**Interpreting the residual wavefield for polarization change in a 4-C shear-wave data matrix**

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**Summary**

Multicomponent shear-wave data recorded at an oblique line-azimuth to the natural coordinate system will display shear-wave splitting (birefringence). After the initial rotation into the natural coordinate system, there is often still significant residual energy left in the off-diagonal components of the 4-C shear-wave data matrix, and these signals, referred to as the residual wavefields, are often ignored in subsequent processing. This work investigates the link between the residual wavefield and polarization changes in a 4-C shear-wave data matrix. Local polarization changes cause local scattering, and the energy in the off-diagonals cannot be eliminated by rotation with an average polarization angle. It is consequently possible to detect the polarization change by analysing the residual wavefield. This is achieved by extracting the instantaneous polarization using a singular value decomposition of the 4-C data matrix into principal and residual components. Both synthetic and real data will be used to illustrate the scheme.

**Introduction**

The purpose of multicomponent data processing is to recover the principal time series as well as to retrieve the anisotropy information, particularly the \( qS1 \) polarization azimuth. Alford (1986), Thomsen (1988), Li and Crampin (1993), and Li (1997) have presented methods for this purpose. All these methods assume that, for orthogonal polarized and vertically propagating split shear waves, shear wave splitting can be simulated by a uniform two-component eigensystem, with the eigenvectors representing the polarization, and the eigenvalues representing the amplitudes, of the two split shear waves. However, depth change of polarization may be common, particularly, in fractured reservoirs. Wintersten and Meadows (1991), and Thomsen et al. (1995), and Li and MacBeth (1997) have presented methods to recover the polarization change. Most of these methods are based on layer-stripping, and they are not applicable to surface data when the polarization change do not associated with an impedance change.

Depth change in polarization azimuth causes variations in the eigensystem. Consequently, the real eigensystem may not be extracted from the data matrix. Instead, an optimum eigensystem in the least-square sense is often obtained. If this is applied to real data, the principal modes \( qS1 \) and \( qS2 \), the diagonal terms of the rotated matrix, are extracted from the real data. However, the off-diagonal (residual) components might not be eliminated. Although they are usually smaller than the diagonals (Li and MacBeth, 1997), they do have some effects on the processing. We refer this residual energy in the off-diagonals after the initial rotation as the residual wavefield. In this paper, we shall investigate the link between the residual wavefield and the \( qS1 \) polarization azimuth, and establish ways to identify and estimate the polarization change from the residual wavefield.

**Theory**

Common field practice in acquiring 4-C shear wave data employs two horizontal sources and two horizontal receivers; the recorded data contain four components arranged in a \( 2 \times 2 \) matrix \( D(t) \). The sources and receivers may be misoriented. This misorientation can be easily corrected by rotating the recorded data matrix:

\[
D(t) = R(T(\alpha_c))D(t)R(\alpha_s),
\]

where superscript \( T \) is the transpose operator, \( \alpha_c \) and \( \alpha_s \) are the geophone and source azimuth, \( R(\alpha) \) is the standard orthogonal rotation matrix:

\[
R(\alpha) = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}
\]

and

\[
R^T(\alpha) = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}
\]

For simplification, in the following discussion, \( D(t) \) is assumed with any misorientation corrected.

If the applied source motion is linearly polarized and any inconsistency in source and geophone response can been compensated for (Li, 1994), an eigensystem exists, and the recorded data matrix can be written as:

\[
D(t) = R(\alpha)\Lambda(t)R^T(\alpha),
\]

where \( \alpha \) is the polarization azimuth of the fast split shear wave, and \( \Lambda(t) \) is the diagonal transfer function for the two split shear waves \( qS1 \) and \( qS2 \). Here we have:

\[
D(t) = \begin{pmatrix} x(t) & y(t) \\ y(t) & y(t) \end{pmatrix}
\]
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\[
\Lambda(t) = \begin{pmatrix}
qS1 & 0 \\
0 & qS2
\end{pmatrix}
\]

and

\[
D(t) = \begin{pmatrix}
qS1 \cos^2 \alpha + qS2 \sin^2 \alpha & -(qS1 - qS2) \sin \alpha \cos \alpha \\
-(qS1 - qS2) \sin \alpha \cos \alpha & qS1 \sin^2 \alpha + qS2 \cos^2 \alpha
\end{pmatrix}
\]

