Multi-scale fracture characterization using fractal frequency-power-law attenuation models
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Summary
We have investigated wave scattering by chaotic fractured systems of fractal geometry with random spatial variation. Specifically, we have examined simple closed-form solutions in fractal poroelastic media. These solutions may be characterized by their frequency-power-law (FPL) signature caused by wave dispersion and attenuation. Numerical results show that the fractal dimension can be estimated from the FPL dependence of the scattered wavefield. It appears that finite-bandwidth signals are delayed with respect to the wavefront in comparable elastic media. Applications to fracture characterization are considered.

Introduction
Since the 1980s, the concept of fractals has been increasingly adopted in geophysics. Realistic models of rock structure, reflection sequences, and scattering phenomena due to fault/fracture patterns over a wide range of scales have been shown to possess fractal properties (Todoeschuck et al., 1992; Shapiro & Faizulin, 1992; Kurianov et al., 2002). The main outcome of experimental studies is that the fractal (Hausdorff-Besicovitch) dimension $D$ can be inferred from the FPL of certain wavefield attributes such as the power spectrum, wave shape, variogram, average radiation patterns, etc. These attributes invariably show a power dependence on frequency, with the exponent related to the parameter $D$. It has become clear that this dependence is a universal effect and that special conditions are not required for the development of fractal properties.

Figure 1: Eq. (3) for $a=0.3,0.4,0.5$.
In fact, many fractal models can be reduced either to the percolation cluster or to the fractal aggregate of mass $M=m_0(R/a_0)^D$ related to its size $R$, where $m_0$ and $a_0$ are the mass and the size of constituent particles, respectively. A union of fractal sets having different dimensions is referred to as a multifractal. See Gould and Tobochnik (1996) for details.

Figure 2: Original source signature versus FPL signature for $a=0.3,0.4$.

Theory
The purpose of this paper is to study scattering by fractal inclusions which causes the energy loss of the directly propagated field under isothermal conditions. If the data follow a fractal model, then the effective sound velocity $c$ averaged over the characteristic length $R$ is $c(\omega) \propto \omega^{1-a}$, related to its size $R$, where $m_0$ and $a_0$ are the mass and the size of constituent particles, respectively. A union of fractal sets having different dimensions is referred to as a multifractal. See Gould and Tobochnik (1996) for details.

Figure 3: Fractured model as a percolation cluster of fractal dimension $D=1.896$. The model size is 0.5 km.

where $\omega$ is the angular frequency and the parameter $a$ characterizes the scale dependence of the bulk or shear modulus $K$. By introducing the characteristic wavelength $\lambda$, one can obtain (Kurianov et al., 2002)

$c(\lambda) \propto \lambda^{(-\eta+D-d)/2}$

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where \( d \) is the space dimension and \( \eta = \log_n[K(l)/K(nl)] \) if scales \( l \) and \( nl \) belong to the self-similarity interval.

The solution (3) convolved with the source signature is referred to as the FPL signature. It represents the wavefield at any distance from \( \Omega \). All the derivatives of the function (3) vanish at \( \Omega \) and \( \tau \to 0 \) as \( a \to 0 \) (Figure 1). Thus, the memory effect causes smoothing of the wavefield as \( \tau \to 0 \) and a rapid amplitude decay when \( \tau \to \infty \), as illustrated in Figure 1. In addition, a significant FPL pulse delay is clearly visible (Figure 2). In practice, this delay can be measured during migration velocity analysis to estimate the fractal dimension. The FPL signature can be used for source deconvolution that accounts for attenuation (Q inverse filtering). In an arbitrary heterogeneous anisotropic medium, the single-mode wavefield can be computed by matching eq. (3) defined inside the minimum sphere with the factorized ray-theory propagator
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(Druzhinin, 2001b, 2002) that expresses the field everywhere outside this sphere.

Figure 5: Plot of $\ln M$ versus $\ln R$ for the fractal model in Figure 3.

