Unexpected shear-wave double reflections
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Summary

A negative Thomsen’s $\epsilon$ can give rise to a specific wave surface shape which allows the generation of a double reflection event at an interface. Other specific combinations of elastic constants can also create this shape. The events arise from the same modal sheet and can occur in acquisitions where a high velocity layer is important, such as sub-salt or sub-basaltic imaging. It is possible that the internal composition of the basalts is suited to the generation of this phenomenon. There are also a number of candidates listed by Thomsen (1986) that are appropriate. This type of anisotropy requires a special underlying mechanism. Apart from being a lithological indicator, the significance of this lies in confusing imaging of target horizons and also influencing the AVO response.

Introduction

Cuspidal features arising from TI anisotropy are not unusual, and have been observed in the field using VSP (Slater et al. 1995; Kerner, Worthington and Dyer 1993). This behaviour arises from the concave nature of one shear wave surfaces. A well known practical consequence of this is multiple arrivals. For certain combinations of elastic constants or a TI medium with a tilted axis of symmetry, a concave surface may be formed, the centre of which lies along the horizontal axis. This can also give rise to an important phenomenon associated with reflection and transmission at a plane interface. For this case, two distinct vertical slownesses on the same modal sheet can correspond to any given incident horizontal slowness. This means that two distinct waves may appear upon reflection. This behaviour can also occur for other types of anisotropy (such as cubic and orthorhombic). Here we examine the consequences of this effect and the practical significance in terms of seismic acquisition. We find that it is important for interfaces with high to low impedance changes. The phenomenon may be of particular relevance to sub-basaltic imaging, but should be considered in AVO studies which utilize large offsets. It is also possible to define the causative mechanism of the anisotropy using this effect.

TI wave surfaces

The study of plane wave propagation in a homogeneous material includes the equation of motion together with the generalized Hooke’s law in a resultant Christoffel equation (Auld 1990)

$$\{p_{j} c_{ijkl} p_{l}\} u_{k} = \rho u_{i},$$

where $\hat{p}$ is the slowness vector, which means the quotient of wavefront normal and phase velocity, and $\hat{g}$ is the polarisation vector. Rock properties are given by density $\rho$ and the stiffness tensor $c$.

Equation (1) may be solved given two horizontal components of slowness, $s_1$ and $s_2$. Even though direct rearrangement of equation (1) no longer gives an eigen-problem, the characteristic equation can be solved for the remaining component.

$$\det |c_{ijkl} p_{j} p_{l} - \delta_{ik} \rho| = 0, \quad \delta_{ik} = \begin{cases} 1 & i = k \\ 0 & i \neq k \end{cases}$$

Generally the characteristic equation has six roots corresponding to vertical slownesses $p_{3m} (m = 1, \ldots, 6)$. These separate into two sets with three wave modes in each for positive and negative vertical slownesses. In the case of VTI medium, which is referred to as the most common type of anisotropy (Thomsen 1986), this equation can be solved within a Cartesian coordinate system, with the 3-axis corresponds to the symmetry axis of the medium

$$p_{3m} = \begin{cases} \pm \sqrt[3]{-P(p_{1,2}) \mp \sqrt{P^{2}(p_{1,2}) - Q(p_{1,2})}} \\ \pm \sqrt{R(p_{1,2})} \end{cases}$$

where

$$P(\xi) = \frac{c_{11} c_{33} - 2c_{13} c_{44} - c_{33}^2 \xi^2 - (c_{44} + c_{33}) \rho}{2 c_{44} c_{33}}$$

$$Q(\xi) = \frac{c_{11} c_{44} \xi^4 - (c_{11} + c_{44}) \rho \xi^2 + \rho^2}{c_{44} c_{33}}$$

$$R(\xi) = \frac{\rho - c_{66} \xi^2}{c_{44}}.$$ 

Due to the chosen symmetry, there are no azimuthal dependencies in the above expressions, and the vertical slowness component $p_3$ can be described as a function of either one horizontal slowness component $p_1$ or the other ($p_2$), without losing generality. Furthermore the transversely isotropic formulations are applicable to all planes of symmetry, provided these planes contain two mutually perpendicular axes of symmetry (Helbig 1994).

Figure 1 illustrates the shapes of slowness surfaces for different degrees of P-wave anisotropy (Thomsen’s $\epsilon$), whereby a common shear-anisotropy of 3% is assumed.
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The qP and qSP (polarization parallel to the symmetry plane) are coupled, whilst the SR (polarization at right angles to the symmetry plane) is separated. With ε becoming increasingly negative from figures a to d, both the coupled wave surfaces begin to distort strongly. But while the inner curve must remain convex, the outer one becomes more concave at each step. Although not the only parameter to cause this affect (see Helbig 1994 for more detailed analysis), this does show that an increase in the qP-wave anisotropy can influence the wave surface shape and hence finally the cuspidal features. This has implications for Snell’s law at an interface between two such materials.

