriminate different fluids with sesimc data. Previous work has demonstrated how fluid-related dispersion results in a frequency-dependent AVO response. In this paper we show how to combine the smoothed pseudo Wigner-Ville distribution with a frequency-dependent AVO inversion scheme to obtain direct estimates of dispersion from pre-stack data. Numerical studies illustrate the application of the method to synthetic data, and we conclude that the work has the potential to be useful for fluid detection.

Introduction

Fluid substitution using Gassmann's theory lies at the heart of most seismic fluid-detection methods. While Gassmann has proven to provide an excellent approximation, the elastic behaviour of fluid-saturated rocks is often frequency-dependent, and this is not accounted for by the theory. Considerable effort has been expended on laboratory studies (Batzle et al. 2006) and theoretical investigations (Jakobsen and Chapman, 2009) with the goal of understanding this frequency-dependence. Being able to measure frequency-dependence of velocity from reflection data would greatly assist fluid discrimination efforts.

Chapman et al. (2005) performed a theoretical study of reflections from layers which exhibit fluid-related dispersion and attenuation, and showed that in such cases the AVO response was frequency-dependent. Application of spectral decomposition techniques allowed the behaviour to be detected on synthetic seismograms. Wilson et al. (2009) extended this analysis, introducing a frequency-dependent AVO inversion concept aimed at allowing a direct measure of dispersion to be derived from pre-stack data.

This paper represents a feasibility study for using the Wilson et al. (2009) technique for inferring dispersion properties from reflection data. We begin by introducing the Smoothed Pseudo Wigner-Ville distribution which we use for time-frequency analysis. We then outline the theory presented by Wilson et al. (2009), and perform a numerical study to demonstrate frequency-dependent AVO inversion on synthetic data. We conclude that the method is able to estimate dispersion properties from seismic data, and we hope that his may aid seismic fluid-detection efforts.

Smoothed Pseudo Wigner-Ville Distribution

We use Wigner-Ville Distribution (WVD) based method for spectral decomposition. WVD is well-recognized as an effective method for time-frequency analysis of nonstationary signals (Cohen, 1995). The WVD of signal $x(t)$ can be defined as:

$$WVD(t,f)=\int_{-\infty}^{\infty} X(t+\tau/2)\bar{X}(t-\tau/2)e^{-j2\pi f\tau}d\tau\,, \tag{1}$$

where τ is the time delay variable, X (t) is the analytical signal associated with the real signal x (t). Since the value is determined by all the values rather than part of the signal limited by a time window, WVD avoids a trade-off between time and frequency resolution. However, this improvement comes at the cost of suffering from cross-terms interference (CTI) due to bilinear feature. We use two smooth windows $g(\nu)$ and $h(\tau)$ to reduce the effect of CTI, namely the Smoothed Pseudo Wigner-Ville Distribution (SPWVD, Claasen and Mecklenbräuker, 1980):

$$SW_{g,h,X}(t,f)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(t+\frac{\tau}{2})\bar{X}(t-\frac{\tau}{2})g(\nu)h(\tau)e^{-j2\pi f\tau}d\nu d\tau\,, \tag{2}$$

where ν is the time delay and τ is the frequency offset, $g(\nu)$ is the time smooth window, $h(\tau)$ is the frequency smooth window on condition that $g(\nu)$ and $h(\tau)$ are both real symmetric functions and $g(0)=h(0)$. SPWVD is a high "cost performance" method in resolution and calculating time which provides higher temporal resolution than STFT and CWT (Wu and Liu, 2009).

Frequency dependent AVO inversion

Smith and Gidlow's (1987) two-term AVO approximation of reflection coefficient R can be re-written as:

$$R(\theta)\approx A(\theta)\frac{\Delta V_p}{V_p}+B(\theta)\frac{\Delta V_s}{V_s}\,, \tag{3}$$

where θ is the angle of incidence and A and B can be derived in terms of the velocities. Following the arguments of Wilson et al. (2009), the reflection coefficient and the P- and S-wave impedances are considered to vary with frequency due to dispersion at the interface, then (3) can be written as:

$$R(\theta, f) \approx A(\theta)\frac{\Delta V_p}{V_p}(f) + B(\theta)\frac{\Delta V_s}{V_s}(f). \tag{4}$$

Expanding (4) as first-order Taylor series around a reference frequency f_0:

$$R(\theta, f) \approx A(\theta)\frac{\Delta V_p}{V_p}(f_0) + (f - f_0)A(\theta)I_a + B(\theta)\frac{\Delta V_s}{V_s}(f_0) + (f - f_0)B(\theta)I_b, \tag{5}$$

where I_a and I_b are the derivatives of impedance with respect to frequency evaluated at f_0:

$$I_a = \frac{d}{df}\left(\frac{\Delta V_p}{V_p}\right); I_b = \frac{d}{df}\left(\frac{\Delta V_s}{V_s}\right). \tag{6}$$

