An improved strategy for exact Zoeppritz equations AVA inversion

Shuangquan Chen\textsuperscript{1,2,*}, Lixia Zhi\textsuperscript{1,2,4}, Xiang-Yang Li\textsuperscript{3}

1. State Key Laboratory of Petroleum Resources and Prospecting, China University of Petroleum (Beijing) 102249, People’s Republic of China
2. CNPC Key Laboratory of Geophysical Prospecting, China University of Petroleum (Beijing) 102249, People’s Republic of China
3. British Geological Survey, Murchison House, West Mains Road, Edinburgh EH9 3LA, UK
4. College of Science, China University of Petroleum (Beijing) 102249, People’s Republic of China

Summary

The intention of this paper is to implement the Zoeppritz equations inversion using a nonlinear inversion strategy to obtain a more accurate density parameter from the prestack inversion and apply it to real field data. An iterative regularizing Levenberg-Marquardt (IRLM) scheme for Zoeppritz-based prestack inversion is presented, by incorporating the Tikhonov regularization method. It regularizes the inversion problem within an algorithm that minimizes the misfit between the observed and the synthetic data at the same time. The method allows us to stabilize the inversion without the use of a priori information or a covariance or scale matrix. Considering the difficulty to obtain accurate prior information, the method gives a valuable and interesting alternative. Numerical examples show that the inversion strategy is a reliable method for accurately estimating the density parameter as well as the P- and S-wave velocities for data with different noise levels. In particular, we also observe that as the noise level decreases, more accurate estimations are obtained. The IRLM scheme provides a robust and promising method for estimating elastic parameters (especially for density estimation) using nonlinear prestack inversion.
Introduction

For the prestack seismic inversion, we mainly use amplitude versus angle (AVA) or offset (AVO) information to invert the elastic parameters of subsurface media. The major aim of AVO inversion is to retrieve the subsurface media parameters by minimizing the misfit between the prediction and the observed seismic dataset, and it is playing an increasingly important role for hydrocarbon detection and lithology identification (Chopra and Castagna, 2007). Considering a plane wave that arrives at an interface between two adjacent media, AVA changes can be described by the exact Zoeppritz equations (Zoeppritz, 1919), which depict the exact reflection and transmission coefficients as a function of incidence angle and six seismic parameters (i.e., P- and S-wave velocities, and densities of the upper and lower media). Prestack AVO inversion based on the exact Zoeppritz equations is becoming a research hotspot for geophysicists. It is a typically nonlinear, multi-parameter inversion problem, complex in mathematical form and highly nonlinear (Zhu and McMechan, 2012; Zhi et. al., 2013; Zong et. al., 2013).

It is well known that the prestack AVO inversion problem is an ill-posed and nonlinear mathematical problem, which implies that a small perturbation in the data may result in a large perturbation in the estimated parameters. This is particularly true when one attempts to estimate the density term. The lack of stability indicates that we cannot just use standard optimization schemes to solve it. To stabilize the inversion, one way is via the inclusion of a covariance matrix or a scale matrix that provides correlation information between the unknown AVO parameters under the classical Bayesian framework. In general, the covariance matrix is only known when we have borehole measurements or a good understanding of the expected AVO classes in the area of study. Moreover, the associated term of the scale matrix can be seen as a regularization term that alleviates the ill-posedness of the inverse problem, but the choice of scale matrix is usually based on purely geological data which has no connection with the stability and fidelity issues of the inverse problem. Therefore, for some unsuitable choices of scale matrix, a risk of instabilities and lack of fidelity can arise.

Developing an effective and stable strategy for nonlinear prestack inversion is very important. In this paper, we incorporate the Tikhonov regularization method in the LM scheme and propose an iterative regularizing LM scheme (IRLM) for nonlinear prestack AVO inversion, which is different to other approaches where the inversion problem is firstly regularized and then optimized with a standard solver. In addition, instead of inverting the velocity and density reflectivity as done in most linearized AVO inversions based on the Bayesian regularization method, we directly invert the true parameter values of P- and S-wave velocities and density, so that no further accumulation error is induced. The performance of IRLM is demonstrated by a synthetic example using well-logging data, which shows that the method is robust for generating accurate estimates of unknown subsurface parameters given seismic data with low noise, especially of densities.