Fig. 1: Intersection of slowness surfaces with the vertical plane of symmetry for a transversely isotropic medium at different degrees of P-wave anisotropy (ε=0.0, 0.08, 0.16, 0.24).

The anisotropic Snell’s law

At a plane horizontal interface, waves scatter such that the horizontal slownesses are preserved. The final direction of the phase slowness depends upon the position of the points on the wave surfaces corresponding to the incident horizontal slowness. Figure 2 illustrates this process. In the first medium the wave normal direction determines the incident horizontal slowness. This is then carried across to the second medium whereupon it determines the number of permissible wave modes by the vertical intersection with the various wave surfaces for the common horizontal slowness. For the general case it can be seen that there are six intersections of this line with the slowness surfaces, corresponding to the allowable phase velocities.

Fig. 2: Two different methods to find permissible slowness vectors and their relation according to the anisotropic Snell’s law.

It is thus common practice to believe, that an incident wave may scatter into three or less reflected waves, and three or less transmitted waves, with modes arising from distinct modal surfaces. However, a surprising feature of the wave surface concavity is that an incident wave may cause two reflected and two transmitted waves arising from the same mode. A magnified view of this is shown in figure 3.

Fig. 3: Scattering of an incident wave into two reflected and two transmitted waves of same mode.

Whenever an incident wave has a horizontal slowness within the range A of the concave slowness surface, there are more than two shear roots to the characteristic equation (3). Due to their energy flow directions the “upper” (a) and “middle lower” (c) solutions in figure 3 belong to reflected waves. The “middle higher” (b) and “lower” (d) solutions belong to transmitted
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waves. By looking at the phenomenon in terms of reflections we thus have two potential arrivals within the same sheet, but with different slowness vectors (directions) created by any one incident wave. It should be emphasized, that this is not a standard mode conversion, and must be considered in all cases where the slowness surface has an concave shape at its intersection with the interface-plane. Is it possible to find this effect in actual seismograms, and how significant is it?

Example

In answer to the question above we illustrate this phenomenon by considering the case of a high velocity layer surrounded by a low velocity material for a marine acquisition. It is believed that this situation is particularly appropriate for the generation of the double reflections. Such conditions may arise in sub-salt (Gulf of Mexico) or sub-basaltic (Rockall margin) imaging, especially at the larger offsets where efficient converted wave processing is possible (Purnell1992). Converted wave types are thought to be important for deep water (900m to 1500m) acquisition in the 3 to 6km offset range. Such raypaths possess horizontal slownesses which lie close to those indicated above.

Synthetic seismograms are computed using the anisotropic reflectivity method of Taylor (1991) for an isotropic and anisotropic high velocity layer. The model used consists of a typical sub-basaltic situation, with a 900m thick water layer, 900m of overlying sediment, and a 400m thick target layer. The modeling has inhibited the sea-bottom and top layer reflection for clarity of illustration. It can be seen in the isotropic case that the normal incidence P-wave decreases in amplitude to be replaced by a wave refracted from the top of the target layer, and a strong converted shear-wave reflecting from the layer bottom. In the anisotropic case, for our particular combination of elastic constants, the converted shear-wave is replaced by strong double reflection also arising from the base of the target layer. They begin to appear in the mid-offset range and consequently may be confused with a additional internal layer. This also highlights its potential significance in AVO studies.

Discussion and Conclusions

A wave surface concavity may be generated by large negative ε or other combinations of elastic constants, which gives rise to a double reflections. It is possible that this phenomenon is important for situations in which high velocity layers exist, as in sub-salt or sub-basaltic imaging or in gas producing dolomites. However, it appear from measurements of anisotropy on salt that such a large negative ε may not occur (Sun et al. 1992). However basalts possess both large P-wave and shear-wave anisotropies (Kiorboe and Peterseon 1995) arising from columnar basalts flows, cracks and joints, but also in particular from vesicles created during the cooling process. The average geometry of these vesicles may be appropriate to generate large P-wave anisotropy due to their aspect ratio (Crampin 1991; Horne and MacBeth 1995). It is intersecting to note that Horne and MacBeth (1995) found an unexpected shear-wave behaviour corresponding to a negative γ. Examples corresponding to both this current situation and their study can be found in Thomsen (1986). The origin of these particular cases merits further investigation to understand the nature and geometry of the causative features, and its possible significance as a lithological indicator. This phenomenon may also have implications in interpreting intra-bed multiples, imaging top target horizons, and in cross-well acquisition.

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