Elastic reverse-time migration using an efficient staggered-grid finite-difference scheme

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SUMMARY

The imaging quality and efficiency of elastic reverse-time migration (RTM) depends strongly on the algorithms used for the solving of wave equations. To improve the quality and efficiency for seismic imaging, an efficient staggered-grid finite-difference (SFD) scheme based on a sampling interpolation method with adaptive variable difference operator lengths is proposed to perform elastic RTM. Numerical analysis shows that the sampling interpolation SFD scheme has higher accuracy than the conventional Taylor-series expansion SFD scheme for solving elastic wave equations. The proposed elastic RTM algorithm is also tested using a theoretical model. The model test demonstrates that elastic RTM using the sampling interpolation SFD scheme can obtain better images than that using the Taylor-series expansion SFD scheme, particularly for PS images. In addition, the application of adaptive variable difference operator lengths can effectively improve the computational efficiency in elastic RTM.

Introduction

Multicomponent seismic exploration can reveal underground elastic information more comprehensively than single P-wave exploration, so vector migration has become an active research field in seismic imaging. In particular, elastic reverse-time migration (RTM) based on the vector wave field can provide a superior way to image complex subsurface structures, and it is becoming more and more important (Yan et al., 2015). Elastic RTM requires forward extrapolation of the source wavefield and backward extrapolation of the recorded receiver wavefield (Yan and Sava, 2008), and the essence of wavefield extrapolation is to solve wave equations. The imaging accuracy and efficiency of RTM depends strongly on the algorithms used for solving wave equations (Yan et al., 2015). Therefore, how to solve elastic wave equations is very important to elastic RTM. At present, the most popular method for solving equations is staggered-grid finite-difference (SFD) schemes. However, numerical approximations of the SFD schemes are required for spatial derivatives, and conventional SFD schemes based on Taylor-series expansion often lead to serious numerical dispersion (Yan et al., 2015; Xin et al., 2015). Accurate numerical approximations are generally achieved by using either relatively very fine computation grids or very long difference operators. However, either approach will dramatically increase the computational cost.

In this paper, we first introduce a sampling interpolation SFD scheme and the adaptive variable-length difference operator, and then analyze the numerical accuracy of the proposed SFD scheme. Finally, we adopt the efficient SFD scheme based on the sampling interpolation method with the adaptive variable difference operator lengths to perform elastic RTM, testing the algorithm using a theoretical model, and the test result shows the advantages of the proposed method.

Theory and Method

The (2*M*)th-order SFD scheme for the first-order derivative of the function *p*(*x*) can be expressed as (Kindelan et al., 1990)

$$\frac{\partial p}{\partial x} \approx \frac{1}{h}\sum_{m=1}^{M} a_m \left[p\left(x+mh-0.5h\right) - p\left(x-mh+0.5h\right) \right], \qquad (1)$$

where h is the grid size and a_m are difference coefficients. From Eq. (1), we can obtain the dispersion relation by using plane wave theory (e.g., Yang et al., 2014; Yan et al., 2015),

$$\beta \approx \sum_{m=1}^{M} a_m \sin\left[\left(2m-1\right)\beta\right], \qquad (2)$$

where $\beta = kh/2$ $(0 \le \beta \le \pi/2)$ and k is the wavenumber.

Generally, the difference coefficients determined by the Taylor-series expansion method are adopted to solve wave equations, but serious numerical dispersion will exist at larger wavenumber, which will affect the seismic modeling and imaging accuracy. We introduce a high precision SFD scheme to solve wave equations. First, we take *M* sampling points for β $(\beta_1, \beta_2, \ldots, \beta_M)$, and they are distributed evenly over the range from 0 to a given *b*, where $0 < b \le \pi/2$. Then we approximate the dispersion relation (Eq. (2)) by interpolation at the *M* sampling points (e.g., Yang et al., 2015; Yan et al., 2015; Xin et al., 2015). Finally, we can use the *M* sampling points to construct a system of linear equations from Eq. (2):

