A practical approach to P-SV prestack time migration and velocity analysis for transverse isotropy
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Summary
We present an accurate double-square-root (DSR) migration equation for P-SV (C-) waves in transversely isotropic media. It is expressed in terms of the migration velocity, P/S velocity ratio, and Thomsen’s parameters. These parameters are estimated using both migration velocity analysis and correlation stacking procedure. We show the results of prestack time migration applied to the Valhall data and compare the isotropic and VTI images produced with C-waves.

Introduction
Advanced P-SV processing techniques now exist to produce credible isotropic images of C-waves using multi-component OBS recordings in complex areas (Zhu et al., 1999; O’Brien et al., 1999; Barkved et al., 1999; Brzostowski et al., 1999; Yuan et al., 1999). Further complications often arise due to the wide occurrence of vertical transverse isotropy (VTI) in marine sediments. Accounting for the anisotropy is thus critical for improving C-wave images (Thomsen, 1999).

Step 1: Isotropic velocity analysis
C-wave processing is more dependent on medium parameters than conventional P-wave processing. Various velocities and velocity ratios are needed to achieve an accurate C-wave imaging. These may include short-spread rms stacking velocities \(v_{pm}, v_{sm}, \) and \(v_{sn}\) for P-, S- and C-waves, respectively, and average vertical velocity ratio \(\gamma_0\), stacking velocity ratio \(\gamma_s\), and effective velocity ratio \(\gamma_{eff}\), which are related to each other by the following relationships (Thomsen, 1999)

\[
v_{sn}^2 t_{\alpha} = v_{pm}^2 t_{\beta 0} + v_{sm}^2 t_{\delta 0} 
\]

and

\[
\gamma_0 = \frac{t_{\alpha 0}}{t_{\beta 0}}, \quad \gamma_s = \frac{v_{pm}}{v_{sm}}, \quad \gamma_{eff} = \frac{\gamma_s^2}{\gamma_0}.
\]

Step 2: Anisotropic velocity analysis
Let us assume a VTI medium defined by the P-wave and S-wave vertical velocities \(v_{p0}\) and \(v_{s0}\) and the Thomsen’s (1986) anisotropic parameters \(\varepsilon\) and \(\delta\). We use two other anisotropic parameters \(\sigma\) and \(\eta\) (Tsvenkin and Thomsen, 1994; Alkhalifah and Tsvenkin, 1995), satisfying

\[
\sigma = \gamma_0^2 (\varepsilon - \delta) \quad (3)
\]

and

\[
\eta = \frac{\varepsilon - \delta}{1 + 2\delta} = \frac{\sigma}{(1 + 2\sigma) \gamma_0^2} \quad (4)
\]

This leads to (Alkhalifah and Tsvenkin, 1995),

\[
v_{pm} = v_{p0} \sqrt{1 + 2\delta}, \quad v_{sn} = v_{s0} \sqrt{1 + 2\sigma} \quad (5)
\]

Assuming \(\gamma_0 = 3\), common in marine sediments, a nominal 5% P-wave anisotropy (\(\varepsilon = 0.05\) and \(\delta = 0.01\))
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yields a 36% ($\sigma = 0.36$) difference between the stacking SV velocity $v_{sn}$ and the vertical velocity $v_{so}$. This indicates that the anisotropy effects can not be ignored during C-wave processing.

If the parameters $v_{pm}$ and $\gamma_0$ are known, we can use the double-scanning procedure by Li and Yuan (1999a) to determine the parameters $\sigma$ and $\gamma_n$. This practical and robust procedure has been tested on real OBS data from the North Sea. See Yuan et al. (1999) for more detail.

