A new approach to derive NMO velocity and moveout for seismic waves in HTI media

Hengchang Dai and Xiang-yang Li
British Geological Survey, Murchison House, West Mains Road, Edinburgh, EH9 3LA, UK

Summary

We have derived anisotropic moveouts of seismic waves in HTI media by a new approach. In this approach, we use the idea of rotating VTI media into HTI media to study the behavior of the velocity and moveout of seismic waves in HTI media. This is achieved by developing a relationship between the incident angle and azimuth angle in HTI media and the angle to the symmetry axis in VTI media. In this approach, we firstly derive two general definitions which easily define the NMO velocity and the quadratic parameter in VTI media. Based on these definitions and the relationship between the angles of VTI and HTI media, we derived the exact formula of NMO velocities and anisotropy parameters of P, SV and SH waves in HTI media. The NMO velocities in HTI media obtained in this approach are exactly elliptical for any strength anisotropy. Based on the NMO velocity and anisotropy parameters, we also derived the exact format of moveout for P and SV waves. For weak HTI media, a unified moveout for P and SV waves is obtained.
Introduction

In recent years, multi-component 3D seismic data have demonstrated their usefulness for characterising fractured reservoirs. In fractured reservoirs, fractures are generally aligned in a preferred direction which depends on the stress history, and this often gives rise to horizontal transverse isotropy (HTI). Knowing the direction of fractures as a priori information before drilling is especially useful for the placement of horizontal or deviated wells. Many theoretical and field studies have shown that azimuthal variation in wavefield attributes (such as velocity and amplitude) can be used for fracture detection. The azimuthal variation of anisotropic moveout is one of the most useful attributes for detecting natural fractures and is mainly determined by the NMO velocity. For weak HTI media, Tsvankin (1997) has derived that the NMO velocity of single mode waves (P and S wave) is elliptical in the azimuthal plane and its maximum is in the direction of fracture strike. Al-Dajani and Tsvankin (1998) have derived the azimuthal variations of the high order term used in the azimuthal moveout for weak anisotropy. However, most applications use only the velocity variation in the moveout. In this paper, we have reinvestigated this using a different approach. Firstly we have derived a general rule which can be used to derived the NMO velocity and anisotropy parameters. Secondly, we consider the HTI medium by rotating the symmetry direction of a VTI medium from vertical to horizontal and derive a relationship between the angles in HTI and in VTI. Based on this relationship, we have derived NMO velocity, and anisotropy parameters for HTI media. For weak HTI anisotropy, we have obtained a unified format of moveout for P and SV waves.

General definitions of NMO velocity and anisotropy parameters in VTI media

In VTI media, the phase velocity of P, SV and SH waves are written as (Thomsen, 1986):

\[ v_p^2 = \alpha_0^2 [1 + \varepsilon \sin^2 \theta + D^*(\theta)] \]  
\[ v_s^2 = \beta_0^2 [1 + \alpha_0^2 \varepsilon \sin^2 \theta - \frac{\alpha_0^2}{\beta_0^2} D^*(\theta)] \]  
\[ v_{sh}^2 = \beta_0^2 [1 + 2\gamma \sin^2 \theta] \]

\[ D^*(\theta) = \frac{1}{2} \left(1 - \frac{\alpha_0^2}{\beta_0^2}\right) \left[1 + \frac{4\delta^*}{(1 - \beta_0^2/\alpha_0^2)^2} \sin^2 \theta \cos^2 \theta + \frac{4(1 - \beta_0^2/\alpha_0^2)\varepsilon}{(1 - \beta_0^2/\alpha_0^2)^2} \sin^4 \theta - 1\right] \]  

where \( \alpha_0 \) and \( \beta_0 \) are the vertical velocity of P and S waves. \( \varepsilon, \gamma, \text{ and } \delta^* \) are Thomsen’s parameters which indicate the anisotropy of the medium. \( \theta \) is the phase angle. Note that the above equations are based on the phase angle. In practice, we can only observe the ray angle (group angle) (Figure 1). The relationship between the phase angle and the group angle is (Berryman, 1979)

\[ \tan(\varphi(\theta)) = [\tan \theta + \frac{1}{v} \frac{dv}{d\theta}] / [1 - \frac{\tan \theta}{v} \frac{dv}{d\theta}]. \]  

