Estimating frequency-dependent seismic attributes by matching pursuit: a case study
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Summary
This paper investigates the use of spectral decomposition for extracting information on fluid properties. Traditional theory for detecting fluid response is based on the pure elastic Gassman theory, and the resultant seismic effects are frequency-independent. Using dynamic fluid substitution, we demonstrate that the frequency response of seismic reflection and its resultant attenuation and dispersion are directly linked to fluid saturation. To extract this information, we develop an accurate two-stage spectral decomposition method by matching pursuit. This allows us to calculate a range of frequency-dependent attributes, such as absorption coefficient and amplitude gradient in the frequency domain. Application to real data shows a good link between the anomalies and hydrocarbon saturation. The results highlight that careful data processing and modeling are necessary to understand the complex effect of different fluids on the spectral response and enable robust interpretation.

Introduction
Most studies of seismic response to fluid saturation are based on Gassmann’s theory and reflectivity modelling with the Zoeppritz equations, which turned out to be frequency-independent. Recent years have seen the development of technologies which exploit the phenomenon of frequency-related seismic attributes, such as attenuation and dispersion, for predicting reservoir fluid (Castagna et al., 2003; Ebrorn, 2004; Odebeau et al., 2006), and the underlying theory for most of these studies is based on empirical knowledge extrapolated from laboratory measurements.

Chapman (2003) developed a multiscale theoretical model to perform dynamic fluid substitution, which yields an effective medium with its elastic properties varying with frequency. Subsequently, Chapman et al. (2006) have discussed how to calculate the frequency-dependent seismic reflection response, and shown that the resultant frequency-related properties, such as attenuation and dispersive characteristics, are directly linked to fluid mobility within the rock frame. These studies provide a theoretical modeling framework to understand and analyze the seismic frequency response to fluid saturation.

To derive frequency-related attributes from seismic data, it is convenient to use spectral decomposition techniques. There are several techniques available to perform spectral decomposition, such as windowed Fourier transform (WFT), continuous wavelet transform (CWT) and matching pursuit, etc. (Avijit and David, 1995). Since WFT and CFT cannot represent the high frequency component with sufficient accuracy due to the restriction between frequency and scale, matching pursuit is considered to be a better alternative although it is relatively computing-intensive (Mallat and Zhang, 1993). Here, using a dictionary of Morlet wavelets (Morlet et al., 1982), we develop an accurate two-stage method by matching pursuit for spectral decomposition. This allows us to calculate a range of frequency-dependent attributes. Testing on synthetics and real data verifies the approach.

Dynamic fluid substitution
In the case of reflection from a single interface, Gassmann’s theory predicts that changing the fluid saturation in one of the layers will change the impedance contrast and therefore the amplitude of the reflection. This effect is independent of frequency. Chapman (2003) have presented the mathematical procedure for calculating the frequency dependent elastic constants, and the resultant seismic reflection and transmission responses are outlined in Chapman et al. (2006). Based on these formulations, the introduction of gas results in a marked increase in attenuation, and associated dispersion. The existence of strong dispersion in a hydrocarbon saturated layer leads to frequency dependence of the impedance contrast at the interface and so makes the reflection coefficient frequency dependent. This tends to markedly shift the reflections to higher or lower frequency compared to the background trend, with the direction of the shift depending on the AVO class of the reflection. This reflection response does not depend on the thickness of the layer and is the main focus of our study.

Figure 1 shows an example of the variations of velocity and attenuation with frequency calculated using Chapman et al., (2006). Figures 1a and 1b show P-wave velocities drifting from high frequency to low frequency for both gas and water saturations for a sand rock. The parameters in sand are $V_p=2790\text{m/s}$, $V_s=1463\text{m/s}$, and $\rho=2.08\text{g/cc}$ with water saturation, $2.06\text{g/cc}$ with gas saturation, porosity=30%. The gas bulk modulus is taken as 400Mpa, water as 2000Mpa. Reference frequency is 10Hz. The Gassmann effect is shown as a decrease in P-wave velocity when gas replaces water. P-wave velocity with gas saturation shows a larger drift from high to low frequency and higher attenuation than the water saturation case, which proves that gas...
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saturation can lead to abnormally high attenuation. Figures 1c and 1d show that there is no Gassmann effect for the S-wave case because at low frequency the shear-modulus is decoupled from the saturating fluid. S-wave attenuation is very small for gas saturation compared to the P-wave case.