Step 3: Prestack time migration

Referring to Figure 1, we consider the wavefield $u(S, R, t)$ as a function of source $S$, receiver $R$, and time $t$. We create the migration output at each image point $Q$ from a weighted sum

$$m(Q) = \int A(S, Q, R) \partial_t u(S, R, t = \tau(S, Q, R)) ds$$

(6)

with appropriate amplitude weights defined elsewhere (Sena and Toksöz, 1993; Solid et al., 1997; Holstad and Sollie, 1998). In the above equation, $\tau(S, Q, R)$ denotes the sum of the traveltimes along the raypaths $SQ$ and $QR$ in Fig.1, that is, $\tau(S, Q, R) = \tau(S, Q) + \tau(Q, R)$. In C-wave migration, $\tau(S, Q)$ is the one-way traveltime along the P-wave incident raypath $SQ$ and $\tau(Q, R)$ is the one-way traveltime along the SV-wave diffracted (not reflected) raypath $QR$ (Fig.1). Traditionally, these traveltimes are calculated assuming an rms velocity approximation to Snell’s law through a stratified medium. Using the anisotropic DSR equation of Li and Yuan (1999b) for P-SV waves, the traveltime along the composite raypath $SQR$ can be written as

$$\tau(S, Q, R) \approx \sqrt{t_{o,\phi}^2 + \frac{h_s^2}{v_{pm}^2} - 2\eta \Delta t_{p}^2}$$

$$+ \frac{\gamma_{n}^2 t_{o,\phi}^2}{(1 + \gamma_0)^2} + \frac{h_s^2}{v_{mn}^2} + 2\sigma \Delta t_{m}^2$$

(7)

with

$$\Delta t_{p}^2 = \frac{h_s^2}{v_{pm}^2} \left[ t_{o,\phi}^2 v_{pm}^2 / (1 + \gamma_0)^2 + (1 + 2\eta) h_s^2 \right]$$

and

$$\Delta t_{m}^2 = \frac{h_s^2}{v_{mn}^2} \left[ t_{o,\phi}^2 v_{mn}^2 / (1 + 2\sigma) / (1 + \gamma_0)^2 + h_s^2 \right].$$

where the source/receiver offsets $h_s, h_r$ and the zero-offset traveltime $t_{o,\phi} = \tau(Q, S_0)$ are explained in Fig.1. Numerical examples by Li and Yuan (1999b) show that this DSR equation is very accurate for both near and far offsets.

![Figure 1: Geometry of the problem. The source is at $S$ and the receiver is at $R$. The image point is $Q$ and the source-receiver offset is $h = h_s + h_r$. The zero-offset ray reaches the sea surface at $S_0$.](image)

Equation (7) contains four independent parameters $\gamma_0$, $v_{pm}$, $v_{sn}$, and $\sigma$. Note that $\eta$ is related to $\sigma$ through $\gamma_n$ [equation (4)]. The double-scanning procedure by Li and Yuan (1999a), along with the isotropic C-wave velocity analysis based on equations (1) and (2), are used to estimate these parameters prior to migration. We update the input parameters during migration using migration stack power maximization. This is done iteratively without the calculation of sensitivities or gradients.

A 4C data example

We apply the above three-step scheme to the OBS data acquired at the Valhall field, North Sea (courtesy of BP-Amoco and the Valhall License partnership). The Valhall field chalk reservoir is difficult to image with conventional processing due to the presence of gas zones and anisotropy (Zhu et al., 1999). The common-receiver images in Figure 2 demonstrate the benefits of including anisotropy into migration process to improve focusing. Figure 3 shows that C-wave anisotropic migration could enhance seismic images over the gas clouds. This supports interpretations made by O’Brien et al. (1999).
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Conclusions

We have presented a practical approach to perform prestack time migration in VTI media. The crucial point is the C-wave DSR equation expressed in terms of Thomsen’s parameters. Initial estimates of the migration velocity, the P/S velocity ratio, and anisotropy parameters are obtained using correlation analysis of PP and PS stacked sections. These parameters are updated iteratively during migration to improve focusing. Application to real data from the Valhall Field shows that this technique yields a better quality of image than the traditional isotropic C-wave processing.

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Figure 3: C-wave images over the gas clouds of Valhall field in the North Sea: (a) isotropy versus (b) anisotropy.