Interpreting data matrix asymmetry and polarization changes with depth in multicomponent reflection surveys

Xiang-Yang Li* and Colin MacBeth, British Geological Survey, Scotland

SUMMARY

There are two main sources of data matrix asymmetry in four-component shear-wave seismics: that arising from the acquisition geometry due to source and geophone mis-orientation, imbalance and cross-coupling, and that arising from polarization changes due to variations in the structure, lithology or stress. The asymmetry caused by acquisition geometry is more significant than that by the medium. Two indices are used to quantify this asymmetry, and reveal a static shift for source and geophone mis-orientation, but a systematic variation for polarization changes. Their behaviour may be used to identify the origin of the asymmetry.

The asymmetry caused by the medium is studied by deriving normal incidence plane wave reflection coefficients for an interface separating two anisotropic media with a polarization change. The degree of asymmetry in the tensor reflectivity is proportional to the product of the degree of anisotropy in the layers above and below the reflector, and is thus small for most realistic cases. Consequently, the reflection coefficients can be approximated by a similarity transform of the principal reflection coefficients using the expected polarization difference. These equations can then be used to formulate a singular value decomposition (SVD) in the time-domain to recover both the principal reflectivity and the changes of polarizations with depth.

INTRODUCTION

The polarization direction of the fast split shear-wave may give information about the orientation of in-situ stresses. In recent years, many processing techniques have been developed to estimate and interpret this polarization azimuth from multicomponent seismic data (e.g. Alford 1986, Thomsen 1988, Winterstein and Meadows 1991, Lefeuvre et al. 1992, Li and Crampin 1993, Zeng and MacBeth 1993). However, the polarization azimuth of the leading split shear-wave, as related to the direction of the maximum in-situ stress, may vary both laterally with structural location and vertically with depth. This has been observed in both multicomponent reflection data (Lewis, Davis and Vuillermoz 1991), and VSPs (Winterstein and Meadow 1991). Although most processing techniques designed for multicomponent VSPs have taken into account polarization changes, for example: Winterstein and Meadow (1991); Lefeuvre et al. 1992; Zeng and MacBeth (1993), those designed for multicomponent reflection data often assume constant polarization azimuth, for example, Alford (1986), Thomsen (1988), and Li and Crampin (1993a).

Here, we investigate the possibility of determining polarization changes with depth from multicomponent reflection data. Our approach is to examine the relative information in the data matrix with respect to changes in acquisition and medium parameters using asymmetry indicators. An SVD (singular value decomposition) may then be applied to separate the shear-waves and recover the changes in polarization azimuth.

MEASURING DATA MATRIX ASYMMETRY

We consider a four-component shear-wave survey with two horizontal sources and two horizontal receivers (Alford 1986), forming a data matrix of traces d(t):

\[ d(t) = \begin{bmatrix} xx(t) \\ xy(t) \\ yx(t) \\ yy(t) \end{bmatrix} \]

where the top row, \( xx(t) \) and \( xy(t) \), are the in-line (x-axis) traces from the in-line and cross-line sources, respectively, and the bottom row, \( yx(t) \) and \( yy(t) \), are the cross-line (y-axis) traces from the in-line and cross-line sources, respectively.

To measure the degree of data-matrix asymmetry, we introduce two linear transforms in the time domain:

\[ \begin{bmatrix} \chi(t) \\ \zeta(t) \end{bmatrix} = \begin{bmatrix} xx(t) + yy(t) \\ xy(t) - yx(t) \end{bmatrix} \]

\[ \chi(t) \] and \( \zeta(t) \) define transform time series, forming a position vector in the plane of \( \chi, \zeta \), and \( \zeta(t) \) being directly related to the instantaneous asymmetry of the data matrix. The coordinate \( \chi, \zeta \) is defined with the trajectory of the particle motion in the displacement plane (Figure 1). If the motion is linear, there will exist a coordinate transform which maximizes \( \chi \) and minimizes \( \zeta \) (Figure 1a), hence leading to a symmetric data matrix; if the motion is nonlinear (elliptical, Figure 1b), \( \zeta \) coordinate is comparable with the \( \chi \) coordinate, hence leading to an asymmetric data

\[ \Delta \theta \]

