A new approach for converted-wave moveout in transversely isotropic media
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Summary
The truncated Taylor series expansion of the converted-wave travel time equation may generate severe error when applied to anisotropic media (Levin 1989). The reason is that the effects of anisotropy on the P-SV wave are usually significant even for very weak anisotropy. To overcome this, we present a new approach based on the equivalent medium concept for transversely isotropic media with a vertical symmetry axis (TIV). The essential step in the new approach is the accurate determination of the conversion point. For near-offset conversions, this is achieved by the development of a new and highly accurate asymptotic approximation, and for middle-to-far offset conversions, by using a fast and accurate numerical scheme. In a single layer medium, after conversion points are determined, the travel times can be calculated directly from the medium properties and the exact equation. For multi-layered TIV media, the P-SV travel time can be approximately modelled using an equivalent single-layer medium with Dix-type root-mean-squared (rms) P- and SV-velocities and anisotropic parameters. The approach provides sufficient accuracy for long spread data and for up to 20% P- and S-wave anisotropy.

Introduction
Recent developments in multicomponent sea-floor recording have refocused interests on converted shear-waves (Berg et al. 1994). One of the great potentials of shear-waves yet to be realized lies in their sensitivity to seismic anisotropy which may provide the vital link to important heterogeneities at the scale-length of the flow units in the reservoir (MacBeth 1995). To explore these potentials, it is important to develop techniques for processing converted waves in the presence of anisotropy.

Most existing analysis techniques for P-SV converted waves are based on the Taylor series expansion of P-SV travel-time equation (e.g. Tessmer and Behle 1988, Tsvankin and Thomsen 1994). This approach may generate severe errors when applied to converted-waves in anisotropic media (Levin 1989), even when the effects of anisotropy are included in the expansion (Berge 1991, Soriff and Sriram 1991). Furthermore the Taylor series approach generates an extra set of effective P-SV properties which are often difficult to interpret and to correlate with other rock properties. To overcome some of these difficulties, we extend the equivalent medium concept of Zhang (1996), and present an alternative approach for P-SV moveouts in TIV medium.

Single layered TIV medium: Conversion point and travel time
Consider a single-layered TIV medium, as shown in Figure 1.

Figure 1: The ray-path of a P-SV converted wave in TIV medium.

The P-SV travel-time can be written as

\[ t_{ps} = \frac{1}{v_p(\phi_p)} \sqrt{x_p^2 + z^2} + \frac{1}{v_s(\phi_s)} \sqrt{(x - x_p)^2 + z^2}, \quad (1) \]

where \( v_p(\phi_p) \) and \( v_s(\phi_s) \) are P- and SV-group velocities along their corresponding raypath with ray angles \( \phi_p \) and \( \phi_s \), respectively. \( x \) is the source-to-receiver offset, \( z \) is the reflector depth, and \( x_p \) is the conversion-point position. It is critical for converted wave processing to find the conversion-point \( x_p \) accurately.

First, we consider the near-offset asymptotic approximation. The P-SV conversion at the interface in Figure 1 satisfies Snell’s law,

\[ \frac{v_p(\theta_p)}{\sin \theta_p} = \frac{v_s(\theta_s)}{\sin \theta_s}, \quad (2) \]

where \( v_p(\theta_p) \) and \( v_s(\theta_s) \) are P- and SV-phase velocities with phase angles \( \theta_p \) and \( \theta_s \), respectively. When \( x \) is near to zero, according to Thomsen (1986), we have

\[ \lim_{x \to 0} \phi_p = \theta_p(1 + 2\delta), \quad (3) \]

\[ \lim_{x \to 0} \phi_s = \theta_s \left( 1 + \frac{v^2_p}{v^2_s} (\epsilon - \delta) \right) \quad \text{and} \quad (4) \]

\[ \lim_{x \to 0} \frac{v_p(\theta_p)}{v_s(\theta_s)} = \lim_{x \to 0} \frac{v_p(\phi_p)}{v_s(\phi_s)} = \frac{v_{p0}}{v_{s0}}, \quad (5) \]

where \( v_{p0} \) and \( v_{s0} \) are vertical P- and SV-wave velocities, and \( \epsilon \) and \( \delta \) are the Thomsen parameters (Thomsen 1986). Substituting equations (3-5) into (2) and...
after some manipulation yields the new asymptotic approximation for $P$-$SV$ conversion point in a TIV medium as

$$x_p = \frac{(1 + 2\varepsilon) \frac{v_{pM}}{v_{pR}}}{1 + (1 + 2\varepsilon) \frac{v_{pM}}{v_{pR}} + 2 \frac{v_{pM}}{v_{pR}} (\varepsilon - \delta)}. \quad (6)$$

For isotropy, the approximation reduces to $x_p = \frac{v_p}{v_{pR}} / (1 + \frac{\varepsilon}{2})$. Results of equation (6) are much more accurate than those of Sena (1993) (Figure 2). Compared with other isotropic approximations, equation (6) is more accurate for the near-offset conversions (offset/depth ratio < 1, Figure 2).

