Simultaneous inversion of elastic and anisotropy parameters for a clay-rich shale formation

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SUMMARY

Anisotropy has a great influence on seismic velocity and amplitude responses. Shale exhibits strongly intrinsic vertical transverse isotropy (VTI) due to the presence of clay mineral, kerogen, and microcracks. Therefore, inversion of seismic data from shale formations needs to estimate multiple parameters. In this paper, we propose a simultaneous inversion of anisotropic $q_Pq_P$-wave reflection coefficients. A quadratic expression is used to model the reflection coefficients of interfaces with strong anisotropy. The model space consists of different classes of parameters, including velocities, bulk density and anisotropy parameters. Practical approaches are proposed to tackle the dimensional difficulties of these parameter classes. This inversion method is demonstrated by an application to real seismic data from a shale formation in the Western Sichuan basin.
Introduction

Anisotropy has a great influence on seismic velocity and amplitude responses. Shale exhibits strongly intrinsic vertical transverse isotropy (VTI) due to the presence of clay mineral, kerogen, and microcracks. Therefore, inversion of seismic data from shale formations needs to estimate multiple parameters. In this paper, we propose a simultaneous inversion of anisotropic qPqP-wave reflection coefficients. A quadratic expression is used to model the reflection coefficients of interfaces with strong anisotropy. The model space consists of different classes of parameters, including velocities, bulk density and anisotropy parameters. Practical approaches are proposed to tackle the dimensional difficulties of these parameter classes. This inversion method is demonstrated by an application to real seismic data from a shale formation in the Western Sichuan basin.

The approximation of reflection coefficients in VTI media

Considering a planar interface separating two anisotropic media with arbitrary impedance contrast and orthorhombic symmetry, Schoenberg and Protazio (1992) presented the plane-wave reflection coefficient in terms of impedance matrices. Zhang et al. (2013) derived the approximation of qPqP- and qPqS-wave reflection coefficients for VTI media by means of a generalized simplifying framework. The expression of qPqP-wave is given as:

$$R_{qPqP}(\theta) \approx R_{pp0}(\theta) + \Delta \delta \sin^4 \theta + \frac{1}{2} \Delta \epsilon \tan^2 \theta \sin^4 \theta + \frac{1}{2} ((1-2\beta^2/\alpha^2) \tan^2 \theta \Delta \zeta_P),$$

(1)

where $R_{pp0}(\theta)$ is the linearized approximation of an isotropic reflection coefficient, $\alpha$ and $\beta$ are the phase velocities of qP- and qS-wave respectively, $\epsilon$ and $\delta$ are anisotropic parameters; $\Delta$ refers to averaging parameters of the upper and lower media, and $\Delta \zeta_P$ refers to differences across the interface; $\Delta \zeta_P$ depends on the deviation between the phase angle and the polarization angle

$$\Delta \zeta_P \approx -2\Delta \epsilon \left[ \Delta \delta + 2(\Delta \epsilon - \Delta \delta) \sin^2 \theta \right] \cos^2 \theta.$$

(2)

Figure 1 (a) An angle gather and Thomsen’s parameter logs for a shale-gas well. (b) Normalized picked amplitude values (red dots) at around 1645ms and their interpolated AVA curve (black curve). (c) Normalized theoretical anisotropic (black) and isotropic (green) AVA curves for comparison. (d) Comparison of the exact expression with different approximations.

Figure 2 Eigenvalues of the Fréchet matrix of the inverse problem using (a) classic linearized qPqP-wave equation (Rüger, 1997) and (b) new qPqP-wave linearized equation versus the maximum angle.
Equation (1) has a similar form to expressions presented in many relevant publications, which allows us to implement the inversion of different parameters. Because the second-order terms regarding the anisotropic parameters are retained, equation (1) shows high accuracy which is demonstrated by comparison with various expressions at strongly anisotropic interfaces (Zhang, 2013). One of the linearized approximations of $qPqP$-wave is derived in Shaw and Sen (2004)

$$R_{qPqP}(\tilde{\theta}) = R_{pp0}(\tilde{\theta}) + \frac{1}{2}\Delta\delta \sin^2 \tilde{\theta} + \frac{1}{2}\Delta\varepsilon \sin^2 \tilde{\theta} \tan^2 \tilde{\theta}$$

This equation agrees with Rüger’s expression (Rüger, 1997) for VTI media. It has generally high accuracy within the small angle range.