The post-stack data matrix \(D(t)\) should be a symmetrical matrix, even in media with a polarization change due to reciprocity (Dellinger and Nolte, 1996), which has two orthogonal eigenvectors at direction \(\alpha\) and \(90^\circ + \alpha\), respectively, and two eigenvalues \(qS1\) and \(qS2\). In theory, \(D(t)\) can be diagonalized. However, in some cases, the instant \(\alpha(t)\) for each time \(t\), is not the same as the average polarization, \(\alpha_0\), for the entire trace, and this causes residual energy in the off-diagonals.

Mathematically, for each time \(t\), \(D(t)\) has an eigensystem with \(qS1(t)\), \(qS2(t)\) and \(\alpha(t) = \alpha_0 + \Delta \alpha(t)\). Then \(D(t)\) can be written as:

\[
D(t) = R(\alpha_0 + \Delta \alpha(t)) \Lambda(t) R(\alpha_0 + \Delta \alpha(t))
\]

where \(\alpha_0\) is the average or optimum polarization azimuth of the entire trace, and \(\Delta \alpha(t)\) is the instant change for each time \(t\). The \(\alpha_0\) can be obtained by other processing methods such as the LTT method (Li and Crampin, 1993).

Applying \(\alpha_0\) to the data matrix \(D(t)\), gives:

\[
\Lambda(t) = R^T (\alpha_0) \Lambda(t) R (\alpha_0)
\]

where

\[
\Lambda(t) = \begin{pmatrix}
qS1(t) & O(t) \\
O(t) & qS2(t)
\end{pmatrix}
\]

It can also be written as:

\[
\Lambda(t) = \Lambda(t) - (qS1(t) - qS2(t)) \sin(\Delta \alpha(t)) \begin{pmatrix}
\sin(\Delta \alpha(t)) & \cos(\Delta \alpha(t)) \\
\cos(\Delta \alpha(t)) & -\sin(\Delta \alpha(t))
\end{pmatrix}
\]

This shows that if the \(\alpha_0\) is applied to each \(D(t)\), the polarization change \(\Delta \alpha(t)\) will contribute to the residual term in \(\Lambda(t)\). If at \(t\), \(\Delta \alpha(t)\) equals zero, \(O(t)\) will be zero too, and the residual terms are eliminated. \(qS1(t)\) and \(qS2(t)\) are also affected by this polarization change. This polarization change has a smaller effect on diagonal terms than on off-diagonal terms. Note that, we do not need to assume the \(\alpha_0\) close to 0 or \(\pi/2\) and \(\Delta \alpha(t)\) is small. The maximum of \(O(t)\) occurs at \(\Delta \alpha(t) = \pi/4\), and its values is \(\frac{qS1(t) - qS2(t)}{2}\).

This residual terms caused by polarization change can be used to measure the polarization change from the seismic trace. It is easy to calculate the instantaneous polarization change at time \(t\). Note this instantaneous polarization change is not stable due to noise or other factors. We can use an average value in a time window. For a time window from \(t\) to \(t + N\), its average polarization can be estimated by an instantaneous SVD of equation (2):

\[
\alpha = \frac{1}{4} \arctan \left( \sum_{t}^{t+N} \frac{2}{\sum_{t}^{t+N} [qS1(t) - qS2(t)]^2 - [2O(t)]^2} \right).
\]

Applying this to the entire trace yields the average polarization \(\alpha_o\). To calculate the \(\Delta \alpha(t)\), we can use a small time window.

Applications and Results

The above analysis can be applied to a surface seismic survey to investigate the polarization changes in shot gathers or in stacked seismic sections. The following procedures can be adapted in the analysis:

1. Use equation (3) to calculate the average polarization \(\alpha_0\) for the entire trace or for a small time window that includes the necessary signals.
2. After \(\alpha_0\) is obtained, apply this \(\alpha_0\) to rotate the 4-C data-matrix.
3. To calculate the \(\Delta \alpha(t)\), we use a small time window sliding across the entire trace, and calculate \(\Delta \alpha(t)\) for every window.

