Interpreting qS1 polarizations due to intersecting fractures
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SUMMARY

Intersecting fracture sets exist in natural fracture systems, and result in complicating estimates of their strikes using seismic anisotropy. As a consequence qS1 polarizations determined from normal incidence shear-waves may not relate directly to the strike of any individual component. For two intersecting fracture sets, the horizontal polarization for qS1 may be simply expressed in terms of their strikes and vertical inclinations. This relation reveals directions where the strike of one fracture set can be measured independently from the other. It also shows that the effect of sub-horizontal fracture planes or beds may be significant, even for small anisotropies.

INTRODUCTION

It is usual for natural fracture systems (Figure 1) to be interpreted as a number of intersecting fracture sets of varying scalelengths formed under different paleo-stress fields and strain conditions (Nelson 1985; Barton 1995). Depending upon the relative stresses present during the generation of each set, such systems may consist of fractures with different inclinations (as in a conjugate pair of shear fractures) or strikes (different paleo-stress directions). Depending upon their origin and subsequent history, some fractures may actively contribute to the seismic anisotropy, but others will not.

Figure 1. A natural fracture network traced from an outcrop (Barton 1995).

The qS1 polarization azimuth, $\phi_H$, is usually interpreted in near-offset VSP as an individual fracture strike, but intersecting fracture sets such as above bias this interpretation. This study develops expressions for $\phi_H$ in the case of vertical fractures intersected by an inclined fracture set. These expressions help provide an understanding of the likely degree of error to be encountered by assuming a single fracture set. They also indicate that along certain directions of propagation it is possible to estimate the individual fracture strikes. Subvertical fracture strikes average more or less linearly. Subhorizontal fracturing or bedding appear to have a more significant and variable influence on the fracture strike.

A SINGLE SET OF FRACTURES

Vertical fractures

A single set of aligned vertical fractures with infinite lateral dimension (Schoenberg and Sayers 1995) or planes containing dilute concentrations of randomly distributed subseismic cracks or ‘crack-like’ contacts can be shown to be equivalent (Liu et al. 1996). I refer to these generically as ‘fractures’.

For a vertical fracture system rotated clockwise by an angle $\phi$ about the vertical (Z) axis, the new elastic constants $c^\Delta$ with respect to the acquisition X-Y-Z axes, may be obtained by using a Bond transformation (Winterstein 1991). This gives the monoclinic arrangement of stiffnesses

$$c = \begin{pmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{12} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}$$

where $c_{23} = c_{22} = 2c_{44}$. This anisotropy has an infinite fold (X-) axis of symmetry, with each raypath in the source-receiver plane lying in a distinct plane of symmetry. The Y-axis corresponds to the fracture strike.

The behaviour of the shear-waves is governed by the Christoffel equation, which permits a determination of the phase velocities and polarizations of the individual wave components. For vertical propagation the normalized qS1 displacement (polarization) may be written as

$$u_1 = \begin{pmatrix} -\sin \phi_H \\ \cos \phi_H \end{pmatrix}$$

where the angle $\phi_H$ refers to the horizontal polarization angle with respect to the Y-axis. For parallel vertical fractures the condition $c_{44} > c_{55}$ normally holds (Horne, MacBeth and Liu 1996), so that qS1 is polarized at right angles to the symmetry plane containing the raypath. As
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This is also true for any raypath in the Z-Y plane, and within 30° of the vertical for the Z-X plane, for a near-offset VSP the angle \( \phi_H \) may be directly interpreted as the fracture strike. The derivation of this may be obtained by inserting the elastic constants from (2) into the Christoffel equation for downward propagation along the vertical (Z) axis. The resultant eigen-equation in terms of the original stiffnesses in (1) is

\[
F^A(\phi)c\bar{u} = \rho u^2 \bar{u}
\]

where \( F^A(\phi;c) \) is defined as

\[
\begin{pmatrix}
c(A\sin\phi B\sin\phi C\cos\phi) & c(A\cos\phi B\sin\phi C) & c(A) \\
c(B\sin\phi A\sin\phi C\cos\phi) & c(B\cos\phi A\sin\phi C) & c(B) \\
c(C\sin\phi A\sin\phi B\cos\phi) & c(C\cos\phi A\sin\phi B) & c(C)
\end{pmatrix}
\]

the eigenvectors being given by (3) with \( \phi_H = \phi \). This result is easily obtained by application of the Jacobi rotation principle detailed in Press et al. (1992).