Berryman (1979) also shows that the scalar magnitude of the group velocity is given in terms of the phase velocity magnitude by

\[ V^2(\varphi(\theta)) = v^2(\theta) + \left(\frac{dv(\theta)}{d\theta}\right)^2 \]  

In reflection seismology, normal moveout (NMO) describes the effect that the distance between a seismic source and a receiver (the offset) has on the arrival time of a reflection in the form of an increase of time with offset. The phase velocity cannot be directly used to measure the normal moveout. To overcoming this problem, one general method is to write it in the form of traveltime VS offset: \( t^2(x^2) \). It may be approximated by the modified Taylor series expansion (Tsvankin and Thomsen, 1994, Tsvankin and Thomsen (1995)) as Equation (1).
\[ t^2 = A_0 + A_2 x^2 + \frac{A_4 x^4}{1 + A_5 x^2}, \]  \hspace{1cm} (7)

where \( A_0 = i_0^2, \ A_2 = \left(\frac{d^2 r}{dx^2}\right)_{t=0}, \ A_4 = \frac{1}{2} \left(\frac{d^2 r}{dx^2}\right)_{t=0}, \) and \( A_5 = \frac{A_4}{1/V_h^2 - 1/V_{nmo}^2}. \) For the phase velocity in the format of

\[ v^2(\theta) = A[1 + B \sin^2 \theta + C \sin^4 \theta + D \sin^6 \theta \sin^2 f(\theta)] \]  \hspace{1cm} (8)

we have derived that

\[ A_2 = \frac{1}{V_{nmo}^2} = \frac{1}{A(1+B)} \]  \hspace{1cm} (9)

\[ A_4 = \frac{1}{2} \frac{d}{dx^2} \left(\frac{d^2 r}{dx^2}\right)_{t=0} = -\frac{1}{A^2(1+B)\gamma^2(0)(1+B)^2} \]  \hspace{1cm} (10)

The high order terms in Equation (8) do not have any contribution to them.

**Relationship between the seismic ray angles in VTI and HTI media**

The HTI medium can be considered by rotating the symmetry direction from vertical to horizontal (Figure 2). Although, physically, the behaviours of seismic waves propagated in VTI and HTI media are the same, the measurement of the seismic waves are different for VTI and HTI media due to the difference of the orientation of the symmetry direction. The velocities and moveout in HTI media can be obtained from those in VTI media by rotating the coordinates. The relationship between the angle (\( \omega \)) of the seismic ray to the symmetry axis in VTI media and the incident angle (\( \theta \)) and the azimuthal angle (\( \varphi \)) of the seismic ray in HTI media can be derived as (Figure 3)

\[ \cos^2 \omega = \sin^2 \theta \sin^2 \varphi \]  \hspace{1cm} (11)

Then based on this relationship, we derive the behaviour of velocities and moveout in HTI media.

**Moveouts and NMO velocities in HTI media**

To derive the velocity variation of the seismic wave in HTI media, first we substitute Equation (11) in Equations (1), (2), (3) and (4), so that we have

\[ v_p^2(\theta, \varphi) = \alpha_0^2[1 + \epsilon(1 - \sin^2 \theta \sin^2 \varphi) + D^*(\theta, \varphi)] \]  \hspace{1cm} (12)

\[ v_h^2(\theta, \varphi) = \beta_0^2 \left[1 + \frac{\alpha_0^2}{\beta_0^2} \epsilon(1 - \sin^2 \theta \sin^2 \varphi) - \frac{\alpha_0^2}{\beta_0^2} D^*(\theta, \varphi)\right] \]  \hspace{1cm} (13)

\[ v_s^2(\theta, \varphi) = \alpha_0^2[1 + 2\gamma(1 - \sin^2 \theta \sin^2 \varphi)] \]  \hspace{1cm} (14)

\[ D^*(\theta, \varphi) = \frac{1}{2} R \left[1 + 4\delta^2 \sin^2 \theta \sin^2 \varphi(1 - \sin^2 \theta \sin^2 \varphi) + \frac{4(R + \epsilon)\delta}{R^2}(1 - \sin^2 \theta \sin^2 \varphi)^2 - 1\right] \]  \hspace{1cm} (15)

