Generalized form of reflection coefficients in terms of impedance matrices of *qP-qP* and *qP-qS* waves in TI media

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Summary

Reflection coefficients of *qP*- and *qS*- incident waves in orthorhombic media can be explicitly expressed by means of impedance matrices. In this paper, we extend previous studies and derive generalized linearized equations of *qP-qP* and *qP-qS* waves. Impedance matrices representing complete reflection properties of the medium can be inverted based on the simplified equations. Besides, these equations are reduced for an isotropic medium with a vertical symmetry axis (VTI) or a horizontal axis (HTI). We discuss their accuracy and applicability by means of three models: two models of VTI/VTI interfaces with moderate and strong anisotropy, and a VTI/HTI model. The approximations have sufficient accuracy for relatively strong anisotropy. Therefore they can be applied in the characterization of unconventional resources, for example, shale gas or coalbed methane.

Introduction

AVO analysis and inversion are essential techniques in the characterization of hydrocarbon reservoirs. Their development is based on the plane wave reflection coefficients from an interface between two subsurface media. The first analytic solution of isotropic media is published by Zoeppritz (1919). Subsequently, a series of approximations of PP- and PS-wave reflection coefficients have been studied for AVO analysis and inversion.

Analytical solutions of reflection coefficients in anisotropic media can be found for wave propagation in symmetry planes, such as by Daley and Hron (1977) for transversely isotropic (TI) media with vertical symmetry axis, and Schoenberg and Protazio (1992) for symmetric plane of orthorhombic media. On the assumption of weak TI media, different linear equations of *qP*-*qP* and *qP*-*qS* waves have been investigated, for example by Ruger (1997) and Jilek (2002).

In this paper, we extend previous studies and derive a generalized equations of *qP*-*qP* and *qP*-*qS* reflection coefficients for orthorhombic media in terms of impedance matrices. These equations can be reduced for an isotropic medium with a vertical symmetry axis (VTI) or a horizontal axis (HTI). We discuss their accuracy and applicability by means of three models: two models of VTI/VTI interfaces with both moderate and strong anisotropy, and a VTI/HTI model. The approximations have sufficient accuracy for relatively strong anisotropy.

Generalized form of reflection coefficients and inversion in orthorhombic media

Orthorhombic anisotropy results from a combination of thin layers and aligned fractures. It is one of the most common forms of anisotropy in sedimentary basin (Bush and Crampin, 1991). In this case, we use *qS* to denote the quasi-SV wave, because the *qP*- and *qSV*-wave are decoupled from the SH wave in the symmetry planes.

An explicit equation for reflection coefficients at a planar interface separating two layers with orthorhombic anisotropy is expressed by

$$\mathbf{R}_i = \begin{pmatrix} R_{qPqP} & R_{qSqP} \\ R_{qPqS} & R_{qSqS} \end{pmatrix} = \left(\mathbf{X}_i^{-1}\mathbf{X}_{i+1} - \mathbf{Y}_i^{-1}\mathbf{Y}_{i+1}\right)\left(\mathbf{X}_i^{-1}\mathbf{X}_{i+1} + \mathbf{Y}_i^{-1}\mathbf{Y}_{i+1}\right)^{-1} \tag{1}$$

where i refers to the ith medium layer, and X_i and Y_i are the frequency independent impedance matrices introduced by Schoenberg and Protazio(1992):

$$\mathbf{X}_i = \begin{pmatrix} \sin\theta_i^* & \cos\varphi_i^* \\ 2\rho'\beta_i \sin\varphi_i \sin\theta_i^* & 2\rho'\beta_i \cos\varphi_i^* \sin\varphi_i \\ -\cos(\theta_i^* - \theta_i)C_{33(i)}/\alpha_i & +\sin(\varphi_i^* - \varphi_i)C_{33(i)}/\beta_i \end{pmatrix}, \mathbf{Y}_i = \begin{pmatrix} -\sin(\theta_i^* + \theta_i)C_{55(i)}/\alpha_i & -\cos(\varphi_i^* + \varphi_i)C_{55(i)}/\beta_i \\ \cos\theta_i^* & -\sin\varphi_i^* \end{pmatrix}. \tag{2}$$