Figure 1. (a) A linearly polarized motion; (b) an elliptically polarized motion. \( \chi \) and \( \zeta \) represent coordinates in the transformed plane; \( \chi' \) and \( \zeta' \) represent the coordinate system along the major and minor axes after rotation. \( \Delta \theta \) defines the angular index.
Interpreting polarization changes with depth

matrix. Thus we may use the degree of non-linearity to define the degree of asymmetry. Note that the degree of non-linearity is often measured using the major and minor eigenvalues of the covariance matrix of the motion (Kanasewich 1981). Thus the non-linearity measurement and the angle of major axis of a particle motion can be used as asymmetry indices:

\[
\gamma(\tau) = \frac{\lambda_{\text{minor}}}{\lambda_{\text{major}}}
\]

and

\[
\Delta \theta(\tau) = \tan^{-1}\left(\frac{\sum \chi_{\tau+1} \zeta_{\tau+1} - \sum \chi_{\tau-1} \zeta_{\tau-1}}{\sum \chi_{\tau+1}^2 - \sum \zeta_{\tau+1}^2}ight)
\]

where \(\lambda_{\text{minor}}\) and \(\lambda_{\text{major}}\) are the minor and major eigenvalues of the covariance matrix for vectors \(\chi(t)\) and \(\zeta(t)\). The summation is over a sliding time window, and \(\tau\) represents the beginning of the window. \(\gamma(z)\) measures the non-linearity of the motion; a larger \(\gamma(\tau)\) implies a greater degree of non-linearity and asymmetry. \(\Delta \theta(\tau)\) is the least square measurement of the angle between the major axis and the axis \(\zeta\) in the transform plane \((\chi, \zeta)\), as shown in Figure 1a. Thus \(\Delta \theta(\tau)\) is the optimum rotation angle that makes the data matrix symmetric in an optimum sense.

INTERPRETING DATA MATRIX ASYMMETRY

MacBeth et al. 1994 has shown that it is possible to distinguish between certain acquisition errors such as misalignment, polarity reversal, and also polarization changes using asymmetry in multicomponent VSPs. In surface seismics the asymmetry indices may respond differently. Here, we investigate two categories of effect: source and geophone mis-orientation, and the reflectivity response in the presence of a depth change of polarizations.

Source and geophone mis-orientation
Consider the simple case of two vertically propagating quasi-shear waves in a homogeneous anisotropic medium with ideal impulse source and geophone response. Assume that the orthogonal source and geophone sets are not aligned in the same directions, for example, the in-line source at a, degrees and the in-line geophone at b, degrees from the qSI polarization direction. We may have in the time domain,

\[
d(t) = C(a_c) \Lambda(t) C^\top(a_g)
\]

where \(C\) is a rotation matrix; \(\Lambda(t)\) is the principal matrix for the split shear-waves. Substituting equation (5) into equations (2) (3) and (4) gives:

\[
\gamma(\tau) = 0; \quad \Delta \theta(\tau) = |a_c - a_g|
\]

which indicates that motion \((\chi, \zeta)\), determined from equation (5), is a linear motion, and reveals that \(\Delta \theta(\tau)\) is a measure of the source and geophone mis-orientations. To verify equation (6), we deliberately rotate the receiver 20° in the common shot records shown in Figure 2 and display the indices as time functions in Figure 3. They show that there is no overall change in \(\gamma(z)\) before and after the rotation (Figure 3a), and \(\Delta \theta(\tau)\) displays a static 20° shift (Figure 3b).

