Inverting anisotropy parameters using split PS converted waves in HTI media

Hengchang Dai and Xiang-yang Li,*
British Geological Survey, Edinburgh UK, EH9 3LA

We propose an approach to use the azimuth variation of velocity of split PS converted waves in HTI media to invert for the Thomsen parameters of the HTI media. We consider an HTI medium as a rotation of the symmetry axis of a VTI medium from vertical to horizontal, and derive formulas for all vertical and NMO velocities of the P, S1 and S2 waves in the HTI medium in terms of the Thomsen parameters of the original VTI medium and the azimuthal angle. Based on the results for P, S1 and S2 waves, we found that the fast PS converted wave is the combination of P and S1 waves, and the maximum of its NMO velocity is in the direction parallel to the fracture strike direction. The slow PS converted wave is the combination of P and S2 waves, and the maximum of its NMO velocity can be in the directions parallel or perpendicular to the fracture direction, depending on the anisotropy parameters of the fractured medium. This work also shows that the Thomsen parameters of the HTI medium can be inverted for, by using the variation of NMO velocities and the vertical velocities of split PS waves. This work creates a theoretical foundation which can be used to develop a practical approach to perform this inversion.

Introduction

In recent years, multi-component 3D seismic data have demonstrated their usefulness for characterizing fractured reservoirs. In fractured reservoirs, fractures are generally aligned in a preferred direction which depends on the stress history, and this often gives rise to horizontal transverse isotropy (HTI). In a HTI medium, a shear wave splits into two orthogonally polarized shear waves (qS1 and qS2). Hence the PS converted wave also splits into two waves (PqS1 and PqS2) which can provide more information about the HTI medium.

Split PS-converted waves have been intensively studied for the prediction of fracture orientation and density (Simmons, 2009; Cheng et al., 2009; Yue et al., 2013), to improve fracture characterization (Mattocks et al., 2005; Dai et al, 2011; Liu et al., 2011) and to distinguish dry fractures from water-saturated fractures (Liu et al, 2014). In this paper, we propose an approach to invert for the anisotropy parameters of a HTI medium using the information obtained from the split shear waves. This work is based on the idea that the HTI medium can be considered as a rotation of the symmetry axis of a VTI medium from vertical to horizontal and then all the vertical and NMO velocities of the seismic waves in the HTI medium can be defined by the Thomson's parameters of the original VTI medium.

Relationship between the angles of seismic ray in VTI and HTI media

The HTI medium can be considered as a rotation of the symmetry axis from the vertical direction to the horizontal direction (Figure 1). Although, physically, the behaviour of seismic waves propagating in VTI and HTI media is the same, the measurements of the seismic waves are different for the VTI and HTI media due to the difference of the orientation of the symmetry axis. The velocities and moveout in HTI media can be obtained from those in VTI media by rotating the coordinates.

Note that after the rotation, the names of the SV and SH waves in the VTI medium have lost their original meaning. Other researchers give them different names. Tsvankin (1997) called the original SH wave as $S_{\parallel}$ wave because its polarized vector is parallel to the isotropy plane and the original SV wave as $S_{\perp}$ because its polarized vector at vertical incidence is perpendicular to the isotropy plane. Crampin (1985) called the $S_{\parallel}$ wave the "fast" shear wave (qS1) since at vertical incidence it propagates faster than the $S_{\perp}$ wave which is called the "slow" shear wave (qS2). In the paper, for convenience, we use the notation of S1 and S2 instead of $S_{\parallel}$ or qS1 and $S_{\perp}$ or qS2.

The relationship between the angle of the seismic ray to the symmetry axis in VTI media and the incident angle and the azimuthal angle of the seismic ray in HTI media can be derived as (Figure 2):

$$\cos^2 \omega = \sin^2 \theta \sin^2 \varphi \tag{1}$$

Then based on this relationship, we can derive the behaviour of velocities and moveout in HTI media.

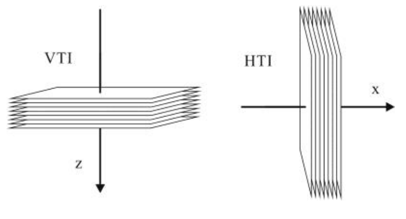


Figure 1. Symmetry plane [x,z] of a transversely isotropic medium with symmetry axis pointing in either z (VTI) or x (HTI) direction.