For a typical CMP gather with n receivers denoted as a data matrix $s(t, n)$. Coefficients A and B at each sampling point, denoted as $A_n(t)$ and $B_n(t)$, can be derived with the knowledge of velocity model through ray tracing. We perform SPWVD on $s(t, n)$ to derive the spectral amplitude $S(t, n, f)$ at a series of frequencies. However, S contains the overprint of seismic wavelet, so we perform spectral balance to remove this effect with a suitable weight function w:

$$D(t,n,f) = S(t,n,f)w(f). \tag{7}$$

In AVO analysis, we usually assume that the recorded amplitudes can be associated with the reflection coefficients through a simple transformation. According to (4), we can obtain $\Delta V_p/V_p$ and $\Delta V_s/V_s$ at the reference frequency f_0 by replacing R with D. Considering m frequencies $[f_1, f_2, \ldots f_m]$, equation (5) can be expressed as matrix form:

$$\begin{bmatrix} D(t,1,f_1) - A_1(t)\frac{\Delta V_p}{V_p}(f_0,t) - B_1(t)\frac{\Delta V_s}{V_s}(f_0,t) \\ \vdots \\ D(t,1,f_m) - A_1(t)\frac{\Delta V_p}{V_p}(f_0,t) - B_1(t)\frac{\Delta V_s}{V_s}(f_0,t) \\ \vdots \\ D(t,n,f_1) - A_n(t)\frac{\Delta V_p}{V_p}(f_0,t) - B_n(t)\frac{\Delta V_s}{V_s}(f_0,t) \\ \vdots \\ D(t,n,f_m) - A_n(t)\frac{\Delta V_p}{V_p}(f_0,t) - B_n(t)\frac{\Delta V_s}{V_s}(f_0,t) \end{bmatrix} \approx \begin{bmatrix} (f_1 - f_0)A_1(t) & (f_1 - f_0)B_1(t) \\ \vdots & \vdots \\ (f_m - f_0)A_1(t) & (f_m - f_0)B_1(t) \\ \vdots & \vdots \\ (f_1 - f_0)A_n(t) & (f_1 - f_0)B_n(t) \\ \vdots & \vdots \\ (f_m - f_0)A_n(t) & (f_m - f_0)B_n(t) \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix}, \tag{8}$$

and I_a and I_b can be solved with least-squares-inversion.

Numerical Example

We consider a two-layer Class III AVO model presented by Chapman et al. (2005), where the top elastic layer had P- and S-wave velocities of $2743ms^{-1}$ and $1394ms^{-1}$. For the dispersive model, the lower layer is defined as a material under water-saturation then substituted with gas by changing the fluid bulk modulus from 2GPa to 0.2GPa. For the elastic model, the P- and S-wave velocities of lower layer were calculated from elastic tensor for the dispersive model at low frequency. Eleven traces for each model are generated using 40Hz Ricker wavelet as our source. The trace space is 100m.

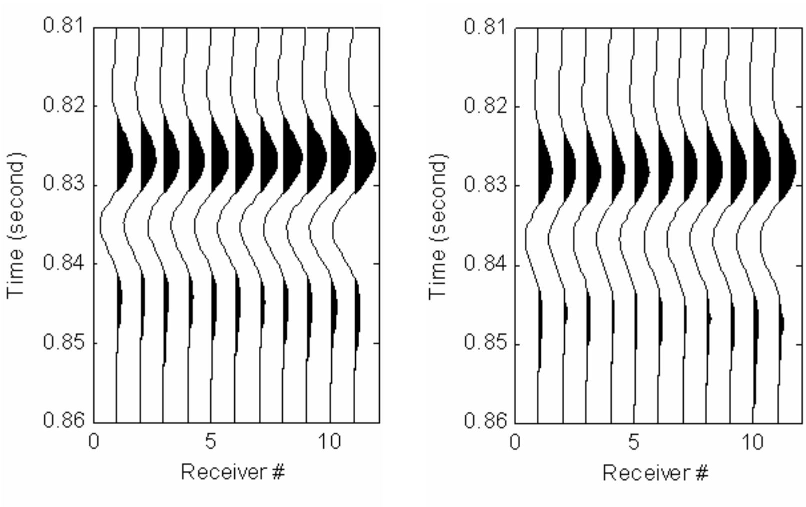

Figure 1 *Synthetic gathers for the two-layer model at the interface with (Left) elastic lower layer and (Right) dispersive lower layer.*

Figure 1 displays the synthetic gathers of the elastic and dispersive models at the interface respectively, both of which the amplitudes increase with the offset gradually. However, compared with the elastic model, the amplitudes for the dispersive model have decreased. Note that the event between 0.82s and 0.83s in the elastic gather arrives earlier than the corresponding event in the dispersive gather. This indicates the attenuation of energy and change of velocity in the dispersive model. SPWVD is performed to calculate the spectral amplitudes at 25, 30, 40, 50, 60, 70 and 80Hz. A set of weights are derived in the elastic model by matching the peak amplitude of the isofrequency trace to the 40Hz amplitude to remove the overprint of wavelet. These same weights are applied to the dispersive model. Figure 2 shows a comparison of isofrequency sections between the elastic and dispersive models at 25, 40, 60 and 80Hz. For the elastic model (upper), similar energy appears on each isofrequency section after spectral balance. In the dispersive case (lower), energy reduces at 25Hz compared with elastic model and decreases markedly with the increase of frequency.