$$\begin{bmatrix} \sin\left[(2\times1-1)\beta(1)\right] & \sin\left[(2\times2-1)\beta(1)\right] & \cdots & \sin\left[(2\times M-1)\beta(1)\right] \\ \sin\left[(2\times1-1)\beta(2)\right] & \sin\left[(2\times2-1)\beta(2)\right] & \cdots & \sin\left[(2\times M-1)\beta(2)\right] \\ \vdots & \vdots & & \vdots \\ \sin\left[(2\times1-1)\beta(M)\right] & \sin\left[(2\times2-1)\beta(M)\right] & \cdots & \sin\left[(2\times M-1)\beta(M)\right] \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_M \end{bmatrix} = \begin{bmatrix} \beta(1) \\ \beta(2) \\ \vdots \\ \beta(M) \end{bmatrix}. \qquad (3)$$

The difference coefficients for first-order derivatives can be obtained by solving Eq. (3). Where, the method is called sampling interpolation SFD scheme.

According to Eq. (2), the parameter $\delta(\beta)$ is defined to describe the numerical dispersion:

$$\delta(\beta)=\left\{\sum_{m=1}^{M}a_m\sin\left[(2m-1)\beta\right]\right\}/\beta. \quad (4)$$

If $\delta(\beta)$ is close to 1, the numerical dispersion is weak. If $\delta(\beta)$ is far from 1, a large numerical dispersion will exist.

To compare the sampling interpolation SFD with Taylor-series expansion SFD schemes, we analyze the numerical dispersion by the dispersion curves. Figure 1 shows the variations of $\delta(\beta)$ with β for different M by the Taylor-series expansion SFD, and the sampling interpolation SFD for numerical dispersion. From Fig.1, we can see that the sampling interpolation SFD scheme has higher accuracy than the Taylor-series expansion SFD scheme over a larger range of wavenumbers under the same spatial difference operator length.

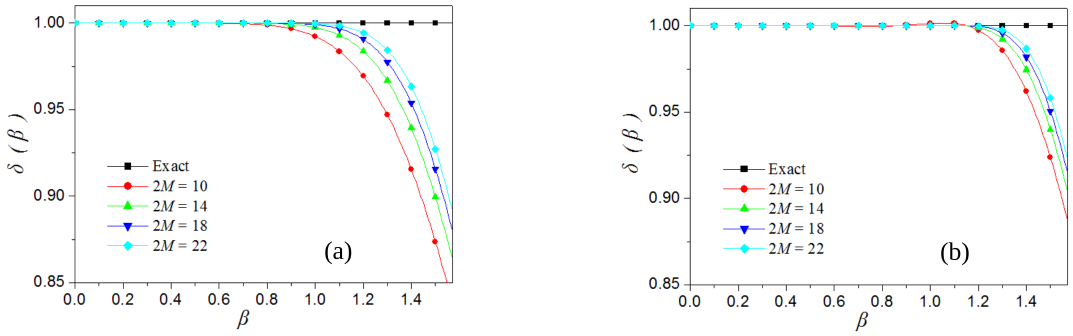

Figure 1 *Plot of numerical dispersion curves by (a) the Taylor-series expansion SFD scheme and (b) the sampling interpolation SFD scheme for different operator lengths.*

In general, the fixed-length difference operator is adopted to compute the spatial derivatives, which leads to more compute time. Based on the idea of the adaptive variable-length finite-difference scheme presented by Liu and Sen (2011), we propose the sampling interpolation SFD scheme with adaptive variable difference operator lengths to solve elastic wave equations. It uses long operators in regions of low velocity and short operators in regions of high velocity to improve computational efficiency in elastic RTM.