In the above equations, α_i and β_i are the phase velocities of the qP- and qS-wave, respectively; $\rho_i' = \left(C_{33(i)} - C_{13(i)}\right)/2\beta_i^2$ reduces to the density ρ_i for isotropic media; the polarization directions are denoted as θ_i^* (*qP*-wave) and φ_i^* (*qS*-wave), and $\sin\theta_i^* = (1+\zeta_{P(i)})\sin\theta_i$; $\sin\varphi_i^* = (1+\zeta_{S(i)})\sin\varphi_i$, where $\zeta_{P(i)} = \frac{C_{11(i)}}{(C_{11(i)} - C_{55(i)})}\left(\delta'\cos^2\theta_i - \varepsilon_P'\right)$ and $\zeta_{S(i)} = -\frac{C_{11(i)}}{(C_{11(i)} - C_{55(i)})}\left(\delta'\cos^2\varphi_i + \varepsilon_S' C_{55(i)}\cos^2\varphi_i/(C_{11(i)}\cos^2\varphi_i)\right)$, representing the polarization deviation from the propagation direction θ_i and φ_i : and $\delta' = (2C_{55(i)} + C_{13(i)} - C_{11(i)})/C_{11(i)}$, $\varepsilon_P' = \rho_i\alpha_i^2/C_{11(i)} - 1$, and $\varepsilon_S' = \rho_i\beta_i^2/C_{55(i)} - 1$. The determinants of the impedance matrices are $|\mathbf{X}_i| \approx C_{33(i)}\begin{vmatrix} \sin\theta_i^* & \cos\varphi_i^* \\ -\cos\theta_i \cos\theta_i^*/\alpha_i & \cos\varphi_i \sin\varphi_i^*/\beta_i \end{vmatrix}$, and $|\mathbf{Y}_i| \approx C_{55(i)}\begin{vmatrix} -\cos\theta_i \sin\theta_i^*/\alpha_i & -\cos\varphi_i \cos\varphi_i^*/\beta_i \\ \cos\theta_i^* & -\sin\varphi_i^* \end{vmatrix}$. In order to evaluate the effect of anisotropy on the AVO response and to invert for the elastic properties, we derive a simple expression for reflection coefficients in terms of the first-order elastic properties. Equation (1) can be approximated for media of small impedance contrast as

$$\mathbf{R}_i \approx \frac{1}{2}\left(\overline{\mathbf{X}}^{-1}\Delta\mathbf{X} - \overline{\mathbf{Y}}^{-1}\Delta\mathbf{Y}\right) \tag{3}$$

In equation (3), $^{-}$ and Δ denotes the average and change of the elastic properties of the *i*th and the (*i+1*)th medium layers, respectively.

Inversion of impedance matrix

The impedance matrices **X** and **Y** are functions of elastic parameters. They represent the complete reflection properties of the medium. If the amplitude of all types of reflection from P and S incident waves are recorded, $\overline{\mathbf{X}}^{-1}\Delta\mathbf{X}$ and $\overline{\mathbf{Y}}^{-1}\Delta\mathbf{Y}$ can be inverted from **R** based on equation (3). Then X and Y can be solved by the recursive matrix-inversion formula that extends the conventional recursive equation:

$$\mathbf{X_i} =_i \mathbf{X_0}\prod_{k=1}^{i}\mathbf{X_k}/\mathbf{X_{k-1}}, \text{ and } \mathbf{Y_i} =_i \mathbf{Y_0}\prod_{k=1}^{i}\mathbf{Y_k}/\mathbf{Y_{k-1}} \tag{4}$$