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(a) P-velocity

(b) P-attenuation

(c) S-velocity

(d) S-attenuation

Figure 1. Predicted P-wave velocity (a) and attenuation (b), and S-wave velocity (c) and attenuation (d) as a function of non-dimensional frequency \( \omega \tau \) under gas and water saturation (solid line-gas, dashed line-water).

From Figure 1, we can see that for class III sand with low impedance, the absolute value of impedance contrast will decrease with frequency in the dispersive case, so reflection shifts toward lower frequency ends. However, for class I AVO with high impedance, the absolute value of impedance contrast will increase with frequency in the dispersive case, so reflection tends to shift towards higher frequency ends.

Spectral decomposition by matching pursuit

We use a dictionary of Morlet wavelets to perform matching pursuit. The Morlet wavelet \( m(t) \) centred at the abscissa \( \mu \) can be defined as (Morlet et al., 1982)

\[
m(t) = \exp\left[-\left(\frac{\ln 2}{\pi^2}\right)\frac{(t-\mu)^2}{\sigma^2}\right] \cos\left[\omega_m(t-\mu) + \phi\right]
\]  

(1)

where \( \omega_m \) is the mean angular frequency, and \( \sigma \) controls the wavelet width, and \( \phi \) is the phase shift. Thus, a Morlet wavelet can be characterized by a set of four parameters, \( \gamma = (\mu, \sigma, \omega, \phi) \). Matching pursuit is implemented iteratively, and each iteration adaptively extracts an optimal form of wavelet \( m_{\gamma_n} \), where \( n \) is the iteration number.

After \( N \) iterations, a seismic trace \( f(t) \) is expanded into the following form:

\[
f(t) = \sum_{n=0}^{N-1} a_n m_{\gamma_n}(t) + R^{(N)} f ,
\]

(2)

where \( a_n \) is the amplitude of the nth wavelet, \( R^{(N)} f \) is the residual, with \( R^{(0)} f = f \).

A two-stage matching pursuit algorithm

We develop a two-stage algorithm to perform matching pursuit accurately and effectively. In stage 1, we first give an approximate estimation of parameters \( (\mu_n, \phi_n) \). Using a Hilbert transform, we can calculate the complex attribute of a real seismic trace (Taner et al., 1979); \( \mu_n \) is taken as the time of the maximum envelope of the complex trace, and \( \phi_n \) as the instantaneous phase of the complex trace.

In stage 2, we search for parameters \( (\sigma_n, \omega_n) \), and update parameter \( \phi_n \) for an optimal wavelet \( m_{\gamma_n} \). The searching is based on the following equation:

\[
m_{\gamma_n}(t) = \arg \max_{g_{\gamma_n}, a_n} \left\|R^{(n)} f, m_{\gamma_n}\right\|,
\]

(3)

where \( D = \{m_{\gamma}(t)\}_{\gamma \in D} \) is a comprehensive dictionary of the constituent wavelets. We use equation (3) to search for the optimal parameter \( (\sigma_n, \omega_n) \) with fixed \( \mu_n \) and \( \phi_n \) values. Then, we optimize the parameter \( \phi_n \) within a sub-dictionary based on equation (3). The searching range for parameter \( \phi \) is \( [\phi - 2\Delta \phi, \phi + 2\Delta \phi] \), where \( \Delta \phi \) is 3 degrees. After obtaining \( \gamma_n = (\mu_n, \sigma_n, \omega_n, \phi_n) \), we compute the amplitude of the wavelet \( a_n \) by
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\[ a_n = \left(R_n^{(n)} f, m_n \right), \]  \( n \) (4)

This two-stage procedure is repeated through all iterations. After \( N \) iterations, the final residual \( R_f(t) \) is regarded as the data noise.

After decomposing a signal \( f(t) \) into a series of wavelets \( m_n(t) \), for \( n = 0, 1, ..., N - 1 \), the time-frequency amplitude spectrum can then be calculated using Wang (2007):

\[
Af(t, \omega) = \sum_{n=0}^{N-1} \frac{\sigma_n}{\sigma_n^*} \frac{1}{\sqrt{2 \ln 2 \omega_n}} \left\{ \left( \frac{\pi}{\sigma_n^*} \right) \left( \sigma_n^* \frac{1}{\left(\ln \frac{\omega}{\omega_n}\right)^2} \right)^{1/2} \exp \left[ -\left( \frac{\ln 2}{\pi^2} \right) \omega_n^2 (t - \mu_n)^2 \right] \exp \left[ -\left( \frac{\pi^2}{4 \ln 2} \right) \sigma_n^2 (\omega - \omega_n)^2 \right] \right\}, \]  \( n \) (5)

where \( \omega_n = \omega_{m,n} \) is the mean frequency of the \( n \)-th wavelet, \( \|m_n\| \) is a normalization factor, and

\[
\|m_n\|^2 = \frac{\pi}{2 \ln 2} \omega_n \left( 1 + \exp \left[ -\frac{\pi^2}{2 \ln 2} \cos \phi \right] \right). \] \( n \) (6)

