Processing P-P and P-S waves for azimuthal anisotropy in multicomponent sea-floor data
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Summary
The azimuthal variations in P-P amplitude, velocity, and interval moveout show elliptical variations in azimuthally anisotropic media. This can be used to determine the fracture strike of the medium and has been verified from real data. However, the P-wave effects only occur at sufficiently large offset with multiazimuths, and are often complicated by other factors. This limits the application of P-wave analysis to some extent. Analysis of P-S waves may thus prove to be beneficial. The azimuthal variations in P-S amplitudes appear to be more sensitive to fracturing, and in particular, for near vertical propagating P-S waves, the polarization and time delay of the shear-wave provide a direct measurement of the fracture orientation and intensity. A robust method is presented for recovering the polarization azimuth of the fast shear-wave for a 3D cross geometry where the source boat sails across the receiver cable.

Introduction
Recently, the use of P-waves has attracted considerable interest because of their relatively low cost in acquisition. These uses include azimuthal P-wave AVO (Lefeuvre 1994, Lynn et al. 1996 and Mallick et al. 1996), and azimuthal variations in P-wave NMO velocity (Sena 1991, Tsvankin 1995), and interval moveout (Li 1997). With the advent of multicomponent sea-floor seismic technology, the study of mode converted shear-waves has become increasingly common in the industry. The mode-converted P-to-S wave retains the benefit of both P- and S-wave surveys (Li et al. 1996) and offers the potential for more cost-effective reservoir characterization and monitoring.

Here, assuming fracture-induced transverse anisotropy with a horizontal symmetric axis (TIH), which is the simplest form of azimuthal anisotropy, I review some of the fundamental effects on wave propagation. These include the amplitude behaviour and velocity effects for P-waves, and the polarization direction and time delay for the P-S waves. For P-wave processing, I will present a new technique to determine the fracture orientation based on the azimuthal variations in the P-wave interval moveout. For P-S wave processing, I will review existing techniques and present new techniques for anisotropy analysis.

AVO analysis
Assume the TIH media are obtained by fracturing isotropic background media, and use \( V_{p0} \) and \( V_{s0} \) as the average P- and S-velocities of the upper and lower background isotropic media. From Li et al. (1996), considering the AVO gradient \( (\sin^2 i) \) and the near-offset \( (\cos i \cos j) \) terms only, the reflection coefficients \( r_{pp} \) and \( r_{ps} \) can be written as,

\[
r_{pp} = r_{p0} + \frac{1}{2} \left[ \Delta \delta_H + \frac{8 V_{s0}^2}{V_{p0}^2} (\gamma_2 - \gamma_1) \right] \sin^2 i, \tag{1}
\]

and

\[
r_{ps} = r_{ps0} + \frac{\sin i}{2 \cos j} r'_{ps} \tag{2}
\]

\[
r'_{ps} = -\frac{V_{p0}^2 - V_{s0}^2}{V_{p0}^2 - V_{s0}^2} \Delta \delta_H + \frac{4 V_{s0}^2}{V_{p0}^2} (\gamma_2 - \gamma_1) \cos i \cos j
- \frac{4 V_{s0}^2}{V_{p0}^2} \Delta \delta_H \cos i \cos j
- \frac{4 V_{s0}^2}{V_{p0}^2} (\gamma_2 - \gamma_1) \sin^2 i + \Delta \delta_H \sin^2 i
- \frac{V_{s0}^2}{V_{p0}^2} (\delta_2 - \delta_1 + 2 \Delta \delta_H) \sin^2 i;
\]

\[
\Delta \delta_H = \delta_2 - \delta_1 - 2 \epsilon_2 + 2 \epsilon_1, \tag{3}
\]

where \( i \) and \( j \) are the average propagation angles of the upper and lower media for \( P \) and \( S \), respectively; \( \epsilon_2, \delta_2, \) and \( \gamma_2 (k = 1, 2) \) are the Thomsen parameters (Thomsen 1986) for the upper \( (k = 1) \), and lower \( (k = 2) \) medium, respectively; \( r_{pp0} \) and \( r_{ps0} \) are the isotropic reflectivities of the background media.

Some immediate observations can be drawn from the first two equations. Firstly, the anisotropy has a first order effect on the AVO response of both P-P and P-S waves through the influence of \( \epsilon, \delta \) and \( \gamma \). Secondly, the anisotropy affects the P-S waves more than the P-P wave, through the near-offset term \( \cos i \cos j \), indicating that the P-S AVO response and its azimuthal variations \( (r_{ps} - r_{p0}) \) are more sensitive to anisotropy, or fracturing, than the P-P AVO response and its azimuthal variations \( (r_{pp} - r_{p0}) \). Thirdly, a smaller angular coverage may be required to reveal the effects of the P-S AVO response because of the presence of the near-offset term \( \cos i \cos j \).

Azimuthal P-wave analysis
P-wave amplitude and velocity. For a fixed offset with incidence angle larger than 15°, the reflection amplitude as a function of azimuth has the following form:

\[
R_{pp}(\phi) = A + B \cos 2 \phi, \tag{4}
\]
where \( A \) and \( B \) are constants. Mallick \textit{et al.} (1996) first presented equation (4) as an empirical expression from numerical modelling, and applied it to 3D land data to quantify fracture strike. Rüger (1996) provided an analytical account of the amplitude variation. Equation (4) forms the basis for azimuthal P- AVO analysis.