Linearization of Reflection coefficients

The matrices of impedance contrast $\Delta\mathrm{X}$ and $\Delta\mathrm{Y}$ can be derived by differentiating $\overline{\mathrm{X}}$ and $\overline{\mathrm{Y}}$ in each of the relevant variables:

$$d = \Delta\rho'\frac{\partial}{\partial\rho'} + \Delta C_{33}\frac{\partial}{\partial C_{33}} + \Delta C_{55}\frac{\partial}{\partial C_{55}} + \Delta C_{13}\frac{\partial}{\partial C_{13}} + \Delta\alpha\frac{\partial}{\partial\alpha} + \Delta\beta\frac{\partial}{\partial\beta} + \Delta\theta\frac{\partial}{\partial\theta} + \Delta\varphi\frac{\partial}{\partial\varphi} + \Delta\zeta_P\frac{\partial}{\partial\zeta_P} + \Delta\zeta_S\frac{\partial}{\partial\zeta_S} \tag{5}$$

The reflection coefficients of *qPqP*- and *qPqS*- waves are expressed by means of the average and contrast of the elastic properties of the adjacent media:

$$R_{qPqP} = \frac{1}{2}\left(\frac{\Delta C_{33}}{\overline{C_{33}}} - \frac{\Delta\alpha}{\overline{\alpha}}\right) + \frac{1}{2}\left[\frac{\Delta\alpha}{\overline{\alpha}} - \frac{2\overline{\rho'}\overline{\beta}^2}{\overline{C_{33}}}\left(\frac{\Delta\rho'}{\overline{\rho'}} + 2\frac{\Delta\beta}{\overline{\beta}}\right) - \frac{2\overline{\beta}^2}{\overline{\alpha}^2}\frac{\Delta C_{55}}{\overline{C_{55}}}\right]\sin^2\overline{\theta} + \frac{1}{2}\frac{\Delta\alpha}{\overline{\alpha}}\sin^2\overline{\theta}\tan^2\overline{\theta} + \frac{1}{2}\left(1 - \frac{2\overline{\beta}^2}{\overline{\alpha}^2}\right)\tan^2\overline{\theta}\Delta\zeta_P \tag{6}$$

$$R_{qPqS} = \frac{\sin\overline{\theta}}{\cos\overline{\varphi}}\left[-\frac{1}{2}\left(\frac{\Delta C_{33}}{\overline{C_{33}}} - 2\frac{\Delta\alpha}{\overline{\alpha}}\right) - \frac{\overline{\beta}}{\overline{\alpha}}\frac{\Delta C_{55}}{\overline{C_{55}}}\cos\overline{\theta}\cos\overline{\varphi} + \frac{\overline{\rho'}\overline{\beta}^2}{\overline{C_{33}}}\left(\frac{\Delta\rho'}{\overline{\rho'}} + 2\frac{\Delta\beta}{\overline{\beta}}\right)\sin^2\overline{\theta} + \frac{1}{2}\left(1 - \frac{\sin 2\overline{\varphi}}{\sin 2\overline{\theta}}\right)\Delta\zeta_P\right] \tag{7}$$

Simplified solution for weak VTI media

First, we present the reflection coefficients for a VTI medium simplified from equations (6) and (7). A VTI medium has a vertical symmetry axis. The anisotropy results from a sequence of thin layers (Bush and Crampin, 1991). For the isotropic media, $\overline{\rho'} = \overline{\rho_0}$, $\alpha = \alpha_0$, $\beta = \beta_0$ and $\Delta\zeta_P = \Delta\zeta_S = 0$; the average elastic constants and their contrast across the interface is given by $\overline{C_{33}} = \overline{\rho_0\alpha_0}^2$, $\overline{C_{55}} = \overline{C_{44}} = \overline{\rho_0\beta_0}^2$ and $\Delta C_{33}/\overline{C_{33}} = \Delta\rho_0/\overline{\rho_0} + 2(\Delta\alpha_0/\overline{\alpha_0})$, $\Delta C_{44}/\overline{C_{44}} = \Delta\rho_0/\overline{\rho_0} + 2(\Delta\beta_0/\beta_0)$. Considering weak VTI anisotropy, Thomsen (1986) proposed a set of parameters which are linked to practical seismic measurements:

$$\varepsilon_i = \frac{C_{11(i)} - C_{33(i)}}{2C_{33(i)}};\ \delta_i = \frac{(C_{13(i)} + C_{44(i)})^2 - (C_{33(i)} - C_{44i)})^2}{2C_{33(i)}(C_{33(i)} - C_{44i)})} \approx \frac{2C_{44(i)} + C_{13(i)} - C_{33(i)}}{C_{33(i)}};\ \gamma_i = \frac{C_{66(i)} - C_{44(i)}}{2C_{44(i)}} \tag{8}$$

and the phase velocities are expressed as

$$\alpha_i \approx \alpha_{0(i)}\left(1 + \delta_i\sin^2\theta_i\cos^2\theta_i + \varepsilon_i\sin^4\theta_i\right);\ \beta_i \approx \beta_{0(i)}\left[1 + \left(\alpha_{0(i)}/\beta_{0(i)}\right)^2(\varepsilon_i - \delta_i)\sin^2\varphi_i\cos^2\varphi_i\right]. \tag{9}$$

Other relevant variables can be written as $\rho_i' \approx \rho\left[1 - \frac{1}{2}\left(\alpha_{0(i)}/\beta_{0(i)}\right)^2\delta_i - 2\left(\alpha_{0(i)}/\beta_{0(i)}\right)^2(\varepsilon_i - \delta_i)\sin^2\varphi_i\cos^2\varphi_i\right]$,

$\Delta\zeta_P \approx \overline{\alpha_0}^2\left[\Delta\delta + 2(\Delta\varepsilon - \Delta\delta)\sin^2\overline{\theta}\right]\cos^2\overline{\theta}/(\overline{\alpha_0}^2 - \overline{\beta_0}^2)$ and $\Delta\zeta_S \approx \overline{\alpha_0}^2\left[\Delta\delta + 2(\Delta\varepsilon - \Delta\delta)\sin^2\overline{\varphi}\right]\cos^2\overline{\varphi}/(\overline{\alpha_0}^2 - \overline{\beta_0}^2)$.

Considering polarization deviations are small, impedance matrices and their contrast can be expressed as

$$\overline{\mathbf{X}} \approx \overline{\mathbf{X}^{(0)}} + \overline{\mathbf{X}^{(1)}},\ \overline{\mathbf{Y}} \approx \overline{\mathbf{Y}^{(0)}} + \overline{\mathbf{Y}^{(1)}},\ \Delta\mathbf{X} \approx d(\overline{\mathbf{X}^{(0)}}) + d(\overline{\mathbf{X}^{(1)}}), \text{ and } \Delta\mathbf{Y} \approx d(\overline{\mathbf{Y}^{(0)}}) + d(\overline{\mathbf{Y}^{(1)}}). \tag{10}$$

Noting that $\mathbf{X}^{(1)}$ and $\mathbf{Y}^{(1)}$ are small, we have

$$\mathbf{R}_i \approx \frac{1}{2}\left\{\overline{\mathbf{X}^{(0)}}^{-1}\left[\Delta\mathbf{X}^{(0)} + \Delta\mathbf{X}^{(1)}\right] - \overline{\mathbf{Y}^{(0)}}^{-1}\left[\Delta\mathbf{Y}^{(0)} + \Delta\mathbf{Y}^{(1)}\right]\right\}. \tag{11}$$

