The frequency dependent azimuthal AVO response of fractured rock.
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Summary
Calculations of the plane wave reflection coefficients at the interface between anisotropic materials typically ignore the possibility of azimuthally varying attenuation, despite evidence that this is a feature of wave propagation in fractured rocks. We extend the method for calculating the reflection coefficients to the case of materials with frequency dependent properties, and study the PP reflection coefficient for materials described by a recently derived theory. Frequency effects are visible in a number of examples. The high and low amplitude azimuths can flip over by 90 degrees as frequency increases, with the frequency at which the crossover occurs being controlled by petrophysical factors. The phase of the reflection coefficient is seen to vary with angle of incidence, azimuth and frequency. However, not all examples show strong frequency effects.

Introduction
The theory of reflection and transmission of plane waves at the interface between anisotropic elastic materials is now well established. This theory forms the basis for the use of amplitude versus offset and azimuth methods for the detection of natural fractures.

Recent work, both observational and theoretical, suggests the possibility of an anelastic earth response. Horne & MacBeth (1997) presented evidence from a number of VSP surveys which is consistent with the existence of strong azimuthally varying attenuation. The analysis of surface seismic data presented by Clark et al. (2001) supports this conclusion. Chapman (2003) derived a poroelastic model which demonstrated how these effects are consistent with fracturing, and Maultzsch et al. (2003) argued that this model was consistent with observations of frequency dependent shear-wave splitting in VSP data.

If this is so, then we must expect that reflection and transmission coefficients should be frequency dependent. This is of importance since a range of theories exist that relate frequency dependence of seismic properties to petrophysical parameters of interest, such as permeability and fracture properties. Moreover, attempts to interpret a frequency dependent response with a frequency independent theory could yield misleading results.

In this paper we outline the theory for extending the standard calculation of the PP reflection coefficient in anisotropic media to the frequency dependent case, and consider results in a number of important cases. We find that frequency effects can exist under a range of circumstances.

Method
We consider the problem of reflection and transmission of plane waves at the interface between two layers whose properties are described by anisotropic, complex and frequency dependent effective elastic tensors. Expressions for such tensors for the case of fractured rock have been given by a number of authors recently (eg. Hudson et al., 1996; Tod, 2001; Chapman, 2003) and in general their properties are different from those predicted by standard theories of viscoelasticity.

Our method is based on the techniques of Schoenberg (1971) and Schoenberg and Protazio (1992). We fix a polar angle $\psi$ and an azimuthal angle $\theta$, so that the direction of propagation of the plane wave is $\mathbf{n} = (\sin \psi \cos \theta, \sin \psi \sin \theta, \cos \psi)$. The axis normal to the interface is taken to be $\mathbf{x}$.

We assume a homogeneous incident P-wave traveling in the direction $\mathbf{n}$. The velocity field of the incident wave will then be of the form:

$$\mathbf{v} = i_0 \mathbf{e}_p \exp(i \omega (s \mathbf{x} - t)), $$

where $\mathbf{e}_p$ is polarization and $i_0$ amplitude. The slowness $\xi$ of this wave is calculated as the value of $\sqrt{\rho / \chi}$ having smallest real part for $\chi$ an eigenvalue of the matrix $C_{ijkl} n_i n_k$ and $\rho$ the density of the rock. We then have $s = \xi n$. The polarization is given by the eigenvector corresponding to $\chi$.

We now calculate expressions for the transmitted and reflected waves. Snell's law states that the horizontal slownesses of all waves interacting at the interface are equal. Since we know $s_1$ and $s_2$, the condition:

$$|C_{ijkl} s_j s_k - \rho \delta_{ij}| = 0,$$
in both the upper and lower media gives a bicubic equation on the vertical slownesses. Solving for the admissible values of \( s_j \), the polarizations, \( e \), satisfy the equation:

\[
(C_{ijkl}s_is_k-\rho\delta_{ij})e_l=0,
\]

and are obtained by singular value decomposition.

The required continuity of displacement and traction of the calculated waves give the conditions on reflected and transmitted amplitudes. We exploit the formalism of Schoenberg and Protazio (1992), who arranged the interface conditions into a convenient matrix representation. Specifically, the 3X3 reflection matrix \( R \), is given by:

\[
R = (X^{-1}X'-Y^{-1}Y')(X^{-1}X'+Y^{-1}Y')^{-1},
\]

where the matrices \( X, X', Y \) and \( Y' \) are as defined in Schoenberg & Protazio (1992), with the slownesses and polarizations taken from the above calculation. The PP reflection coefficient corresponding to the angles \((\theta, \psi)\) is given by \( R_{11} \).

**Examples**

We specialize our discussion to the case of a linearly elastic isotropic layer overlying a fractured layer having transverse isotropy with a horizontal axis of symmetry (HTI) and frequency dependent properties. Table 1 outlines the three models we will consider; in each case, we give the background P- and S- velocities and densities for each layer. An aligned fracture set of density 0.1 is then inserted into the lower layer with the algorithm of Chapman (2003). In all cases we assume a 5\% porosity, a fluid bulk modulus of 2 GPa and a “squirt-flow” frequency of 200 kHz. Notice that permeability dependence is implicit in the squirt-flow frequency.

Model A is the conceptually important case where there is no impedance contrast between the layers and the reflection coefficient is due to the presence of fractures alone. Models B and C are taken from Sayers & Rickett (1997). In the absence of fractures, Model B is a Class I AVO example, while Model C exhibits Class III AVO behaviour.