Dipping fractures

Rotating the vertical fracture system clockwise about the Y-axis by an angle \( \theta \) and then about the vertical Z-axis by an angle \( \phi \), a full triclinic arrangement of elastic constants may be obtained by again using Bond transformations. Inserting these into the Christoffel equation for downward propagation along the Z-axis leads to an eigen-equation in terms of the original stiffnesses with the matrix on the left-hand side of (4) now being \( F^B(\phi, \theta;c) \)

\[
\begin{pmatrix}
sin^2\phi c_{44}^{B}B + \cos^2\phi c_{55}^{B}B & (c_{44}^{B} - c_{55}^{B})\sin\phi \cos\phi & \cos\phi c_{35}^{B} \\
(c_{44}^{B} - c_{55}^{B})\sin\phi \cos\phi & \cos^2\phi c_{44}^{B} + \sin^2\phi c_{55}^{B} & -\sin\phi c_{35}^{B} \\
\cos\phi c_{35}^{B} & -\sin\phi c_{35}^{B} & c_{33}^{B}
\end{pmatrix}
\]

The stiffnesses \( c^B \) are functions of the original stiffnesses \( c \) and \( \theta \), and I have also used inequalities for weak fracture anisotropy

\[
|c_{12} - c_{23}| < |c_{11} - c_{22}| < |c_{44} - c_{55}| < 0.30c_{44}.
\]

which holds for ellipsoidal cracks with an aspect ratio of less than 0.2 and thin fracture planes. For conciseness we replace \( (c_{44} - c_{55}) \) by \( e \), which, when divided by \( 2c_{44} \) gives a measure of the velocity anisotropy (the percentage is obtained by multiplying by 100). Within the weak anisotropy regime of Hudson’s formulations \( f = e/2c_{44} \) is also equal to the ‘crack density’. Eigenanalysis of this matrix gives \( \phi_H = \phi \) through Jacobi rotation about the Z-axis. After this procedure, a small off-diagonal residual of \( c_{35}^{B} \) remains, which may be eliminated by a further rotation about the Y-axis. As this term is much smaller than \( |c_{33}^{B} - c_{35}^{B}| \), the resultant deviation from the normal (horizontal) plane is at most a few degrees.

\section{Intersecting fracture sets}

The technique of Hudson (1986) can be employed to calculate the equivalent elastic constants for intersecting fracture sets of different orientations. The stiffness tensor, \( c \), corresponding to two or more fracture sets embedded in an isotropic matrix is

\[
c = c^0 + c^1 + c^2
\]

where \( c^0 \) is the stiffness tensor for the isotropic matrix, \( c \), the sum of all first-order terms \( c(i) \) (i=1, N) due to the N different fracture sets, and \( c^2 \) is a second order term due to crack-crack interactions. Considering only weak anisotropy, as defined in (5), this second order term may be neglected and the linear sum of first order terms from the different fracture sets may be considered.

\section{Intersecting vertical fractures}

For vertical fractures with strikes \( \phi_1 \) and \( \phi_2 \), the horizontal plane containing the fracture normals is the symmetry plane for a monoclinic system. Following the previous section by inserting the stiffnesses due to two fracture sets, \( c = c^0 + c^{(1)} + c^{(2)} \), into the Christoffel equation for downward propagation along the Z-axis

\[
(F^A(c^0) + F^A(\phi_1)c^{(1)}) + F^A(\phi_2)c^{(2)})\bar{u} = \rho u^2 \bar{u}
\]

where \( F^0(c^0) \) is the contribution from the background isotropic matrix. The isotropic contributions cancel, giving the resultant eigenvectors in (3) with

\[
\tan 2\phi_H = \frac{e_1 \sin 2\phi_1 + e_2 \sin 2\phi_2}{e_1 \cos 2\phi_1 + e_2 \cos 2\phi_2}
\]

\[
- \frac{f_1 \sin 2\phi_1 + f_2 \sin 2\phi_2}{f_1 \cos 2\phi_1 + f_2 \cos 2\phi_2}
\]

\[
\phi_H = \text{the average of } \phi_1 \text{ and } \phi_2 \text{ only if } e_1 = e_2. \text{ A simplification of this equation to the crack density weighted mean}
\]

\[
\phi_H = \frac{f_1 \phi_1 + f_2 \phi_2}{(f_1 + f_2)}
\]

is valid for \( \phi_1 < 45^\circ \) and \( \phi_2 < 60^\circ \), or \( \phi_1 > 45^\circ \) and \( \phi_2 > 60^\circ \), within an accuracy of 5° for individual crack densities in the range 0.01 to 0.06. Equation (9) agrees with the empirical result of Liu et al. (1993) for a range of intersecting vertical fracture systems.