In the case of VTI media, the impedance matrices that can be inverted are now $\overline{\mathbf{X}^{(0)}}^{-1}\left[\Delta\mathbf{X}^{(0)}+\Delta\mathbf{X}^{(1)}\right]$ and $\overline{\mathbf{Y}^{(0)}}^{-1}\left[\Delta\mathbf{Y}^{(0)}+\Delta\mathbf{Y}^{(1)}\right]$. $\mathbf{X}^{(0)}$ and $\mathbf{Y}^{(0)}$ are the isotropic impedance matrices

$$\overline{\mathbf{X}^{(0)}}=\begin{pmatrix}\sin\bar{\theta} & \cos\bar{\varphi}\\ \bar{\rho}\bar{\alpha}(2\sin^2\bar{\varphi}-1) & \bar{\rho}\bar{\beta}\sin 2\bar{\varphi}\end{pmatrix} \text{ and } \mathbf{Y}_{\mathbf{i}}^{(0)}=\begin{pmatrix}-\bar{\rho}\bar{\beta}^2/\bar{\alpha}\sin 2\bar{\theta} & -\bar{\rho}\bar{\beta}\cos 2\bar{\varphi}\\ \cos\bar{\theta} & -\sin\bar{\varphi}\end{pmatrix}; \tag{12}$$

$$\Delta\mathbf{X}^{(0)}=\begin{pmatrix}\cos\bar{\theta}\tan\bar{\theta}\Delta\alpha/\bar{\alpha} & -\sin\bar{\varphi}\tan\bar{\varphi}\Delta\beta/\bar{\beta}\\ \sin^2\bar{\varphi}(2\bar{\alpha}\Delta\rho'+2\Delta\alpha+4\overline{\rho'}\bar{\alpha}\Delta\beta/\bar{\beta}) & \sin 2\bar{\varphi}(\bar{\beta}\Delta\rho'+\overline{\rho'}\Delta\beta)+\\ -\overline{C_{33}}/\bar{\alpha}(\Delta\alpha/\bar{\alpha}-\Delta C_{33}/\overline{C_{33}}) & 2\overline{\rho'}\bar{\beta}\cos 2\bar{\varphi}\tan\bar{\varphi}\Delta\beta/\bar{\beta}\end{pmatrix}, \Delta\mathbf{X}^{(1)}=\begin{pmatrix}\Delta\zeta_P\sin\bar{\theta} & -\Delta\zeta_S\cos\bar{\varphi}\tan^2\bar{\varphi}\\ 2\Delta\zeta_P\overline{\rho'}\bar{\alpha}\sin^2\bar{\varphi} & \Delta\zeta_S(2\overline{\rho'}\bar{\beta}\cos^2\bar{\varphi}+\overline{C_{13}}/\bar{\beta})\tan\bar{\varphi}\end{pmatrix},$$

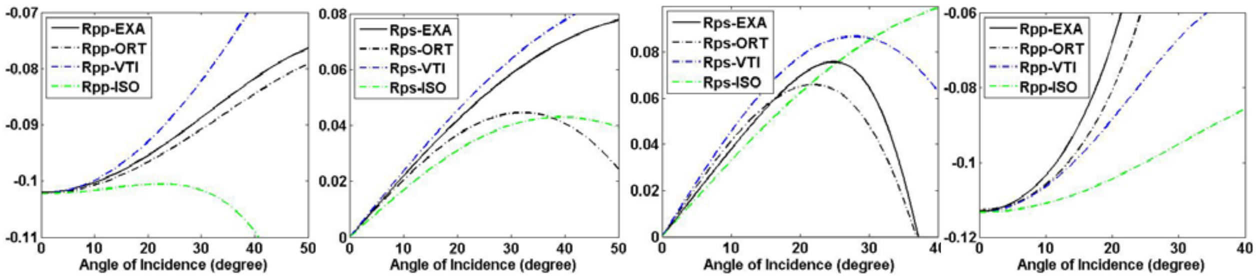
$$\Delta\mathbf{Y}^{(0)}=\begin{pmatrix}\overline{C_{55}}/\bar{\alpha}[2\tan\bar{\theta}\Delta\alpha/\bar{\alpha}- & \overline{C_{55}}/\bar{\alpha}[4\sin^2\bar{\varphi}\Delta\beta/\bar{\beta}-\\ \sin 2\bar{\theta}(\Delta\alpha/\bar{\alpha}+\Delta C_{55}/\overline{C_{55}}) & \cos 2\bar{\varphi}(\Delta\beta/\bar{\beta}+\Delta C_{55}/\overline{C_{55}})\\ -\sin\bar{\theta}\tan\bar{\theta}\Delta\alpha/\bar{\alpha} & -\cos\bar{\varphi}\tan\bar{\varphi}\Delta\beta/\bar{\beta}\end{pmatrix} \text{ and } \Delta\mathbf{Y}^{(1)}=\begin{pmatrix}-\Delta\zeta_P\overline{C_{55}}/\bar{a}\cos 2\bar{\theta}\tan\bar{\theta} & 2\Delta\zeta_S\overline{C_{55}}/\bar{\beta}\sin^2\bar{\varphi}\\ -\Delta\zeta_P\cos\bar{\theta}\tan^2\bar{\theta} & -\Delta\zeta_S\sin\bar{\varphi}\end{pmatrix}.$$