Figure 1 shows the predicted frequency dependence of the PP reflection coefficient in model A for a range of azimuthal angles, \( \theta \), and for an incidence angle of \( \psi = 30 \) degrees and different fracture sizes. It is clear that what appears to be the high amplitude azimuth at low frequency becomes the low amplitude azimuth at high frequency. Moreover, the “crossover point” is dependent on the fracture size and squirt-flow frequency. In principle, therefore, this effect provides access to petrophysical information which cannot be obtained by conventional azimuthal AVO analysis. It is also important when estimating fracture orientation from P-wave amplitude versus azimuth data to appreciate that whether the fracture strike lies in the direction of maximum or minimum amplitude could depend on the frequency being used.

A similar behaviour is present in Model B. In Figure 2 we reproduce the amplitude of the reflection coefficient as a function of frequency for propagation in the two symmetry planes; parallel to the fractures (\( \theta=90 \) degrees) and normal to the fractures (\( \theta=0 \) degrees) for an incidence angle of \( \psi = 45 \) degrees. The fracture size in the example shown is 10 cm. We find once again that the high and low amplitude azimuths flip as frequency increases, with the crossover point being controlled by the squirt-flow frequency and fracture size.

The phase behaviour is interesting in this case. We plot the phase of the reflection coefficient as a function of incidence angle \( \psi \) for propagation parallel and normal to the fractures at 50 Hz. We see that the phase is zero for propagation parallel to the fractures. For propagation normal to the fractures, however, a different type of behaviour is evident. In the corresponding elastic case (Sayers & Rickett, 1997), the reflection coefficient normal to the fractures changed sign at an incident angle \( \psi \) of around 40 degrees. In our case, however, the phase is in general neither 0 nor 90 degrees, and depends strongly on angle of incidence, particularly in the region where fracture related effects are important according to Sayers & Rickett (1997).

We plot the phase versus azimuth for incidence angle \( \psi = 45 \) degrees, and observe approximately sinusoidal dependence. Both the magnitude and form of the azimuthal variation depend on frequency. Alternately, for a given frequency, the effect depends on fracture size and squirt-flow frequency, and so could be interpreted in terms of petrophysical parameters. For the low frequency limit, in the sense that the fracture model reduces to the Gassmann-Brown-Korringa limit with no attenuation, we find no such phase behaviour. Similarly, the effect does not occur when we let the frequency become so high that the model reduces to the Hudson equations. The phase effect which we demonstrate could therefore in principle be used to discriminate between static and dynamic fracture models.

We repeat the analysis for Model C, but find that the frequency and phase effects are very much smaller in this case. Figure 3 shows the variation in amplitude and phase with frequency for angle of incidence of \( \psi = 40 \) degrees. Considering a range of azimuths and incidence angles, we can find little evidence for the effects exhibited in Model B.
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In retrospect this is unsurprising since Sayers & Rickett (1997) demonstrated that in this case the behaviour depends only weakly on the exact properties of the fractures.

Conclusions

Azimuthal AVO analysis is typically based on the analysis of the plane wave reflection coefficients at the interface between linearly elastic anisotropic materials. This ignores the role of attenuation and possible frequency dependence. A number of recently derived theories for the seismic response of fractured rock suggest that frequency dependent anisotropy and azimuthally varying attenuation are indicative of fractures.

We demonstrate that the amplitude of the PP reflection coefficient for an isotropic linearly elastic material overlying a fractured HTI medium with frequency dependent properties can vary strongly with frequency. Specifically, the high and low azimuths can flip over by 90 degrees as frequency increases. The frequency at which this crossover occurs depends on the fracture size and squirt-flow frequency, and therefore could give access to important petrophysical parameters such as permeability. Moreover, if this effect is not understood, misleading conclusions may be drawn from conventional fracture detection studies.

In contrast to the elastic case, the phase of the reflection coefficient can show continuous variations with incidence angle and azimuth. The effect depends strongly on frequency and petrophysical parameters.

Not all cases show clear frequency effects. We study a Class III AVO example in which frequency effects are not visible. This emphasizes the importance of careful modeling for the interpretation of azimuthal AVO effects.

References


Acknowledgements

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<table>
<thead>
<tr>
<th>Model A</th>
<th>Model B</th>
<th>Model C</th>
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<tbody>
<tr>
<td>Vp (km/s)</td>
<td>5.10</td>
<td>5.10</td>
</tr>
<tr>
<td>Vs (km/s)</td>
<td>2.95</td>
<td>2.95</td>
</tr>
<tr>
<td>Density (g/cm$^3$)</td>
<td>2.35</td>
<td>2.35</td>
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</tbody>
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Table 1: Physical properties of the unfractured materials used in the calculations.
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Figure 1: Amplitude of the reflection coefficient against frequency for azimuths of 0, 30, 60 and 90 degrees with the parameters from Model A. Left hand diagram is for 1m fractures, center is 20 cm fractures and right hand diagram is 2cm

Figure 2: Calculations for Model B. Left hand diagram is amplitude of reflection coefficient against frequency for propagation parallel and normal to the fractures, center is phase of reflection coefficient against angle of incidence for propagation parallel and normal to the fractures, right hand diagram is phase against azimuth for an incidence angle of 45 degrees for frequencies of 1Hz, 30Hz and 60 Hz.

Figure 3: Calculations for Model C. Left hand diagram is reflection coefficient amplitude against frequency for propagation parallel and normal to the fractures, right hand diagram demonstrates the lack of phase effects in this case.