According to Eq. (4), the numerical error ε is defined as follows

$$\varepsilon=\delta(\beta)-1=\left\{v\sum_{m=1}^{M}a_m\sin\left[(2m-1)\pi fh/v\right]\right\}/(\pi fh)-1, \quad (5)$$

where v is velocity and f is the frequency. We adopt the S-wave velocity to calculate the numerical error because the numerical dispersion of the S-wave is more serious than that of the P-wave in elastic wave propagation. From Eq. (5), we find that ε is a function of v, M and f. Therefore, for the given maximum numerical error η and maximum frequency $f_{\max}$, the following inequality is satisfied (e.g., Liu and Sen, 2011)

$$\left|\varepsilon(v,M,f)\right|\le\eta, \quad (6)$$

where $f\le f_{\max}$. We can use Eq. (6) to compute the adaptive variable SFD operator lengths.

Numerical Example

To demonstrate the imaging quality and examine the efficiency of difference scheme with adaptive variable difference operator lengths, we test the elastic RTM algorithms using the modified 2D SEG/EAGE salt model. The salt model size is 11000 m × 3600 m, with the grid intervals of 20 m × 20 m. Figure 2 shows P-wave velocity information. The ratio of S-wave velocity and P-wave velocity is a fixed value of 1/1.732.

We perform elastic RTM using the Taylor-series expansion SFD scheme with fixed difference operator lengths, the sampling interpolation SFD scheme with fixed difference operator lengths, and the sampling interpolation SFD scheme with adaptive variable difference operator lengths. Figure 3 shows the fixed and the variable sampling interpolation SFD operator lengths *M* used in elastic RTM. Figure 4 shows the final elastic RTM result using the Taylor-series expansion SFD scheme with fixed difference operator lengths. Figure 5 shows the final elastic RTM result using the sampling interpolation SFD scheme with fixed difference operator lengths. Figure 6 shows the final elastic RTM result using the sampling interpolation SFD scheme with adaptive variable difference operator lengths. From Figs. 4 and 5, we can see that the RTM result using the sampling interpolation SFD scheme are all very clear and their interfaces along the salt are well imaged. However, the RTM result using the Taylor-series expansion SFD scheme has some artifacts, especially in areas of the PS image within the white ellipses, and they are fuzzy. Comparing Figs. 5 and 6, we can hardly see any difference and find that the RTM results using the sampling interpolation SFD scheme with the fixed and the adaptive variable difference operator lengths have similar imaging accuracy, but their computation time is obviously different. On the same computer, the computational efficiency of the migration using the adaptive variable difference operator lengths improves by about 29%, compared with that using the fixed difference operator lengths.