Figure 2: Error analysis of asymptotic approximations for the five anisotropic sedimentary rocks listed in Table 1. Error shows the average difference between the accurate solution and the approximation. The reflector depth is 1000 meters. The star-line shows the results of equation (6), the circle-line shows those of Sena’s (1993), and triangle- and diamond-lines show the two isotropic approximations with velocity ratios $v_{pM}/v_{pR}$ and $v_{pM}/v_{pR}^M$, respectively.

Secondly, we consider the middle-to-far offset. Under the assumption of weak anisotropy (|\epsilon| \ll 1, |\delta| \ll 1), Tsvankin (1996) gave the $P$-wave phase velocity as

$$v_{p}^2(\theta_p) = \alpha_0^2 (1 + 2\varepsilon \sin^2 \theta_p \cos^2 \theta_p + 2\varepsilon \sin^4 \theta_p). \quad (7)$$

The further linearization of equation (7) leads to Thomsen’s (1986) approximation

$$v_{p} = \alpha_0 (1 + \delta \sin^2 \theta_p \cos^2 \theta_p + \varepsilon \sin^4 \theta_p). \quad (8)$$

For $SV$-wave, under the weak anisotropy assumption, we have

$$v_{s}^2(\theta_s) = \beta_0^2 \left( 1 + 2 \frac{\beta_0^2}{\alpha_0^2} (\varepsilon - \delta) \sin^2 \theta_s \cos^2 \theta_s \right). \quad (9)$$

The anisotropic term in equation (9) indicates that $SV$-wave anisotropy is jointly controlled by the anisotropic parameters and by the square of the velocity ratio. For most sedimentary rocks with velocity ratios varying between 1.5 and 4.0, the effects of anisotropy on the $SV$-wave velocity will be very significant even for very weak anisotropy. In such case, further linearization of equation (9) will give a noticeable inaccuracy. For this reason, current analytic approximations for $SV$-waves often lack sufficient accuracy for middle-to-far offset conversion. Here we present a numerical scheme for calculating the conversion-point position:

1. given the phase angles for $P$- and $SV$-waves, calculate the phase and ray velocities and ray angles, and build a phase-to-ray velocity table and a phase-to-ray angle table;
2. given the ray parameter $p = \sin \theta_p / v_{pR}(\theta_p)$ for $P$-wave, find the corresponding $SV$-wave ray velocity and angle, calculate the corresponding offset and conversion-point, and build an $x$-to-$x_p$ table;
3. for given offset, calculate the conversion-point by interpolation from the $x$-to-$x_p$ table.

Figure 3: Difference between the table interpolation and the ray tracing [ANRAY by Gajewski and Pientak, 1987]. The star-line is the difference of conversion points [in meter], and the diamond-line is the travel-time difference [in ms].

The error resulted from the above scheme based on table-interpolation is very small even for very long offset (offset-depth ratio up to 3, Figure 3). After building up the tables, the calculation is very fast and efficient. Thus, the scheme can be used for velocity analysis and movenent correction. This scheme can also be applied for calculating the reflection and transmission point for a dipping interface and anisotropic medium with arbitrary symmetry axis.

Four medium parameters $(v_{pR}, v_{sR}, \epsilon, \delta)$ may be determined for TIV medium using both $P$-$P$ and $P$-$SV$ reflection data. Two parameters can be determined from $P$-$P$ wave velocity analysis using the technique given by Alkhalifah (1996). Using equation (1), the remaining parameters may be obtained from $P$-$SV$ converted waves.

**Multi-layered TIV media**

Consider a stack of $n$ layers and assume the interval properties for the $i$-th layer as $z_i$, $\alpha_{ci}$, $\beta_{si}$, $\epsilon_i$, $\delta_i$ (Figure 4).
At the effective depths $\tilde{z}_p$ and $\tilde{z}_s$ defined by RMS $P$- and $SV$-velocities will not agree with each other.

If the stack of layers has relatively smooth variations of interval vertical velocities through the layers, and if the difference between the average $P$- and $S$- velocity ratio and RMS velocity ratio is negligible, one can prove that the error generated by the single-layered equivalent-medium method will be quadratic in the anisotropy parameter. Thus it is small and negligible for weak anisotropy. We find that the single-layer approach does give results with sufficient accuracy as demonstrated by Zhang (1996) for its isotropic counterparts. This may be not surprising from a data processing point of view. The aim of processing is to identify a best set of parameters which fit the data.

$$\tilde{v}^2_p = \frac{1}{t_{p0}} \sum_{i=1}^{n} t_{pi0} v^2_{p,i}; \quad \tilde{v}^2_s = \frac{1}{t_{s0}} \sum_{i=1}^{n} t_{si0} v^2_{s,i}$$

$$\tilde{v}^2_{ps} = \frac{1}{t_{ps0}} \sum_{i=1}^{n} t_{p0i} v^2_{ps,i}; \quad \tilde{v}^2_{sm} = \frac{1}{t_{s0i}} \sum_{i=1}^{n} t_{s0i} v^2_{sm,i}$$

$$\tilde{v}^2_{pm} = \frac{v^2_{p0} + v^2_{s0}}{t_{ps0}} \approx \tilde{z}_p = \frac{v^2_{p0}}{t_{p0}} \approx \tilde{z}_s = \frac{v^2_{s0}}{t_{s0}}$$