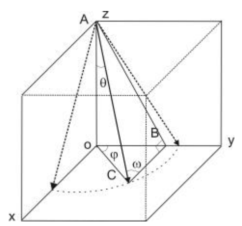


Figure 2. The relationship between the incident angle θ, azimuth angle φ and angle ω (to the asymmetry axis x). Axis Y is the fracture direction.

Phase velocities and anisotropic parameters in HTI media

To derive the velocity variation of a seismic wave in a HTI medium, we substitute Equation (1) into the phase velocities of P, SV and SH waves in a VTI medium (Dai and Li 2015). We have

$$v_p^2(\theta,\varphi)=\alpha_0^2[1+\varepsilon(1-\sin^2\theta\sin^2\varphi)+D^*(\theta,\varphi)] \quad (2)$$

$$v_{S2}^2(\theta,\varphi)=\beta_0^2[1+\frac{\alpha_0^2}{\beta_0^2}\varepsilon(1-\sin^2\theta\sin^2\varphi)-\frac{\alpha_0^2}{\beta_0^2}D^*(\theta,\varphi)] \quad (3)$$

$$v_{S1}^2(\theta,\varphi)=\alpha_0^2[1+2\gamma(1-\sin^2\theta\sin^2\varphi)] \quad (4)$$

$$D^*(\theta,\varphi)=\frac{1}{2}R[\sqrt{1+\frac{4\delta^*}{R^2}\sin^2\theta\sin^2\varphi(1-\sin^2\theta\sin^2\varphi)+\frac{4(R+\varepsilon)\varepsilon}{R^2}(1-\sin^2\theta\sin^2\varphi)^2}-1] \quad (5)$$

where $R=(1-\beta_0^2/\alpha_0^2)$, and α_0 and β_0 are the velocity of P and S waves of the isotropic medium. ε, γ, and δ^* are Thomsen's parameters which indicate the anisotropy of the medium. θ is the incident angle and φ is the azimuth angle.

NMO and vertical velocity of seismic waves in HTI media

Based on these velocities, we have derived the vertical, horizontal, and NMO velocity for P and S1 and S2 waves. The vertical velocities are: $v_{p0}^2=\alpha_0^2(1+2\varepsilon)$, $v_{S20}^2=\beta_0^2$, and $v_{S10}^2=\beta_0^2(1+2\gamma)$. The horizontal velocities in direction $\varphi=0^o$ are: $v_p^2(90{,}0)=\alpha_0^2(1+2\varepsilon)$, $v_{S2}^2(90{,}0)=\beta_0^2$ and $v_{S1}^2(90{,}0)=\beta_0^2(1+2\gamma)$. At $\varphi=90^o$, they are: $v_p^2(90{,}90)=\alpha_0^2$, $v_{S2}^2(90{,}90)=\beta_0^2$, and $v_{S1}^2(90{,}90)=\beta_0^2$. For the anisotropy induced by fractures, generally, we assume that the velocities of P and S waves should reach a maximum in the direction parallel to the fracture strike. To satisfy this assumption, for P waves, it should be $\varepsilon>0$; for S1 waves, it should be $\gamma>0$. However, due to $v_{S2}^2(90{,}90)=v_{S2}^2(90{,}0)=\beta_0^2$, the S2 wave cannot satisfy this assumption.

The NMO velocities can be derived from the phase velocities of P, S1 and S2 waves. For simplicity, we consider the weak anisotropy case. The NMO velocities for a weakly HTI medium are (Dai and Li, 2015):

$$v_{pnmo}^2(\varphi)=\alpha_0^2(1+2\varepsilon)[1-2\frac{(2\varepsilon-\delta)}{1+2\varepsilon}\sin^2\varphi] \quad (6)$$

$$v_{S2nmo}^2(\varphi)=\beta_0^2[1-2\frac{\alpha_0^2}{\beta_0^2}(\delta-\varepsilon)\sin^2\varphi] \quad (7)$$

$$v_{S1nmo}^2(\varphi)=\beta_0^2(1+2\gamma)[1-\frac{2\gamma}{1+2\gamma}\sin^2\varphi] \quad (8)$$