Methodology and strategy

Assuming an elastic model of two semi-infinite isotropic homogeneous spaces in welded contact, the conversion of one elastic wave, either P-wave or S-wave, into another upon reflection or transmission is described by the Zoeppritz equations (Aki and Richards, 1980), which can be written in matrix form as

\[ AR = B, \]

where,

\[
A = \begin{bmatrix}
\sin \alpha & \cos \beta & -\sin \alpha' & \cos \beta' \\
\cos \alpha & -\sin \beta & \cos \alpha' & \sin \beta' \\
\cos 2\beta & -\frac{v_{S1}^2 \sin 2\beta}{\rho_{p1}} & -\rho_{S2} \frac{v_{S2}^2 \cos 2\beta}{\rho_{p1}} & -\rho_{S2} \frac{v_{S2}^2 \sin 2\beta}{\rho_{p1}} \\
\sin 2\alpha & \frac{v_{S2}^2 \cos 2\beta}{\rho_{p1}} & \rho_{S2} \frac{v_{S2}^2 \sin 2\beta}{\rho_{p1}} & -\rho_{S2}^2 \frac{v_{S2}^2 \sin 2\beta}{\rho_{p1}} \\
\end{bmatrix},
\]
\[ \mathbf{R} = \begin{bmatrix} R_{PP} & R_{PS} & T_{PP} & T_{PS} \end{bmatrix}^T, \quad \mathbf{B} = \begin{bmatrix} -\sin \alpha & \cos \alpha & -\cos 2\beta & \sin 2\alpha \end{bmatrix}^T. \]

In the equation, \( v_{p1}, v_{p2}, v_{s1}, v_{s2}, \rho_1, \rho_2 \) are the P-wave velocity, S-wave velocity and density of the upper and lower layers across the interface, \( \alpha \) and \( \alpha' \) are the incidence and transmission angles of the P-waves, and \( \beta \) and \( \beta' \) are the reflection and transmission angles of the S-waves. \( R_{PP} \) and \( R_{PS} \) are P- and S-wave reflection coefficients, \( T_{PP} \) and \( T_{PS} \) are P- and S-wave transmission coefficients, respectively.

For seismic exploration, Zoeppritz equations describe the prestack reflection coefficients, depending on the incident angle and elastic parameters of the subsurface, which makes it possible for us to extract these parameters from the observed seismic data. Assuming that the earth structure can be represented by a series of horizontal layers of constant material properties separated by planar interfaces, the earth model can be represented by a discretized model parameter vector \( \mathbf{m} \). According to the convolution model, given the seismic trace in discrete form \( \mathbf{d}_{\text{obs}} \), the seismic forward model can be expressed as follows:

\[ \mathbf{d}_{\text{obs}}(\theta) = \mathbf{w} \ast \mathbf{r}(m, \theta) + \mathbf{e}(\theta), \]

where \( m = [v_{p1}, v_{p2}, \ldots, v_{pN}, v_{s1}, v_{s2}, \ldots, v_{sN}, \rho_1, \rho_2, \ldots, \rho_N] \), subscript \( N \) is the number of layers and superscript \( T \) denotes transverse operator, \( \mathbf{d}_{\text{obs}}(\theta) \) is the seismic trace corresponding to the incidence angle \( \theta \), and \( \mathbf{w} \) is the source wavelet, \( \mathbf{r}(\theta) \) is the reflectivity coefficient series, \( \mathbf{e}(\theta) \) is the random noise, and the asterisk denotes the convolution operator. We define the model data as

\[ \mathbf{G}(m, \theta) = \mathbf{w} \ast \mathbf{r}(m, \theta). \]

The aim of the prestack AVA inversion is to estimate \( m \) from \( \mathbf{d}_{\text{obs}} \) (Tarantola, 1986). Classical seismic inversion is often formulated as optimization of an objective function \( \mathbf{F} \), e.g., the squared data error

\[ \mathbf{F}(m) = \frac{1}{2} \| \mathbf{d}_{\text{obs}} - \mathbf{G}(m) \|^2. \]  

Equation (4) can also be derived from the perspective of a Bayesian framework. If the noise term \( \mathbf{e}(\theta) \) is assumed to be zero mean Gaussian with covariance \( \mathbf{\Sigma} \), that is \( \mathbf{e}(\theta) \sim \mathcal{N}(0, \mathbf{\Sigma}) \). The maximum likelihood solution of (4) is defined by minimizing as

\[ \mathbf{F}(m) = \frac{1}{2} (\mathbf{d}_{\text{obs}} - \mathbf{G}(m))^T \mathbf{\Sigma}^{-1} (\mathbf{d}_{\text{obs}} - \mathbf{G}(m)) = \frac{1}{2} (\| \mathbf{d}_{\text{obs}} - \mathbf{G}(m) \|^2_{\mathbf{\Sigma}^{-1}}. \]

This is identical to a weighted least square problem. If the noise term represents white noise such that \( \mathbf{\Sigma} = \sigma^2 \mathbf{I} \), the problem reduces to an ordinary nonlinear least square problem. Considering the reflection coefficient is a nonlinear function in term of unknown parameters \( m \), the prestack inversion is a typically nonlinear, multi-parameter inversion problem.