where $t_{p0i}$ and $t_{s0i}$ are the $P$-$P$ and $SV$-$SV$ zero-offset one-way travel times of the $i$-th layer; $t_{p0}$ and $t_{s0}$ are the total zero-offset one-way travel time, and $t_{ps0} = t_{p0} + t_{s0}$ is the effective $P$-$SV$ zero-offset travel time. Note that RMS velocities $\tilde{v}_p$ and $\tilde{v}_s$ are slightly greater than the corresponding average velocities, and that the effective depth $\tilde{z}$ is slightly deeper than actual reflector depth as shown by Figure 4. The tilde variables represent the effective parameters for the equivalent medium. Thus we can calculate the travel time and the conversion point using the single layer equation (1) by substituting these effective parameters into the equations.

The inversion process in the multi-layered case is performed similarly with the single-layered model. After all four anisotropic parameters are obtained, based on equations (10) and (11), Dix-type formula can be performed to inverse anisotropic parameters of each individual layer.

### Accuracy and limits

Equations (10) to (13) define the equivalent medium, and form the basis for our analysis of $P$-$SV$ waves in multi-layered medium. However, strictly speaking, the definitions of the single-layered medium are only accurate for multi-layered medium with constant vertical velocity ratio. If the vertical velocity ratio varies with depth, the average velocity ratio is not exactly equal to the RMS velocity ratio. Thus the effective depths $\tilde{z}_p$ and $\tilde{z}_s$ defined by RMS $P$- and $SV$-velocities will not agree with each other.

Table 1: Model parameters for examining the accuracy of the equivalent medium.

<table>
<thead>
<tr>
<th>rock type</th>
<th>$v_{p0}$</th>
<th>$v_{s0}$</th>
<th>$\epsilon$</th>
<th>$\delta$</th>
<th>$z_i$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dog creek shale</td>
<td>1875</td>
<td>826</td>
<td>0.225</td>
<td>0.100</td>
<td>200</td>
</tr>
<tr>
<td>Pierre shale</td>
<td>2202</td>
<td>969</td>
<td>0.015</td>
<td>0.060</td>
<td>200</td>
</tr>
<tr>
<td>Sandstone shale</td>
<td>3060</td>
<td>1654</td>
<td>0.033</td>
<td>-0.001</td>
<td>200</td>
</tr>
<tr>
<td>Shale-limestone</td>
<td>3366</td>
<td>1819</td>
<td>0.134</td>
<td>0.000</td>
<td>200</td>
</tr>
<tr>
<td>Taylor sandstone</td>
<td>3368</td>
<td>1829</td>
<td>0.110</td>
<td>-0.035</td>
<td>200</td>
</tr>
</tbody>
</table>

Figure 5: Accuracy of the equivalent medium approaches. It shows the difference between the anisotropic ray tracing results and the results calculated using the equivalent medium approach for the five-layer model in Table 1. The star-line shows the conversion-point difference in metres, and the diamond-line shows the travel-time difference in ms.

To give some indications of the accuracy of the equivalent medium approach, we construct a five-layered model with different impedance contrast and anisotropic parameters. Using a spread of 30 geophones with 100m interval, we compare the travel time calculated from the equivalent single-layer model with the exact anisotropic ray tracing time. The conversion point and travel time from the equivalent single-layer model matches the ray tracing results very well and
is sufficiently accurate up to offset-to-depth ratio 1.8 (Figure 5).

Discussion and conclusion
We have presented a more accurate asymptotic approximation of the P-SV conversion point for near-offset conversions and a fast numerical scheme for mid-to-far offset conversions. These lead to the development of a new approach for calculating P-SV moveout in multi-layered TIV media based the equivalent medium concept. The calculation uses the asymmetric raypath, and to some extent, limits the source of error only to layering effects. With either P-properties, or SV-properties given from independent sources, the equation allows directly inversion of the unknown set of parameters. This avoids the introduction of wavetype-dependent properties which complicates the interpretation in subsequent applications as in the Taylor series expansion. We can also utilize this approach to develop velocity analysis and stacking procedure for P-SV waves in anisotropic media.

When this equivalent-medium approach is used for data processing, independent P- or SV-information must be supplied. Otherwise it will be very difficult to implement due to the large number of parameters. In contrast the conventional approach of Taylor series expansion allows the processing of P-SV waves independently even if there is no P- and SV-wave data available by the use of wavetype-dependent parameters.

We conclude from our study that multi-layered horizontal TI V media can be modelled using an equivalent single layer for P-SV wave propagation. The effective travel time depends on four parameters: the RMS P-wave NMO velocity, vertical velocity ratio, and anisotropy parameters $\epsilon$ and $\delta$. When combined with P-, or SV-wave analysis, the unknown P- or SV-wave properties can be generated directly from the converted data using this approach. Interval velocities and anisotropic parameters can be determined from their corresponding RMS measurements using Dix-type equations.

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Reference


MacBeth, C., 1995, How can anisotropy be used for reservoir characterization?: First Break, 13, 1, 31-37.