Results

Let us consider the percolation cluster in Figure 3 generated by the numerical algorithm of Gould and Tobochnik (1996). In contrast to regular fractals, the percolation model is constructed upon use of the theory of percolation pertaining to stochastic geometry. This model is universal due to the fact that it arises from randomly distributed particles provided their concentration is sufficiently high. This is why the use of percolation models allows quantitative characterization of real fracture/fault patterns (Shapiro & Faizullin, 1992) and multi-scale interpretation of log data (Druzhinin, 2001). Here, we focus on frequency-dependent changes in the spectral and amplitude characteristics of the scattered original or migrated wavefield (Figure 4) that could be associated with the presence of fractal fracture patterns. Figure 5 shows the plot of the $\ln M$ versus $\ln R$ for the model in Figure 3. The dashed line is a linear least squares fit to the data. The slope of this line is 1.828. According to eq. (1), this slope is an estimate of $D = 1.896$. To examine the FPL dependence of direct body waves propagating in a stratified medium containing fractal inhomogeneities in Figure 3, we compute acoustic finite-difference (FD) snapshots in the frequency range $f = 20 - 200$ Hz. Figure 4 is the example of snapshots and corresponding depth images for $f = 60,80,100$ Hz. The fractal structure of the model in Figure 3 is very prominent because of multiple scattering with intrinsic correlation in a wide range of scales. Following Todoeschuck et al. (1992), we assume that the angular dependence of scattering intensity is described by the FPL $1/k^D$, $k$ being the spatial frequency. We start by computing the F-K spectrum $P$ for the scattered data in Figure 5, as shown in Figure 6. Next, the peak envelope of the $k$-averaged spectrum $\langle P \rangle$ as a function of the frequency $f$ is computed (Figure 7). In Figure 7, the case $\beta \approx 0$ corresponds to the frequencies $f \geq 100$ Hz. This case is white noise, with no correlation due to high-frequency scattering on isolated inclusions. For $f < 100$ Hz, the spectrum $\langle P \rangle$ is correlated since $\beta > 0$. The power-law exponent is estimated by measuring the slope after fitting the function $\log \langle P \rangle$ versus $\log f$ in a least-squares sense to a polynomial of degree $N \geq 1$. The slope of this polynomial in the log-log domain gives the estimate of $D$. Note that power-law estimation in the full frequency range yields $D \approx 1.56$. Recall that the fractal dimension of the Wiener process is $D = 3/2$. This means that our analysis is equally applicable to Gaussian processes implicit in classical models of statistical deconvolution and least-squares inversion. Finally, the estimate of $D$ is used to compute the Green’s function (3). The subsequent reverse-time FD migration of the scattered data produces multi-scale exploding reflector images shown in Figure 4 (right panels). The slant-stack or dip decomposition coherency analysis of these images create maps of fracture orientation.

Figure 6: F-K spectrum of the wavefield in Figure 4b.

Figure 7: Peak envelope versus frequency $f = 20 - 200$ (Hz) and Mandelbrot-Richardson (MR) log-log plot in the frequency range $20 - 95$ Hz. The MR slope is 1.88. The exact value is $D = 1.896$. 
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(Figure 8). Also plotted are the histograms of the fracture dip angle $\theta$ (in degrees). The present dip-scan algorithm is a replacement of the dip filters used in the standard DMO approach. It takes very many local slant stacks of different angles to include all the directions of fracture set orientation. The values $\theta = 0$ and $\theta = 90^0$ correspond to the stratified background medium and vertical fractures, respectively. In Figure 8, correlation between large fracture sets ($\theta \approx 0, 45^0, 90^0$) as well as scale-dependent fracturing (e.g. $\theta = 30, 55^0, 70^0$) are observed. Clearly, the scale-invariant anisotropic model should assume that the plane of fractures is the vertical plane (HTI model).

Conclusions

This paper solves the problem of wave scattering in fractal media which cannot be described by conventional deterministic models. FPL dispersion properties of certain wavefield attributes have been numerically examined. Extensions to scales exceeding the wavelength are possible. Results are important for inelastic depth imaging, inverse $Q$ filtering, fracture detection, and integrated geophysical reservoir monitoring.

References


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Figure 8: Maps of fracture orientation (left panels) and corresponding histograms (right panels): (a) $f = 60$ Hz, (b) $f = 80$ Hz, and (c) $f = 100$ Hz. The histogram functions count the number of fractures within a range of dip angles (in degrees) and display each range as a rectangular bin. The length scale is normalized on 0.5 km.