\section{Intersecting vertical and inclined fractures}

For fractures with strikes \( \phi_1 \) and \( \phi_2 \), the latter having a vertical inclination of \( \theta \), (4) becomes

\[
(F^A(\phi_1)c^{(1)}) + F^B(\phi_2, \theta)c^{(2)})\bar{u} = \rho u^2 \bar{u}
\]

Eigenanalysis again yields the eigenvectors in the form of (3) with the horizontal polarization \( \phi_H \) being defined by

\[
\tan 2\phi_H = \frac{e_1 \sin 2\phi_1 + e_2 G(\theta) \sin 2\phi_2}{e_1 \cos 2\phi_1 + e_2 G(\theta) \cos 2\phi_2}
\]
where $G(\theta) = \cos^2 2\theta - \sin^2 2\theta$. This may be easily generalized for two inclined fracture sets (a conjugate set)

$$\tan 2\phi_H = \frac{e_1 G(\theta_1) \sin 2\phi_1 + e_2 G(\theta_2) \sin 2\phi_2}{e_1 G(\theta_1) \cos 2\phi_1 + e_2 G(\theta_2) \cos 2\phi_2}$$ (12)

**DISCUSSION**

**Behaviour of bi-fracture system**

Figure 2 shows the qS1 polarization ($\phi_H$) variations for a hemisphere of directions plotted using an equal area projection, and for a small selection of crack densities $f$ and inclination $\theta$. I focus on the central region corresponding to vertical propagation (open square), with the $\phi_H$ variation predicted by (11) being shown in Figure 3. For both fracture sets essentially vertical, the polarization azimuth lies between $\phi_1$ and $\phi_2$, tending more towards $\phi_2$ for larger crack densities in the inclined set. When the inclination is within 20° of the vertical, this variation becomes quite flat, being given within 5° by (9). As the vertical inclination increases, the polarization direction is directed more towards the vertical fracture strike $\phi_1$. The qS1 polarization gives exactly $\phi_1$ at an inclination of 30°. There are also other ranges of directions (some of these being marked by arrows) where this occurs, these lying at specific points near to the strike direction for each fracture set. The bounding directions for these regions are determined by simple expressions related to the strikes and dip.

Beyond 30°, the intersecting fracture set continues to move the polarization clockwise at a more dramatic rate. Consequently, this means that subhorizontal fractures, horizontally lying shales or formations with a fine-layered internal structure may have a strong influence. As it is not expected that dipping formation anisotropy should possess the same strike as the fractures, being formed by distinctly different geological processes, this could contribute a major source of error in VSP interpretation. It appears that this strong behaviour is present even for small degrees of anisotropy. For example, 4 percent anisotropy is enough to bias the polarization direction by as much as 10° for inclinations of 70°.

**Solution to an apparent discrepancy at the Conoco testsite?**

Horne, MacBeth, Queen and Rizer (1996) analysed two near-offset VSPs with shot points at approximately diametrically opposite azimuths to determine the anisotropy related to fracturing at the Conoco testsite facility in Oklahoma. Inversion of the shear-wave observations indicated a TI medium which corresponds to a fracture set striking N50°E and vertical inclination of 18° towards the SE. In a separate study at the same site, Horne and MacBeth (1994) inverted shear-waves from an 290m offset azimuthal VSP, and interpreted the results as parallel fractures striking N69°E with an inclination of 10° towards the SE. The second set of results were consistent with a priori information resulting from an extensive analysis of the natural fractures occurring at the site, and evidence from dipmeter and BHTV readings suggesting dipping fractures. One possible explanation is that a third fracture set exists inclined by 10° to the NW. It has a strong influence on the qS1 polarization for the near-offset VSPs, but little influence on the dipping set due to $G(0)$ being close to zero.

**CONCLUSIONS**

There are three major conclusions from this study:

1. Sets of subvertical ($\theta < 20°$) fractures affect the interpretation of the qS1 polarization by combining to form an average strike;
2. Specific directions exist where it is possible to determine the individual fracture strikes of a bi-fracture system;
3. Weak sub-horizontal fractures or bedding anisotropy may have a significant effect on the strike determination of a vertical fracture set using the qS1 polarization angle estimated from a near-offset VSP.

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Figure 2. qS1 polarization behaviour for intersecting vertical and dipping fracture sets with different strikes plotted on an equal area projection. The vertical fracture set has an $f_1$ of 0.03 and a strike $\phi_1$ of 0°, whilst the inclined set has a strike $\phi_2$ of 60° and a variable crack density $f_2$. Open square boxes indicate the directions sampled by a near-offset VSP, and arrows indicate some ranges of directions where individual fracture strikes may be directly measured (in this case $\phi_1$).

$\phi_1 = 0°; \phi_2 = 60°$

Figure 3. Variation of qS1 horizontal polarization angle $\phi_H$ versus vertical inclination $\theta$ of an intersecting fracture set with a range of ‘crack densities’ from 0.02 to 0.10. The vertical fracture set is fixed with $f_1$ and $\phi_1$ as in Figure 2.