To satisfy the assumption that the wave travels fast in the direction parallel to the fracture strike, it should be $2\varepsilon-\delta>0$ and $\gamma>0$. Since the S2 wave cannot satisfy this assumption because $v_{S2}^2(90{,}0)=v_{S2}^2(90{,}90)=\beta_0^2$, the direction of its maximum NMO velocity should be decided by the parameter, $\delta-\varepsilon$. If $\delta-\varepsilon>0$, the maximum of the S2 NMO velocity is at $\varphi=0^o$. If $\delta-\varepsilon<0$, the maximum of the S2 NMO velocity is at $\varphi=90^o$. Note that at $\varphi=0^o$, $v_{S1nmo}^2(0)=\beta_0^2(1+2\gamma)>v_{S2nmo}^2(0)=\beta_0^2$. So the S1 wave is the fast shear wave. At $\varphi=90^o$, $v_{S1nmo}^2(90)=\beta_0^2$ and $v_{S2nmo}^2(90)=\beta_0^2[1-2\frac{\alpha_0^2}{\beta_0^2}(\delta-\varepsilon)]$, and which wave is the fast wave is determined by $\delta-\varepsilon$.

The NMO velocity and moveout of split PS waves in a VTI medium

In a HTI medium, the S wave converted from a P wave will split into two waves. We know that one wave (S1) is polarized in the direction parallel to the fracture direction and the other one (S2) is polarized in the direction perpendicular to the fracture direction. Note that due to the difference between the vertical velocities of the S1 and S2 waves, we can define two vertical Vp/Vs ratios as: $R_{PS10} = \frac{v_{P0}}{v_{S10}} = \frac{t_{S10}}{t_{P0}}$, and $R_{PS20} = \frac{v_{P0}}{v_{S20}} = \frac{t_{S20}}{t_{P0}}$. The NMO velocities of PS1 and PS2 in HTI media are the combination of the velocities of the P wave and the corresponding S waves. They can be written as:

$$V^2_{PS1'nmo} = \frac{1}{t_{P0} + t_{S10}}[v^2_{Pnmo} t_{P0} + v^2_{S1nmo} t_{S10}] \tag{9}$$

$$V^2_{PS2nmo} = \frac{1}{t_{P0} + t_{S20}}[v^2_{Pnmo} t_{P0} + v^2_{S2nmo} t_{S20}]. \tag{10}$$

For the PS1 wave, the NMO velocity can be rewritten as:

$$V^2_{PS1nmo} = V^2_{PS1nmo0}(1 - \Delta_{PS1nmo} \sin^2 \varphi) \tag{11}$$

Where $V^2_{PS1nmo0} = \frac{v^2_{P0} + R_{PS10} v^2_{S10}}{1 + R_{PS10}}$ and $\Delta_{PS1nmo} = \frac{2R_{PS10}}{1 + R_{PS10}}(\frac{2\varepsilon - \delta}{1 + 2\varepsilon} + \frac{\gamma R_{PS10}}{1 + 2\gamma})$.

Note that Δ_{PS1nmo} is always positive, so at $\varphi = 0^o$, its NMO velocity reaches a maximum. For PS2 waves, the NMO velocity can be rewritten as:

$$V^2_{PS2nmo} = V^2_{PS2nmo0} - \Delta_{PS2nmo} \sin^2 \varphi \tag{12}$$

Where $V^2_{PS2nmo0} = \frac{v^2_{P0} + R_{PS20} v^2_{S20}}{1 + R_{PS20}}$ and $\Delta_{PS2nmo} = \frac{2R_{PS20}}{1 + R_{PS20}} \frac{[(2\varepsilon - \delta) - R_{PS20}(\varepsilon - \delta)]}{(1 + 2\varepsilon)}$.

Note that Δ_{PS2nmo} can be positive or negative, so the maximum of its NMO velocity can be at $\varphi = 0^o$ or $\varphi = 90^o$. We also note that in Δ_{PS2nmo}, $(\varepsilon - \delta)$ is enlarged by R_{PS20}. If $(2\varepsilon - \delta)$ and $(\varepsilon - \delta)$ are of the same order of magnitude, the sign of Δ_{PS2nmo} is determined by $(\varepsilon - \delta)$ (S2 wave). This phenomenon is confirmed by a numerical study. Liu et al (2014) performed a study using a numerical model which contains three layers (Figure 4).The first layer and third layer are all isotropic. The second layer is a HTI layer. HTI is induced by incorporating vertically aligned fractures, based on Hudson's theory. The fracture direction is 120° in the azimuthal plane. The fractured layer is saturated with gas or water.