Substituting these equations into equations (6) and (7) yields

$$R_{qPqP(i)}(\bar{\theta})\approx R_{PP0(i)}(\bar{\theta})+\frac{1}{2}\Delta\delta\sin^2\bar{\theta}+\frac{1}{2}\Delta\varepsilon\sin^2\bar{\theta}\tan^2\bar{\theta} \tag{13}$$

$$R_{qPqS(i)}(\bar{\theta})\approx R_{PS0(i)}(\bar{\theta})+\frac{1}{2}\sin\bar{\theta}/\cos\bar{\varphi}\left[\Delta\delta\sin^2\bar{\theta}+2(\Delta\varepsilon-\Delta\delta)\sin^4\bar{\theta}\right]+\frac{1}{2}\left(\sin\bar{\theta}/\cos\bar{\varphi}-\sin\bar{\varphi}/\cos\bar{\theta}\right)\Delta\zeta_P \tag{14}$$

Figures 1 and 2 compare the exact *PP*- and *PS*-wave reflection coefficients with the different approximations introduced above. Two single-interface models with VTI anisotropy are used for the evaluation. The first model has moderate anisotropy (20% for P-wave velocity and 10% for S-wave velocity), while the second model has strong anistoropy (26% for P-wave velocity and 48% for S-wave velocity). For both cases, the generalized approximations of *qP-qP* (equation 6) agree well with the exact numerical results, even at a large angle of incidence. The reduced PP-wave approximation shows good accuracy when the angle is smaller than 30°. A similar situation is found in the case of R_{qPqS}, except the accuracy of the generalized approximation in Figure 1b is relatively lower.



(a) (b) (c) (d)

Figure 1 *(a) PP- and (b) PS-wave reflection coefficients for a VTI/VTI interface. The Thomsen parameters (ε,δ,and γ) for the upper and lower layers are 0.02, 0.02 and 0.0, and 0.15, 0.2 and 0.1, respectively. (c) PP- and (d) PS-wave reflection coefficients for a strong VTI interface. The Thomsen parameters (ε,δ,and γ) for the upper and lower layers are 0.256, -0.25 and 0.481, and 0.0, 0.0 and 0.0, respectively.The exact reflectivities (solid black) are compared with different approximations (equations (5) and (6) are dotted black curves; equations (7) and (8) are dotted green curves; equations (14) and (15) are dotted blue curves)*