To demonstrate the accuracy of the expression, we use it to model the AVA (amplitude versus angle) of a real PP-wave angle gather (Figure 1a). The amplitude between $0^\circ$ to $35^\circ$ at around 1645ms (2953m) is picked and plotted with its interpolated AVA curve Figure 1(b). A poor match is seen between the interpolated amplitude and the exact isotropic AVA (green curve in Figure 1c) even within this small angle range, while the interpolated amplitude agrees with the anisotropic reflection coefficients (black curve in Figure 1c) calculated using the exact expression presented in Schoenberg and Protazio (1992). The elastic parameters used for AVA modelling are extracted from the well logs in Figure 1a, and the Thomsen’s parameters ($\varepsilon = 0.3$, $\delta = 0.05$) are estimated based on a rock physics model. For convenience of comparison, all the AVAs are normalized by the amplitude at normal incidence. The exact expression of anisotropic reflection coefficients is approximated by equation (1) and the classic VTI approximation ((Rüger, 1997; Shaw et al., 2004) in Figure 1d). Both the approximations have generally good accuracy within the range of small angle, but the classic approximation deviates from the exact solution at large angles.

**Viability of inversion using different approximations of reflection coefficients**

In this section, we discuss the viability of inversion using different approximations for elastic parameters and anisotropic parameters, in the case of VTI media. Considering a simple linear inverse problem using seismic reflection amplitudes, we have

$$\Delta d = F \Delta m,$$

where $\Delta d$ is the data residue, $\Delta m$ is a small model perturbation to be solved, and

$$F = [F^\alpha \ F^\beta \ F^\rho \ F^\delta \ F^\varepsilon ]$$

represents the sensitivity matrix with the Fréchet derivatives $F^i$ in terms of five model parameters $(\Delta\alpha/\alpha, \Delta\beta/\beta, \Delta\rho/\rho, \Delta\delta$ and $\Delta\varepsilon$) to be solved. We analyze the matrix $F$ estimated using different $qPqP$-wave approximations. The eigenvalues of different sensitivity matrices are compared in Figure 2, where Figure 2(a) and 2(b) corresponds to the results of using the classic equation and equation (1), respectively. The data ranges from vertical incidence to a maximum angle, and is sampled uniformly over the maximum angle range. Corresponding eigenvalues of $\Delta\alpha/\alpha$, $\Delta\beta/\beta$ and $\Delta\rho/\rho$ of both cases have similar value range, but the fourth and fifth eigenvalues show different situation. In Figure 2(a), both the fourth and the fifth eigenvalues are very small and do not increase with the maximum angle. The fourth eigenvalue in Figure 2(b) is much larger than that in 2(a), and it approaches the range of the first three eigenvalues increasing with the maximum angle; the fifth eigenvalue is still small. Although there is an obvious improvement in the fourth eigenvalue of $F$ estimated using the quadratic approximation, the condition number (the ratio of the maximum eigenvalue to the minimum eigenvalue) of the inverse problem is still large. It implies that in order to solve all the five parameters, we need to deal with different sensitivities of model parameters and apply an appropriate regularization to the inversion, even when using the quadratic approximation of the reflection coefficient.
Simultaneous inversion using approximations with high-order terms

The inversion of model parameters $m$ is considered as minimising the difference between observed $d(m)$ and synthetic data $d_0$, with a regularisation in terms of an initial model $m_0$

$$J(m) = \Delta d^T C_m^{-1} \Delta d + \Delta m^T C_m^{-1} \Delta m$$

(6)

where $\Delta d$ is the data residue $\Delta d = d(m) - d_0$; $\Delta m$ is the model perturbation vector, $\Delta m = m - m_0$; $C_m^{-1}$ represents the inverse data covariance matrix; $C_m$ is the model covariance matrix.