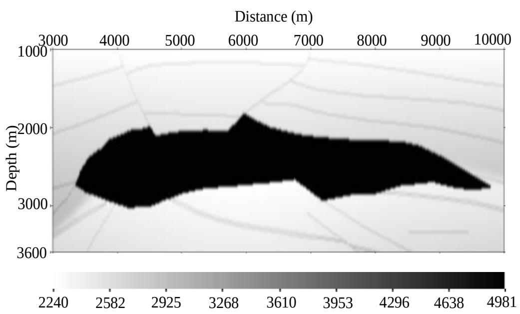

Figure 2 *P-wave velocity for the 2D salt model.*

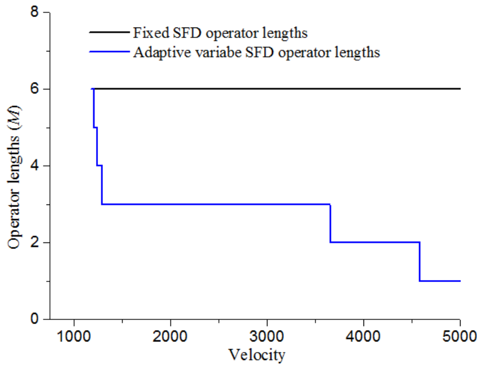

Figure 3 *Fixed and variable SFD operator lengths*

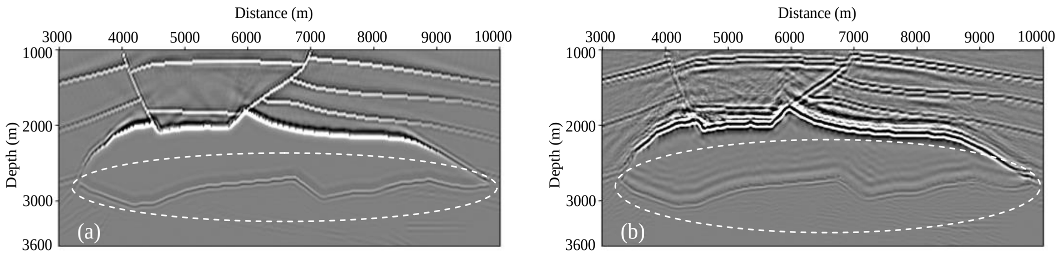

Figure 4 *Elastic RTM result using the Taylor-series expansion SFD scheme with fixed difference operator lengths for the modified salt model: (a) PP image, (b) PS image.*

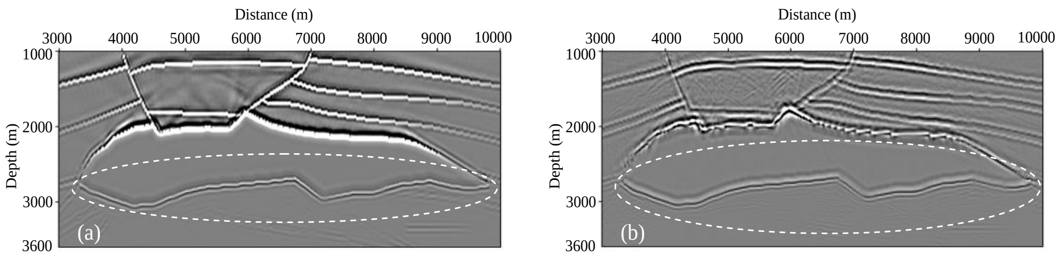

Figure 5 *Elastic RTM result using the sampling interpolation SFD scheme with fixed difference operator lengths for the modified salt model: (a) PP image, (b) PS image.*

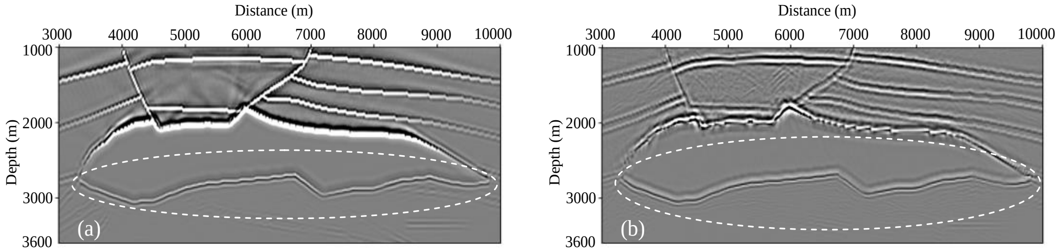

Figure 6 *Elastic RTM result using the sampling interpolation SFD scheme with adaptive variable difference operator lengths for the modified salt model: (a) PP image, (b) PS image.*

Conclusions

A sampling interpolation SFD scheme with adaptive variable difference operator lengths has been proposed to perform elastic RTM. Numerical analysis demonstrates that the sampling interpolation SFD scheme has higher accuracy than the Taylor-series expansion SFD scheme. The proposed elastic RTM algorithm has also been tested. The model test shows that the elastic RTM using the sampling interpolation SFD scheme can obtain better images than that using the Taylor-series expansion SFD scheme, especially the PS images. Additionally, the adaptive variable difference operator lengths can effectively improve computational efficiency in elastic RTM, compared with the conventional fixed difference operator lengths.

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