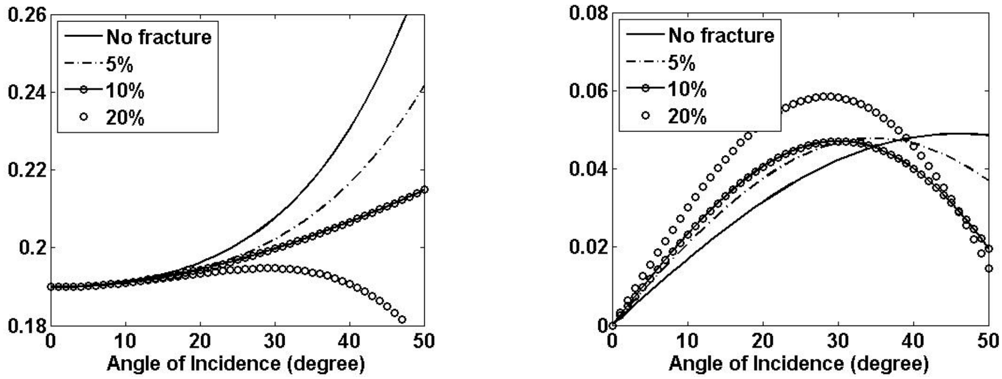
AVO response in fractured HTI media

A medium containing aligned vertical fractures gives rise to effectively HTI anisotropy. The derivation of the reflection coefficients for weak HTI media is similar to that of VTI media.

$$R_{qPqP(i)}(\bar{\theta})\approx R_{PP0(i)}(\bar{\theta})+\frac{1}{2}\left[\Delta\delta-2\Delta\varepsilon+8\left(\frac{\overline{\beta_0}}{\overline{\alpha_0}}\right)^2\Delta\gamma\right]\sin^2\bar{\theta}+\frac{1}{2}\Delta\varepsilon\sin^2\bar{\theta}\tan^2\bar{\theta} \tag{15}$$

$$\begin{aligned}R_{qPqS(i)}(\bar{\theta})\approx R_{PS0(i)}(\bar{\theta})&+\frac{1}{2}\sin\bar{\theta}/\cos\bar{\varphi}\Big[2\overline{\beta_0}/\overline{\alpha_0}\Delta\gamma\cos\bar{\theta}\cos\bar{\varphi}+(\Delta\delta-2\Delta\varepsilon)\sin^2\bar{\theta}\\&+4\left(\overline{\beta_0}/\overline{\alpha_0}\right)^2\Delta\gamma\sin^2\bar{\theta}+2(\Delta\varepsilon-\Delta\delta)\sin^4\bar{\theta}\Big]+\frac{1}{2}\left(\sin\bar{\theta}/\cos\bar{\varphi}-\sin\bar{\varphi}/\cos\bar{\theta}\right)\Delta\zeta_P\end{aligned} \tag{16}$$

where the Thomas parameters are defined in different way, due to the 90° rotation of the symmetry axis. In order to evaluate the effects of fractures on the multicomponent AVO response, we build a model with a overburden shale and four types of sand. The shale has weak VTI anisotropy (ε=0.01,δ=0,and γ=0.01); the gas sand is discussed in Gassmann (1951) and Castagna (1993). This model has a high/low impedance contrast, and is common for gas reservoirs (solid curve in Figure 3). Gas sands with different fracture density are simulated by Hudson's equation (Hudson 1981). The fracture-induced anisotropy reduces the amplitude of the *qP-qP* wave, especially for wide incident angles. In contrast, the amplitude of the *qP-qS* wave increases with increasing fracture density, from 0° up to nearly 35°).



(a) (b)

***Figure** 3 (a) PP- and (b) PS-wave reflection coefficients for four VTI/HTI interfaces. The overburden shale has weak VTI anisotropy (ε=0.01,δ=0,and γ=0.01). Gas sands with different fracture density from 5% to 20% are simulated.*

Conclusions

We have introduced a generalized form of the *qP-qP* and the *qP-qS* wave reflection coefficients for TI media in terms of impedance matrices. The equations are derived, based on the exact solution for orthrhombic media. Reduced equations for VTI and HTI media are derived by considering weak anisotropy. We use three single-interface models to evaluate the equations. The generalized equations have sufficient accuracy even for relatively strong anisotropy. Simplified equations for VTI media show good accuracy when the angle of incidence is small. We use the reflection coefficients for HTI media to evaluate the effects of fractures on gas sand AVO responses. The amplitude of the *qP-qP* wave at wide angle apertures, and that of *qP-qS* at small angle apertures, is sensitive to the presence of fractures.

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