If the object function $J$ is a smooth function of the model parameters we can make a locally quadratic approximation about the current model $m_0$

$$J(m_0 + \Delta m) = J(m_0) + \nabla_m J(m_0) \Delta m + \frac{1}{2} \Delta m^T \nabla_m^T \nabla_m J(m_0) \Delta m$$

(7)

and the model perturbation vector is determined by minimizing above equation

$$\Delta m = (H + C_m^{-1})^{-1} g$$

(8)

where $H = F^T C_m^{-1} F$ is the Hessian matrix, $g = F^T C_m^{-1} \Delta d$ is the data gradient vector. The Hessian matrix is regularized by the model covariance matrix $C_m$, which can be estimated from well logs or the initial models. Equation (8) can be solved iteratively using the classic conjugate gradient method or the subspace method (Kennet et al., 1998). The problem remaining is how to balance the different sensitivities of the model parameters.

![Figure 3](image1.png)  
(a) Comparison of real values of five model parameters (blue) and their estimates (red) using equation (8).  
(b) Comparison of real values of five model parameters and their estimates using equation (10).

![Figure 4](image2.png)  
Figure 4 Comparison of well logs (black), initial models (grey) and estimates (red) of five model parameters: $\Delta \alpha / \alpha$, $\Delta \beta / \beta$, $\Delta \rho / \rho$, $\Delta \delta$ and from the top to the bottom.

We apply a practical approach by rescaling the Fréchet matrix with a weighting matrix $W$. The weighting matrix is introduced in Wang (1999) to balance the contributions of model parameters of amplitude and traveltime. It is modified here as

$$\text{diag}(W) = [w_\alpha I \quad w_\beta I \quad w_\rho I \quad w_\delta I \quad w_\epsilon I]$$

(9)

where the weighting factor $w_j$ ($j = \alpha, \beta, \rho, \delta, \epsilon$) represents the ratio of eigenvalues summation for the varied sub-matrix $F_j$. The model perturbation is now
\[
\Delta m = W (W^T HW + C_m^{-1})^{-1} W^T g
\]  
(10)

An numerical example resulting from the inversion of synthetic data is shown as
\[
\begin{align*}
w_\alpha & = 2.719 \\
w_\beta & = 1.626 \\
w_\rho & = 13.963 \\
w_\epsilon & = 74.264 \\
\end{align*}
\]

Figure 3 compares the solution of equation (8) and (10). An obvious improvement is seen in the estimate of \( \Delta \alpha \), which represents the P-wave anisotropy and usually has a large variation. Not all the model parameters could be equally well solved if using the unweighted inversion scheme. The weighting matrix can efficiently balance the sensitivities of different parameters and therefore stabilize the inversion.

### Inversion example using real data

The simultaneous inversion for elastic parameters is demonstrated using real seismic data (Figure 1a) from the Western Sichuan basin, southwest China. The shale formation of interest (2750m-3200m) contains a great quantity of clay and is characterized as strongly anisotropic. The inversion of real seismic data requires an appropriate initial model, especially for the anisotropy parameters. An initial model is obtained by the smoothing the well logs. The logs of anisotropy parameters (Figure 1b) are estimated using the volumetric fractions of minerals, porosity and estimated aspect ratio based on a rock physics model, and they are demonstrated in Zhang et al. (2014). Five datasets are used for the input with maximum angle 35°. Mixed-phase wavelets are estimated using well constraints for each dataset, and they are used to estimate the reflectivities. Inversion results for a single CDP near the borehole are shown in Figure 4. The inverted parameters (red dashed curves) are compared with well logs (black curves) and initial models (grey curves). All five parameters are equally well solved and the solution shows a good agreement with well logs.

### Conclusions

A simultaneous inversion is developed for elastic parameters and anisotropy parameters, where only \( qPqP \)-wave reflection coefficients are used in the inversion. We apply the approximation of reflection coefficient with high-order accuracy to a contrast in anisotropy parameters. The inverse problem is well regularized and weighted to solve the dimensional difficulties between these parameter classes. This inversion method is demonstrated using both synthetic data and real seismic data from a shale formation.

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### References


Wang, Y., 1999, Simultaneous inversion for model geometry and elastic parameters: Geophysics, 64, 182-190.