Polarization change at an interface
Consider a planar interface separating two anisotropic media, for which the qSI polarizations of the downgoing waves are different. The plane wave reflection matrix can be written as:

\[
R = \begin{bmatrix}
X_{11} & X_{12} \\
X_{21} & X_{22}
\end{bmatrix} = \begin{bmatrix}
x_{1}^{-1} x_{3} - y_{1}^{-1} y_{3} \\
x_{3}^{-1} x_{1} + y_{3}^{-1} y_{1}
\end{bmatrix}^{-1}
\]

where \(r_{ij}(i,j=1,2)\) is the reflection coefficient from the i-th mode (incident wave) to the j-th mode. \(X_i(1=i\leq 2)\) are frequency independent impedance matrices for the upper and lower medium (Schoenberg and Protazio 1992). For normally incident shear-waves on an interface between media with orthorhombic symmetry or higher, we have:

\[
X_1 = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}, \quad Y_1 = \begin{bmatrix}
0 & \rho_1 (1-\delta) v_1 \\
\rho_1 v_1 & 0
\end{bmatrix}
\]

\[
X_2 = C(\Delta \alpha) \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}, \quad Y_2 = C(\Delta \alpha) \begin{bmatrix}
0 & \rho_2 (1-\delta) v_2 \\
\rho_2 v_2 & 0
\end{bmatrix}
\]

Figure 2. A shot data matrix selected from a multicomponent surface line in south Texas. Here it is used to illustrate the asymmetry in field data.
Interpreting polarization changes with depth

Figure 3. Asymmetry indices calculated from the shot data matrix in Figure 2. The dotted lines are values calculated from the original data; the solid lines are values from the modified data matrix whose geophone directions are deliberately rotated by 20°. Note that there is no change in the index, but there is a static shift in the angular index after rotation. The y index is generally small except for some changes in the near-surface, and the original angular index A0 is close to zero, suggesting that the data matrix in Figure 2 is generally symmetric, and the source and geophone are aligned very well.

where \( \delta_1, \rho_1, \) and \( \nu_1 \) are the degree of anisotropy, density and matrix velocity in layer 1, respectively; \( \delta_2, \rho_2, \) and \( \nu_2, \) the corresponding parameters in layer 2; \( \Delta \alpha, \) the angular changes in \( qS1 \) polarization from layer 1 to layer 2. If the \( qS1 \) polarization in the \( i \)-th layer is \( \alpha_i, \) then \( \Delta \alpha=\alpha_i-\alpha,. \)

Substituting equation (8) into equation (7), we can obtain the explicit plane wave reflection coefficients which agree with the formulation of Li and Crampin (1993b). The asymmetry term in the explicit equation is

\[
\delta_2 \left[ \begin{array}{c}
-\sin^2 \Delta \alpha \\
-\sin 2 \alpha_1 \sin^2 \Delta \alpha \end{array} \right] (\rho_1, \nu_1, \nu_2). \tag{9}
\]

This term is proportional to the product of \( \delta_1 \) and \( \delta_2, \) the anisotropy in the upper and lower layer. Thus the asymmetry due to a polarization change is a second order effect with respect to the degree of anisotropy, and is expected to be negligible. This is contrary to our expectations for VSP data, where such changes produce a significant effect on the data (MacBeth et al. 1994).

INTERPRETING POLARIZATION CHANGES

The purpose of multicomponent data processing is to recover the principal time series and thus the principal reflectivity, as well as to retrieve the anisotropy information, particularly the \( qS1 \) polarization azimuth. Alford (1986), Thomsen (1988), and Li and Crampin (1993a) have presented methods for a uniform medium with azimuthal anisotropy. Here we extend these methods for multicomponent reflection data to accommodate polarization changes.

Approximate tensor reflectivity equations

For small \( \Delta \alpha, \) the reflectivity matrix of equation (7) can be approximated by:

\[
R = C(\Delta \alpha) R_0 C^T(\Delta \alpha) = C(\Delta \alpha) \begin{bmatrix} R_0^1 & 0 \\ 0 & R_0^2 \end{bmatrix} C^T(\Delta \alpha); \tag{2}
\]

for \( \Delta \alpha \) close to 90° \( (\Delta \alpha-\pi/2) \) is small, approximated by:

\[
R = C(\Delta \alpha) R_0^1 C^T(\Delta \alpha) = C(\Delta \alpha) \begin{bmatrix} R_0^1 & 0 \\ 0 & R_0^2 \end{bmatrix} C^T(\Delta \alpha), \tag{3}
\]

where \( R_0^1 \) and \( R_0^2 \) are the principal tensor reflectivities at, respectively, \( \Delta \alpha=0\) (no polarization change) and \( \Delta \alpha=90\)°, following Thomsen (1988). After numerical comparison with the exact solution, we find that equation (10) is good for \( \Delta \alpha<30\)°, and that equation (11) is good for \( \Delta \alpha>50\)°.