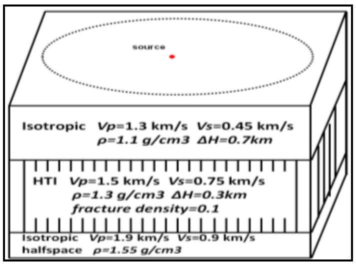
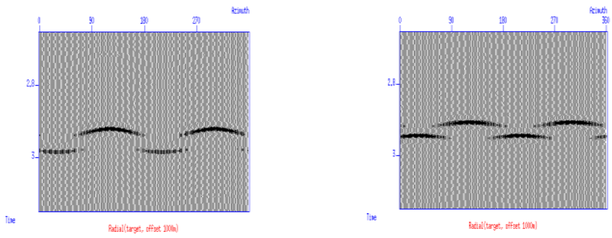


Figure 4: Synthetic model used to show the effects of dry and water-saturated fractures.



(a) (b)

Figure 5: Part of the azimuth gathers of the radial component of the dry model (a) and water-saturated model (b) when the X/Z ratio is 1.0.

This study found that the fast PS wave has similar azimuthal variations in the dry and water-saturated cases. The NMO velocity reaches a maximum in the direction parallel to the fracture direction. However, the slow PS wave has different variations for the two cases (Figure 5). For the dry fractures, the NMO velocity reaches its maximum in the direction parallel to the fracture direction; and for the water-saturated fractures, it reaches its maximum in the direction perpendicular to the fracture direction.

Estimating anisotropy parameters of HTI media.

The split PS converted waves provide more information which can be used to invert for the Thomson parameters. There are four parameters in Δ_{PS1nmo} and three parameters in Δ_{PS2nmo}. We will need three more conditions to determine them. They can be obtained from the vertical velocity ratios and vertical travel time.

Because there are two vertical velocities of S1 and S2 waves, after processing, the two converted waves are located at different vertical times in the zero offset (stacked section or migrated image). The time difference between the two shear waves is:

$\Delta t = t_{PS20} - t_{PS10} = t_{S20} - t_{S10} = \frac{h}{\beta_0} - \frac{h}{\beta_0 \sqrt{1+2\gamma}}$. Since $t_{S10} = \frac{h}{\beta_0 \sqrt{1+2\gamma}}$, so we can have:

$$\gamma = (\frac{\Delta t}{t_{PS10}} \frac{1+R_{PS10}}{R_{PS10}}) + \frac{1}{2}(\frac{\Delta t}{t_{PS10}} \frac{1+R_{PS10}}{R_{PS10}})^2 \tag{13}$$

Here R_{PS10} can be obtained from PP and PS wave correlation. Once we have measured Δt, t_{PS10}, and R_{PS10}, γ can be calculated using Equation 13. Based on azimuthal velocity analysis of the HTI medium, we can measure Δ_{PS1nmo} and Δ_{PS2nmo}. Then based on Equation 11, we can obtain:

$$\frac{2\varepsilon - \delta}{1+2\varepsilon} = \Delta_{PS1nmo} \frac{1+R_{PS10}}{2R_{PS10}} - \frac{\gamma R_{PS10}}{1+2\gamma}. \tag{14}$$

Based on Equation 12, we can obtain:

$$\frac{(\varepsilon - \delta)}{(1+2\varepsilon)} = [\frac{(2\varepsilon - \delta)}{(1+2\varepsilon)} - \Delta_{PS2nmo} \frac{1+R_{PS20}}{2R_{PS20}}] \frac{1}{R_{PS20}} \tag{15}$$

Then, ε and δ can be obtained by solving Equations 14 and 15.

Conclusions

In this work, we have proposed an approach to use the azimuth variation of velocity of the split PS converted waves in HTI media to invert for the Thomsen parameters of the HTI media. Based on the time-delay of the PS1 and PS2 wave, PS1 wave vertical travel time, and the vertical Vp/Vs1 ratio, we can invert for the Thomsen parameter γ. Then, based on this γ and the NMO velocity of the PS1 and PS2 waves, we can calculate the Thomsen parameters ε and δ. This work creates a theoretical foundation. In future work, we will develop a practical approach to perform this inversion and apply it to synthetic and real data to evaluate it.

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