Then take the Taylor series for \( D^*(\theta, \varphi) \) and substitute it in Equations (12) and (13), and we have
For the SH wave, we have derived that

$$v_{3\text{sham}(\varphi)}^2 = \beta_0^2 (1 + 2\gamma)[1 - \frac{2\gamma}{1 + 2\gamma} \sin^2 \varphi]$$

(18)

and

$$A_{sh4} = 0.$$  

(19)

Equation 19 means the moveout for the SH wave is hyperbolic. For P and SV waves, we have derived that

$$v_{\text{pmno}}^2(\varphi) = \alpha_0^2 (1 + 2\epsilon)[1 + 2\frac{(\delta - 2\epsilon)}{1 + 2\epsilon}] \sin^2 \theta \sin^2 \varphi + \frac{\mu}{(1 + 2\epsilon)} \sin^4 \theta \sin^4 \varphi + O(\sin^3 \theta \sin^2 \varphi)$$

(16)

$$v_{sv}^2 = \alpha_0^2 \left[1 - 2\frac{\alpha_0^2}{\beta_0^2} \frac{1 - 2\epsilon(\delta - \epsilon)}{(R + 2\epsilon)} \sin^2 \theta \sin^2 \varphi - \frac{\alpha_0^2}{\beta_0^2} \mu \sin^2 \theta \sin^2 \varphi + O(\sin^3 \theta \sin^2 \varphi)\right]$$

(17)

where

$$\mu = 2(\epsilon - \delta) + 2\frac{6\epsilon^2 - 2\epsilon\delta + 2\delta^2}{(R + 2\epsilon)} - 4\epsilon^2 + 5\delta^2 + 16\delta^2 \frac{1}{(R + 2\epsilon)^2} + 16\epsilon^2 \frac{5\epsilon^2 - 2\epsilon\delta + \delta^2}{(R + 2\epsilon)^2}.$$ 

For the SH wave, we have derived

$$v_{\text{sham}}^2(\varphi) = \beta_0^2 (1 + 2\gamma)[1 - \frac{2\gamma}{1 + 2\gamma} \sin^2 \varphi]$$

(18)

and

$$A_{sh4} = 0.$$  

(19)

Equation 19 means the moveout for the SH wave is hyperbolic. For P and SV waves, we have derived that

$$v_{\text{pmno}}^2(\varphi) = \alpha_0^2 (1 + 2\epsilon)[1 + 2\frac{(\delta - 2\epsilon)}{1 + 2\epsilon}] \sin^2 \theta \sin^2 \varphi + \frac{\mu}{(1 + 2\epsilon)} \sin^4 \theta \sin^4 \varphi$$

(20)

$$v_{sv}^2(\varphi) = \beta_0^2 \left[1 - 2\frac{\alpha_0^2}{\beta_0^2} \frac{(\delta - \epsilon)}{(R + 2\epsilon)} \sin^2 \varphi + \frac{2\epsilon(\delta - \epsilon)}{(R + 2\epsilon)} \sin^2 \varphi\right]$$

(21)

$$A_{p4} = -\frac{1}{V_{\text{pmno}}^4 t_{p0}^2} \frac{\mu}{(1 + 2\epsilon)} \sin^4 \varphi$$

(22)

$$A_{sv4} = -\frac{1}{V_{\text{pmno}}^4 t_{s0}^2} \frac{-(\alpha_0^2/\beta_0^2) \mu \sin^4 \varphi}{(1 + 2\epsilon)}$$