**Synthetic tests**

We use a sinc function and Morlet wavelets to generate two sets of synthetic data: 1) \( \sin (2 \pi \cdot 100 \cdot t) + m(t)_{f=50} \), and 2) \( m(t)_{f=20} + m(t)_{f=40} + m(t)_{f=80} \). We decompose the data using windowed Fourier transform (WFT), continuous wavelet transform (CWT) and the two-stage matching pursuit (MP). Figure 2 shows the comparison on the second set of the synthetic data, where the vertical axis is time, and the horizontal axis is amplitude in Figure 2a, frequency (Hz) in 2b and 2d, and scale in 2c, respectively. The scale of CWT is inversely proportional to the frequency of WFT.

**Real data comparison**

The real data are from the Xinchang gas field, in Southwest China. The reservoir formation is tight sandstone and buried at around 5,000 meters depth from the surface (Tang et al., 2008). Due to the very low porosity and matrix permeability, the production is entirely dependant on fractures. In 2005, a 3D3C multicomponent survey was acquired in order to characterize these fractures, and here we use this dataset to test our method.

Figure 3a shows the input data which is a vertical component P-wave section. We have calculated iso-frequency sections at 10, 20, 30 Hz, respectively using both WFT and the two-stage MP for comparison. Figures 3b and 3c show the corresponding iso-frequency spectra at 20Hz respectively. Note that the vertical axis is two-way time in seconds, and the horizontal axis is trace number.

These comparisons using real and synthetic data confirm that the two-stage MP decomposition can illustrate a signal in time-frequency space more accurately, and produce a more subtle time-frequency spectra than WFT and CWT (Figure 2). It also provides better resolution and isolates the anomalies more clearly than WFT (Figure 3c).
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Figure 3. Comparison of WFT and MP for real data: (a) input P-wave section; iso-frequency sections at 20 Hz from (b) WFT and (c) the two-stage MP, respectively; (d) the calculated gradient p-section using equation (8).

Estimating frequency-dependent attributes

Assuming a medium with constant quality factor Q, the amplitude spectrum of a seismic pulse propagating in the medium at frequency f obeys (White, 1992):

\[
\left| A_1 (f) \right| = \left| A_0 (f) \right| \exp(-\pi ft/Q) .
\]  

(7)

where \( t \) is the travel-time, and \( \left| A_0 (f) \right| \) is the initial amplitude spectrum of the pulse. Equation (7) may be rewritten as,

\[
\ln \left| A_1 (f) \right| = p \cdot f + k,
\]

(8)

where \( p = -\pi t / Q \), and \( k = \ln \left| A_0 (f) \right| \). For any \( f \), if \( k \) is a constant, \( p \) indicates the attenuation/dispersion at time \( t \) for the propagating seismic wave. For any given \( t \), \( p \) can be solved from two or more iso-frequency sections, and is referred to as the frequency-amplitude gradient. An example of \( p \)-section calculated from the iso-frequency data at 10Hz, 20Hz and 30Hz (Figure 3c) is shown in Figure 3d, and the warm colours represent a high negative gradient (Figure 4).

For gas-saturated sand in the study area, reflection amplitudes shift to low frequency (Figure 1), yielding a negative gradient with frequency. Figure 4 shows a map view of the amplitude gradient estimated for the target horizon, and the zones with high negative values (warm colours) reveal the distribution of the gas sands, which correlate well with the known reservoirs (Tang et al. 2008).

Figure 4. Inverted frequency-amplitude gradient from the PP data by the proposed frequency-dependent analysis for the target horizon.

Conclusions

We have presented a two-stage algorithm for performing matching pursuit. Compared with other techniques, the algorithm is both accurate and computationally efficient. Frequency-dependent attributes can then be calculated from the decomposed iso-frequency data. Application to both synthetic and real data verifies the approach.

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References