Inverting for polarization changes

Assume a uniform anisotropic overburden with the \( qS1 \) polarization direction at an angle \( \alpha \) to the inline source direction, and a polarization change of \( \Delta \alpha \) in the subsurface. Following the convolution model and the reflectivity method, the multicomponent reflection data matrix can be written as:

\[
D(\omega) = G(\omega) M(\omega) S(\omega); \]

\[
M(\omega) = C(\alpha) \Lambda_\theta(\omega) \Lambda_\alpha(\omega) C^T(\alpha \cdot \Delta \alpha), \]

where \( S(\omega) \) and \( G(\omega) \) are the source and geophone response, respectively; \( M(\omega) \) is the medium response for the two shear modes \( qS1 \) and \( qS2, \) \( R_0, \) is a diagonal matrix containing either of the principal reflectivities \( R_0^1 \) or \( R_0^2, \) \( \Lambda_\theta(\omega) \) and \( \Lambda_\alpha(\omega) \) are up- and down-going propagator, and are equal with the reciprocity assumption. After proper compensation for the source and geophone response by amplitude corrections (Li 1994), we can obtain \( M(\omega) \) from \( D(\omega). \)

The predominant effect of the polarization change is to introduce extra rotation operators dependent upon \( \Delta \alpha, \) Noting that \( \Delta \alpha \) is close to 0° or 90°, we may re-write \( M(\omega) \) in equation (12) as:

\[
M(\omega) = C(\alpha + \Delta \alpha) \Lambda_\theta(\omega) C^T(\alpha + \Delta \alpha), \tag{5}
\]

where the diagonal phase-shift operators have now been absorbed into the diagonal reflectivity term \( R_0, \) It is now possible to process and interpret the data matrix using a singular value decomposition (SVD) to determine the polarization azimuth \( \alpha, \) its changes with depth \( \Delta \alpha, \) the average time delay \( \Delta t, \) and the principal shear-wave reflectivity. Initial application to full wave synthetics and real data reveals the potential of this technique (Figure 4).
CONCLUSIONS

The reflection data matrix is more sensitive to asymmetry induced by the acquisition than by the reflectivity. The asymmetry due to polarization changes is proportional to the product of the anisotropy in the layers above and below a reflector, and is small for most cases. Although the reflection matrix from surface surveys is often less informative than the transmission matrix derived from VSPs, it is still possible to recover some polarization changes. For this purpose, we derived approximate equations of the tensor reflectivity for vertically propagating plane shear-waves, which can be approximated by rotating the principal reflectivity with the angle of polarization changes. Changes in polarization azimuth and the principal shear-wave reflectivity may then be recovered by using a simple SVD procedure.

ACKNOWLEDGEMENTS

We thank Mike Mueller of Amoco Production Company for providing the reflection data. We thank Stuart Crampin for helpful discussion during this work. This work was supported by the Edinburgh Anisotropy Project and the Natural Environment Research Council, and is published with the approval of the Director of the British Geological Survey (NERC) and EAP sponsors.

REFERENCES


Li, X.-Y., 1994, Amplitude corrections for multicomponent surface seismic data: 64th SEG Meeting, Los Angeles, Expanded Abstracts, 1505-1508.
Li, X.-Y. and Crampin, S., 1993b Linear-transform techniques for processing shear-wave anisotropy in four-component seismic data: Geophysics, 58, 240.256.

Figure 4. Contour map of polarization section showing the polarization changes calculated from the multicomponent seismic line in Figure 2. The section is dominated by the background light grey colour, indicating an average background polarization azimuth of 40°. There are significant polarization changes in the near surface and along the Austin Chalk, marked by the zones of dark grey colour. The polarization anomalies along the Austin Chalk trend may be correlated with the fracture swarms within the Chalk.