(23)

$$A_x = \frac{A_4}{1/V_h^2 - 1/V_{nmo}^2}$$ is complex. However for weak VTI media, we can approximately define

$$\eta(\varphi) \approx \frac{(\epsilon - \delta)/(1 + 2\epsilon) \sin^4 \varphi}{[1 + 2(\delta - 2\epsilon)/(1 + 2\epsilon) \sin^2 \varphi]}$$

and

$$\zeta(\varphi) \approx \frac{-(\alpha_0^2/\beta_0^2)(\epsilon - \delta) \sin^2 \varphi}{[1 + 2(\alpha_0^2/\beta_0^2)(\epsilon - \delta) \sin^2 \varphi]}.$$ Then we have

$$A_{p5} = \frac{1 + 2\eta(\varphi)}{V_{\text{pmno}}^2 t_{p0}^2}$$ and

$$A_{sv5} = \frac{1 + 2\zeta(\varphi)}{V_{\text{svmo}}^2 t_{s0}^2}.$$ Then the moveouts are derived as:

$$t_p^2 = t_{p0}^2 + \frac{x^2}{V_{\text{pmno}}^2(\varphi)} - \frac{2\eta(\varphi)x^4}{V_{\text{pmno}}^4(\varphi)V_{\text{pmno}}^2(\varphi)t_{p0}^2 + (1 + 2\eta(\varphi))x^2}$$

(24)

$$t_s^2 = t_{s0}^2 + \frac{x^2}{V_{\text{svmo}}^2(\varphi)} - \frac{2\zeta(\varphi)x^4}{V_{\text{svmo}}^4(\varphi)V_{\text{svmo}}^2(\varphi)t_{s0}^2 + (1 + 2\zeta(\varphi))x^2}$$

(25)

Note that the NMO velocities we derived in this paper are exactly elliptical for all strengths of anisotropy. They are different from the results obtained by Tsvankin (1997). Also $A_{p4}$ and $A_{sv4}$ in this paper are slightly different from those obtained by Al-Daiani and Tsvankin (1998). The anisotropy parameters for P and SV waves vary with the azimuth angle. For $\varphi = 0^\circ$ (the fracture direction), $\eta(0^\circ) = 0$ and $\zeta(0^\circ) = 0$; so the moveouts are hyperbolic. Note that as the azimuth angle increases from $0^\circ$ to $90^\circ$, $\sin^4 \varphi$ increases slowly at the beginning and rapidly at the end. So the anisotropy parameters also follow this trend: increasing slowly in the beginning and then rapidly at the end. For
\( \varphi = 90^\circ \), the absolute values of both anisotropy parameters reach a maximum. This phenomenon implies that the anisotropy parameters have lesser effects on the moveouts in the direction close to the fracture direction and larger effects in the directions close to the fracture normal direction. When the offset is large, we may not be able to ignore it. The format of the moveout for P and SV waves is the same for weak HVI media. For the SV wave, the moveout is different from that obtained by Yuan (2002). Figure 3 shows the accuracy of the moveouts. It is an example for the direction: \( \varphi = 90^\circ \).

![Figure 3: The residual moveout \((\Delta t = t - t_{true})\) of P and SV waves in HTI media. \( \alpha = 2000 m/s \), \( b = 1000 m/s \), \( \epsilon = 0.15 \), \( \delta = 0.1 \). The red line is residual moveout calculated using Equations (24) and (25). The black line in (b) is the residual moveout calculated using the Yuan (2002) formula.](image)

**Conclusions**

In this paper, we use the idea of rotating VTI media into HTI media to study the behavior of the velocity in HTI media. This is achieved by developing a relationship between the incident angle and azimuth angle in HTI media and the angle to the symmetry axis in VTI media. Firstly we derived two general definitions which can easily define the NMO velocity and the quadratic parameter in VTI media. Based on these definitions and the relationship between the angles in VTI and HTI media, we derived the exact formula of NMO velocities of P, SV and SH waves and anisotropy parameters in HTI media. NMO velocities in HTI media are exactly elliptical for any strength anisotropy. The anisotropic parameters also vary with azimuthal angle. The exact forms of moveout for P and SV waves are derived by using the NMO velocity and anisotropic parameters. For weak HTI media, we have derived a unified moveout for P and SV waves.

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**References**


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