Equation (4) or (5) is valid for a single prestack gather. For \( n \) prestack gathers, the objective function, also known as the data misfit cost function which measures the mismatch between the observed seismic data and the predicted data, is the summation over all traces for the PP data \((i=1,2,\ldots,n)\) as given in the following equation

\[ \arg \min_m \mathbf{F}(m) = \frac{1}{2} \| (\mathbf{d}_{\text{obs}}(\theta) - \mathbf{w}(\theta) \ast \mathbf{r}(m, \theta)) \|^2_{\mathbf{\Sigma}^{-1}} = \frac{1}{2} \sum_{i=1}^{n} \| (\mathbf{d}_{\text{obs}}(\theta) - \mathbf{G}(m, \theta)) \|^2_{\mathbf{\Sigma}^{-1}}, \]

where \( \mathbf{w}(\theta) \) in the equation (6) means that we can use a different wavelet for each of the P-wave angle stacks.

Equation (6) is a highly nonlinear and ill-posed problem, so that generating an accurate and stationary solution in the case of noisy observed data \( \mathbf{d}_{\text{obs}} \) is not easy work to do. We proposed an iterative regularizing Levenberg-Marquardt (IRLM) inversion strategy to solve the nonlinear AVO inversion problem (equation 6). In contrast to other approaches where the problem is first regularized (e.g., under Bayesian framework or by Tikhonov’s method) and then optimized with a standard solver, we adopt an iterative regularization technique, where the aim is to regularize the problem within an algorithm that also provides an approximation to a minimizer of equation (6) (Kaltenbacher et. al., 2008).
Numerical example

We present a numerical example to show the capabilities of the IRLM scheme to invert directly the P- and S-wave velocities, and density parameters using the prestack seismic gathers. The numerical model is extracted from real field well-logging data, shown in Figure 1 including P-, S-wave and density curves. The synthetic seismic prestack gathers are generated in the angle domain by convoluting the prestack P-wave reflectivity calculated using the exact Zoeppritz equations and a zero-phase Ricker wavelet. In the inversion processing, we use the seismic record time ranges from 1508 to 2420ms with a sampling interval of 2ms, and the range of the prestack incident angle is from 10 to 45 degrees.

Figure 2a shows the inversion results using the IRLM scheme, and figure 2b shows the inversion results using the iterative regularizing LM scheme is superior to the conventional GLI technique for nonlinear AVA inversion based on the exact Zoeppritz equation. This verifies that we can obtain more reliable elastic parameters, especially for density, using the IRLM scheme than the conventional GLI scheme in exact Zoeppritz equations inversion processing.

For testing the noiseproof feature of the IRLM method in the prestack seismic inversion, we add Gaussian noise with different noise levels into the synthetic gathers. Different signal-to-noise ratio (S/N) synthetic gathers are used to invert the elastic parameters, as shown in Figure 3a-c. The correlations between the inverted and the real model for P-velocity and S-velocity for different noise levels are all very pretty good, as illustrated in Figure 3, facilitating visualization. The correlation for density is relatively poor, but in general it is still in good agreement with the true model.

Figure 1. Three well-logging curves extracted from one of the study area wells, showing P- and S-wave velocities and density from left to right.

Figure 2. Comparison of the inversion results (black dotted line), initial model (grey line) and true model (black line). Three parameters are inverted and illustrated in the figure, showing P-wave velocity (left panel), S-wave velocity (middle panel) and density (right panel). (a) IRLM and (b) GLI schemes are used for the exact Zoeppritz equations inversion.
ar prestack AVA inversion. As shown in the numerical examples, the proposed method allows us to stabilize the inversion without the use of a priori information or the use of a covariance or scale matrix. The iteratively regularizing Levenberg-Marquardt scheme provides a robust and promising method for estimating elastic parameters (especially density) for nonlinear prestack inversion.

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