Here, we give two examples using this method to extract the polarization changes.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>100m</td>
<td>X 20</td>
<td>Y: 3%</td>
</tr>
<tr>
<td>Layer 2</td>
<td>200m</td>
<td>X 20</td>
<td>Y: 3%</td>
</tr>
<tr>
<td>Layer 3</td>
<td>100m</td>
<td>X 40</td>
<td>Y: 10%</td>
</tr>
<tr>
<td>Layer 4</td>
<td>300m</td>
<td>X 20</td>
<td>Y: 3%</td>
</tr>
<tr>
<td>Layer 5</td>
<td>Halfspace</td>
<td>X 20</td>
<td>Y: 3%</td>
</tr>
</tbody>
</table>

**Figure 1.** Schematic model containing a region with 10% shear-wave anisotropy in layer 3 simulating a fracture reservoir.

Figure 1 shows a model that includes a polarization change in its third layer. Figure 2a is the 4-C synthetic
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seismic stack sections created from this model by using the reflectivity method (Taylor, 1990). Figure 2b shows the same data after being rotated 20° degrees. It clearly shows the polarization change corresponding to the layer 3 in the model. Figure 2c and 2d show examples of instant polarization changes for CDP 7. It is easier to identify the changes from Figure 2d than from Figure 2c.

Table 1: Isotropic parameters used in the model shown in Figure 1, where the anisotropy parameters are annotated.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Density $\rho$ (g/cm$^3$)</th>
<th>$v_p$ (km/s)</th>
<th>$v_s$ (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>2.1</td>
<td>3.00</td>
<td>1.70</td>
</tr>
<tr>
<td>Layer 2</td>
<td>2.2</td>
<td>3.46</td>
<td>2.00</td>
</tr>
<tr>
<td>Layer 3</td>
<td>2.2</td>
<td>4.33</td>
<td>2.50</td>
</tr>
<tr>
<td>Layer 4</td>
<td>2.2</td>
<td>3.46</td>
<td>2.00</td>
</tr>
<tr>
<td>Layer 5</td>
<td>2.5</td>
<td>5.00</td>
<td>2.88</td>
</tr>
</tbody>
</table>

This method is also applied to a dataset acquired by Amoco Production Company in 1996 in Sandstone, South-central Wyoming. Figure 3a shows a part of the stack section and Figure 3b shows the final rotated 4-C section. Note that, the main signal energy is in the XX and YY components and most energy in the XY and YX components is eliminated. This indicates that the instantaneous rotation is very effective. There are possible some local polarization changes for event at 2s (Figure 3b).

Conclusion

In this work, we investigate the residual wavefield in 4-C matrix recordings and the links to polarization changes. The residual field in the off-diagonal of the 4-C matrix is caused by the polarization changes that differ from the average polarization. Results from the synthetic and real data example show that the residual wavefield is a good indicator of the polarization change, and an instantaneous SVD decomposition can be used to detect the change.

Acknowledgement

This work is presented with the approval from the Director of the British Geological Survey and EAP sponsors: Amerada Hess, Amoco, BG plc, Conoco, Elf UK, Fina, Mobil, PGS, Phillips, Saga Peroleum, Schlumberger, Shell, and Texaco.

Figure 2. (a) Synthetic 4-C data matrix for normal incident shear wave calculated for the model in Figure 1. (b) the matrix after rotating (a) by 20 degrees. (c) is the instant polarization change calculated from CDP 7 in (a)) and (d) is the change calculated from CDP 7 in (b).
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![Image](image_url)

Figure 3. (a) A 4-C stack section of the data acquired by Amoco Production Company in 1996 in Sandstone, Wyoming; (b) the corresponding section for (a) after rotation by the instantaneous polarization.

References


Li, X. Y., 1997, Fractured reservoir delineating using multicomponent seismic data, Geophysical Prospecting, 45, 39-64.

Li, X. Y. and MacBeth, C., 1997, Data-matrix asymmetry and polarization changes from multicomponent surface seismic, Geophysics, 62, 630-643


