Seismic wave propagation in media with interconnected cracks and pores

Tim Pointer*, Enru Liu, British Geological Survey; John A. Hudson, University of Cambridge; Stuart Crampin, University of Edinburgh

Summary

The effects of movement of interstitial fluids within a cracked solid are examined. We consider two distinct mechanisms: flow through connections between otherwise isolated cracks, and diffusion into a porous matrix material (Hudson, Liu & Crampin 1996). These models are used to obtain the overall effective elastic constants for a medium whose parameters are considered geologically realistic. We determine the phase velocities and attenuations for seismic waves passing through such a rock and analyse their dependence on the permeability and the background porosity. We find that the properties of P and S waves are very sensitive to the transport of fluids through interconnected pathways. The limit of zero flow is equivalent to isolated fluid-filled cracks, the opposite extreme of free flow is that of dry or gas-filled cracks. The transitional behaviour which is observed occurs over a range of permeabilities that encompasses those of most oil and gas bearing lithologies. The introduction of fluid flow causes P-waves to travel slower in the direction across the cracks than parallel to them, as expected intuitively. There is also a marked increase in the attenuation $Q^{-1}$ from negligible values to the order of 1. Similar effects arise from the introduction of an equant porosity.

Introduction

The concept of interconnections between cracks and pores is based upon a model in which the material contains a random distribution of penny-shaped circular cracks. The theory has been developed in a series of papers starting with Hudson (1980). It is assumed that the wavelengths involved are large compared with the size of the cracks, and that the crack density $\varepsilon$ is less than 0.1. This study is simply an application of the recent advances made by Hudson, Liu & Crampin (1996), in which the transfer of fluids between cracks by otherwise invisible pathways, or through a network of pores is considered.

The expression for the effective elastic constants, accurate to second order in $\varepsilon$, is given by Hudson (1986) as

$$C = c^0 + 2c^1 + c^2$$  (1)

in which $c^0$ are the parameters for the matrix material, $c^1$ are the first-order terms which account for single scattering, and the second-order terms $c^2$ give the crack-crack interactions. The crucial part is the calculation of $c^2$ which can be expressed as a function

$$c^2 = c^2(U_{11}, U_{33})$$  (2)

where $U_{33}$ is the response parallel to the direction of shear stress, and $U_{11}$ is the response parallel to the direction of normal stress, for vertically aligned cracks (Hudson 1986). $c^2$ can then be written in terms of $c^1$ as follows:

$$c_{ijkl}^2 = \frac{1}{\mu} c_{ijk\alpha}^1 \chi_{rstu} c_{tukl}^1$$  (3)

where

$$\chi_{ijkl} = \left\{ \frac{\delta_{ij} \delta_{kl} (4 + \beta^2/\alpha^2) - (\delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} (1 - \beta^2/\alpha^2))}{15} \right\},$$  (4)

and $\alpha$ and $\beta$ are the respective P- and S-wave speeds of the matrix material.

In the presence of connections between cracks or an equant porosity the response of each crack to an imposed normal stress, $U_{11}$, changes. The parameter $U_{33}$ remains unaffected since a shear stress applied to a crack face does not produce a volume change. The resulting new expression given by Hudson, Liu & Crampin (1996) for the case of interconnected cracks is

$$U_{11} = \frac{4(\lambda + 2\mu)}{3(\lambda + \mu)} \sqrt{(1 + K)},$$  (5)

where,

$$K = \left\{ \frac{\alpha \kappa_f (\lambda + 2\mu)}{\pi c \mu (\lambda + \mu)} \right\} \left\{ 1 - \frac{3i\kappa_f k^2 D_r}{4\pi v a^2 \omega} \right\}^{-1},$$  (6)

$\lambda$ and $\mu$ are Lame’s parameters for the solid matrix, $\kappa_f$ is the bulk modulus of the fluid infill, $a$ is the aspect ratio, $\omega$ is the angular frequency, $k$ is the wavenumber, and lastly $D_r$ is the diffusivity of the rock.

In the case of an equant porosity $\phi_m$ the following expression for $K$ is obtained:

$$K = \frac{1}{\alpha \kappa_f} \left\{ \frac{\lambda + 2\mu}{\pi c \mu} \right\} \left\{ 1 + 3(1 - i)J/2c \right\}$$  (7)
where,

\[ J^2 = \omega \phi_m \kappa_f D_m / 2, \quad J > 0, \quad (8) \]

in which \( D_m \) is the diffusivity of the fluid in the rock matrix.

Numerical results

The parameters used in the following analysis are presented in Table 1. The P- and S-wave speeds and density of the rock matrix were measured from a Mesaverde sandstone at a depth of 3805 m (Lin 1985).

Table 1. The model parameters used to obtain the \( |U_{11}| \), velocity and attenuation variations in Figures 1-4.