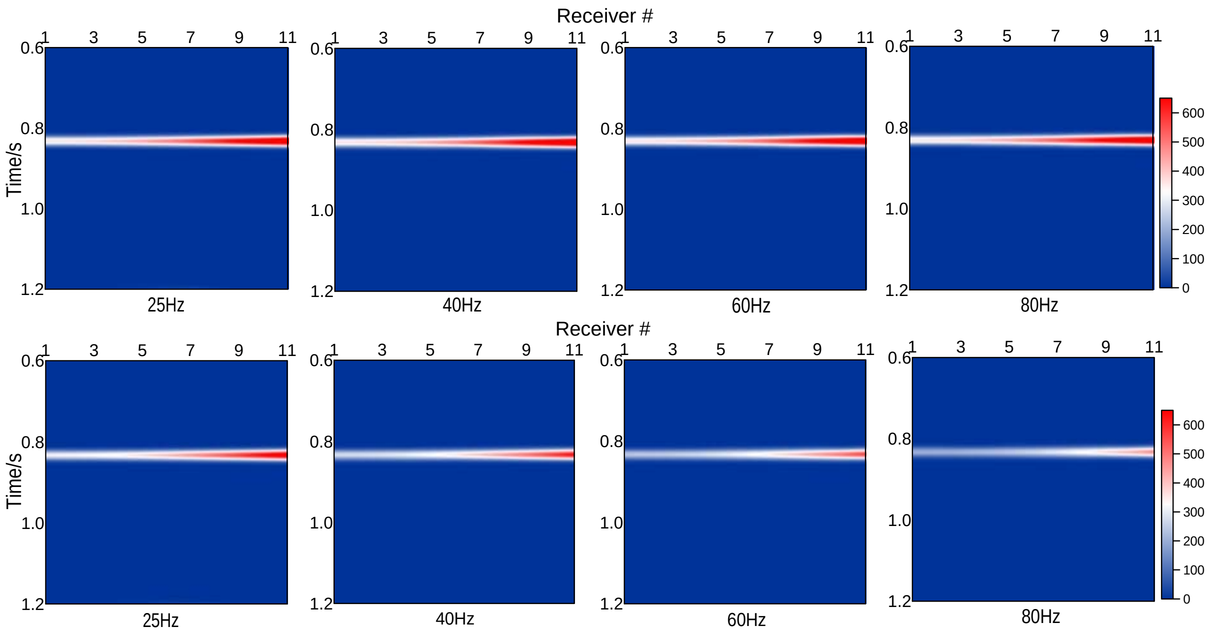

Figure 2 *Isofrequency sections of (upper) the elastic gather and (lower) dispersive gather at 25Hz, 40Hz, 60Hz and 80Hz. Spectral balance based on the input wavelet has been carried out with 40Hz as reference frequency.*

Figure 3 displays the inverted $\left|\frac{d}{df}(\frac{\Delta V_p}{V_p})\right|$ for the elastic and dispersive model. The magnitude of P wave dispersion for the dispersive model is more profound than the elastic model. We can also see the time delay between the dispersive and elastic arrivals due to the high temporal resolution of SPWVD.

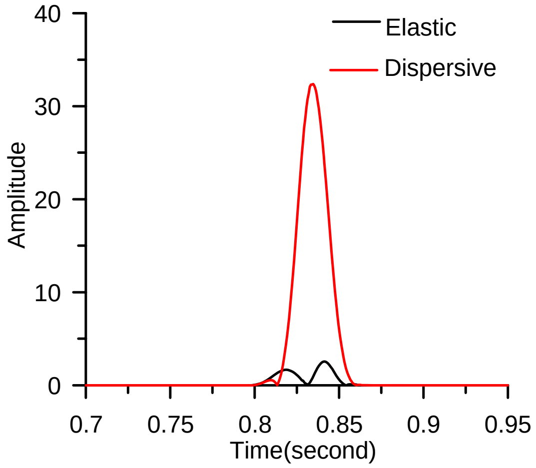

Figure 3 *The inverted magnitude of P wave dispersion for the elastic and dispersive models.*

Conclusions

Fluid-related dispersion and attenuation gives rise to a frequency-dependent reflection coefficient, and in many cases this can have a strong effect on reflection data. This paper combines a modern time-frequency analysis technique with a frequency-dependent AVO inversion scheme to demonstrate the possibility of inferring direct dispersion properties from pre-stack data. We believe that the technique has the potential to improve our ability to detect fluids with seismic data.

Acknowledgements

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