<table>
<thead>
<tr>
<th>Matrix material</th>
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<tr>
<td>P-wave speed</td>
<td>3962 ms(^{-1})</td>
</tr>
<tr>
<td>S-wave speed</td>
<td>2926 ms(^{-1})</td>
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<td>density</td>
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</tr>
<tr>
<td>S-wave speed</td>
<td>0 ms(^{-1})</td>
</tr>
<tr>
<td>density</td>
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<tr>
<td>aspect ratio</td>
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<tr>
<td>crack density</td>
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<td>frequency</td>
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</tr>
<tr>
<td>P-wavelength</td>
<td>79 m</td>
</tr>
<tr>
<td>S-wavelength</td>
<td>59 m</td>
</tr>
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</table>

(i) Interconnected cracks

In the case of interconnected cracks the response \( U_{11} \) is dependent on the additional parameter \( D_r \), or equivalently the permeability of the rock \( K_r \) through the relation (Sheriff 1991)

\[ D_r = \frac{K_r}{\eta_f}, \quad (9) \]

where \( \eta_f \) is the fluid viscosity (\( 10^{-3} \) Pas or 0.01 Poise for water, Table 1).

The amplitude of \( U_{11} \) increases smoothly over a narrow range (about 1000 mD) in \( K_r \) (Figure 1). When \( K_r \) is equal to about 1 mD the quantity \( |U_{11}| \) is very small and the situation is similar to the case of isolated fluid filled cracks as there is negligible flow between the cracks, when \( K_r \) is about 1000 mD then the pressure relaxation is so large that the cracks are effectively dry. The permeability of ‘good’ oil-bearing rocks is of the order of 1000 to 10000 mD and for gas-bearing rocks is 1 mD or less. The effect on the behaviour of the body wave phase velocities of \( qP \), \( qS_1 \) (faster shear wave) and \( qS_2 \) (slower shear wave) are shown in Figure 2. The part of the graphs for \( qS_1 \) and \( qS_2 \) which does not change with permeability corresponds to \( qSH \) motion which is dependent only on \( U_{33} \). The \( qSV \) component however, changes significantly and consequently the incidence angle at which the two shear wave slowness sheets intersect.

Figure 1. Variation of \( |U_{11}| \), the amplitude of the crack response to an imposed normal stress, with the diffusivity \( D_r \) (or permeability \( K_r \)) of the whole rock.
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Figure 2. Variation in body wave phase velocity with the angle of incidence at a vertical crack face, produced for different permeabilities in a medium containing interconnected cracks. \(qP\) denotes the quasi-P-wave, \(qS_1\) and \(qS_2\) are the faster and slower split shear waves respectively. The values of \(K_P\) for the individual plots are: (a) 1 mD, (b) 10 mD, (c) 30 mD and (d) 1000 mD.

A criticism of Hudson’s initial model (Hudson 1980), has been that for thin cracks with a vanishing aspect ratio P-waves travel across the crack face at the same speed as in a parallel direction for the saturated case (Thomsen 1995). From Figure 2 it can be seen that as the permeability departs from zero the \(qP\) velocity travels at a reduced speed perpendicular to the crack faces which is expected as there is a higher impedance to wave motion.

Figure 3. The variation of body wave attenuation with the angle of incidence at a vertical crack face, produced for different permeabilities in a medium containing interconnected cracks. The values of \(Q^{-1}\) for the individual plots are: (a) 1 mD and (b) 30 mD (see Figure 2).

Another observation has been that the early model (Hudson 1980) predicts a negligible amount of attenuation due to viscous and scattering absorption. Figure 3 shows that the attenuation of \(qP\) and \(qS\) waves becomes significant due to the dissipation of energy by the movement of crack fluids. The magnitude of \(Q^{-1}\) reaches the order of 1. The P-wave attenuation shown in Figure 3(b) shows a similar trend to that obtained in the laboratory experiments by Rathore et al. (1995), also shown in Thomsen (1995), where the pulse broadening and amplitude decrease is greatest for P-waves travelling perpendicular to the crack faces,

(ii) Equant porosity

Similarly, the presence of an equant porosity affects only the \(U_{11}\) component. Figure 4 shows the phase velocity and attenuation variations for a medium with a porosity of 30% and can be compared with Figures 2(a) and 3(a). There is a similar effect to that described above for interconnected cracks, however, the dependence of \(U_{11}\) on \(\phi_m\) is different. An attempt is made to compare the results with the model of equant porosity proposed by Thomsen (1995), and the labora-
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tory experiments of Rathore et al. (1995).

Figure 4. The variation of body wave velocity and attenuation with the angle of incidence at a vertical crack face, for a medium with a background or 'equant' porosity of 30% (see Figures 2 and 3).

Conclusions

The introduction of fluid flow either through interconnected cracks or a porous matrix tends to weaken the rock. There is a large effect on the rock's elastic properties even though the volume occupied by the cracks and pores may be small. The degree of fluid saturation strongly controls the velocity and attenuation of $qP$ and $qSV$ seismic body waves. The fluid 'squirt' (Dvorkin, Mavko & Nur 1995) between cracks or through pores may be a dominant attenuation mechanism and is also strongly dependent on the crack size and frequency.

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References