For a single reflector, the shot spread normal move- out velocity \( v_{nmo} \) for a given ray at azimuth \( \phi \) can be written as,

\[
v_{nmo}^2(\phi) = v_{p0}^2[1 + 2(\delta - 2\epsilon) \sin^2 \phi]
= C + D \sin^2 \phi, \tag{5}
\]

which yields,

\[
v_{nmo}^2(\phi) = v_{nmo}(\phi = 0) \cos^2 \phi + v_{nmo}^2(\phi = 90^\circ) \sin^2 \phi. \tag{6}
\]

Equation (5) has a similar form to the P-wave azimuthal AVO equation (4), and equation (6) reveals a simple elliptical variation of the NMO velocity along the azimuthal direction. Grechka and Tsvankin (1996) generalized equation (6) for generally inhomogeneous anisotropic media, and Corrigan \textit{et al.} (1996) applied equations (5) and (6) to real data.

\[
\Delta t(\phi, x) = \frac{x - 2x}{v_{p0}^2} \cos 2\phi \sin i_2 \left[ 2\epsilon - \delta - (\epsilon - \delta) \sin^2 i_2 \right] = A(x, \epsilon, \delta) \cos 2\phi,
\]  
\[
\left( \Delta t_1 - \Delta t_2 \right) = \frac{\sin 2\phi}{\cos 2\phi}; \tag{8}
\]

\[
\tan 2\phi = \frac{\sin 2\phi}{\cos 2\phi} = \frac{\Delta t'_2}{\Delta t'_1}. \tag{9}
\]

Thus, for the four line configuration in Figure 2, the cross-plot of \( \Delta t_1 \) versus \( \Delta t'_2 \) shows a linear trend, indicating the direction of \( 2\phi \). A least square analysis of the cross-plot can be used to estimate the fracture strike as,

\[
\phi = \frac{1}{4} \tan^{-1} \left\{ \frac{2 \sum_{x}^{\Delta t_1 \Delta t'_2}}{\sum_{x}^{\left( \Delta t'_1 - \Delta t'_2 \right)^2}} \right\}. \tag{10}
\]

\textbf{P-P interval moveout.} Assume a fractured layer with azimuthal anisotropy overlain by a weakly anisotropic overburden (Figure 1). Consider two orthogonal line- azimuths at angles \( \phi \) and \( \phi + \pi/2 \) to the fracture strike, respectively (Lines 1 & 3, Figure 2). The azimuthal difference of the interval moveout for the fractured layer between these two lines can be written as (Li 1997)

\[
\Delta t_1 = \Delta t(\phi, x) = A \cos 2\phi; \quad \Delta t_2 = \Delta t(\phi + \varphi_0, x) = A \cos 2(\varphi_0 - \phi).
\]

This leads to

\[
\Delta t'_1 = A \sin 2\phi \quad \text{and} \quad \Delta t'_2 = \left( \Delta t_2 - \cos 2\varphi_0 \Delta t_1 \right) / \sin 2\varphi_0; \tag{8}
\]

\[
\tan 2\phi = \sin 2\phi / \cos 2\phi = \Delta t'_2 / \Delta t'_1. \tag{9}
\]

\textbf{P-S polarization analysis}

P-wave anisotropic effects occur only at sufficiently large offsets with wide azimuthal coverage. Consequently, the effects are subtle and often difficult to recover from the data. In contrast, shear-wave anisotropic effects occur at near vertical propagation and are relatively stable and robust.
When a shear-wave enters a fracture-induced anisotropic medium, it splits into two modes which travel with different speeds (Crampin and Lovell 1991). For near vertical propagation, the fast shear-wave polarizes parallel to the fracture strike, and the slow wave polarizes perpendicular to the strike. Furthermore the normalised time-delay between the fast and slow split shear-wave:

\[
\epsilon_{td} = \frac{t_2 - t_1}{t_1} = \frac{C_{44} - C_{66}}{2C_{66}} - \frac{1}{2}\gamma^2 \approx \gamma, \quad (11)
\]

is a measure of the Thomsen parameter \(\gamma\) which is in turn related to the fracture intensity (porosity) in the medium. Thus \(P-S\) wave polarization analysis provides an effective way to determine the fracture strike and density.

\[\begin{bmatrix} V_r(t) \\ V_t(t) \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} S_r(t) \\ S_t(t) \end{bmatrix} = R(\phi) \begin{bmatrix} S_r(t) \\ S_t(t) \end{bmatrix}, \quad (12)\]

where \(R(\phi)\) is the 2D rotation matrix, and \(S_r(t)\) and \(S_t(t)\) represent the amplitudes of the fast and slow waves, respectively.

Assuming that the fast and slow shear-wave have similar waveforms with only a time delay \(\Delta t\), \(S_r(t) = S(t)\), \(S_t(t) = S(t - \Delta t)\), equation (12) can be solved by rotation analysis based on an objective function which measures the similarity of the two waveform. Rotating the radial and transverse components with an angle \(\alpha\) gives the rotated components \(R_{1r}(\alpha, t)\) and \(R_{1t}(\alpha, t)\) as,

\[
\begin{bmatrix} R_{1r}(\alpha, t) \\ R_{1t}(\alpha, t) \end{bmatrix} = R^T(\alpha) \begin{bmatrix} V_r(t) \\ V_t(t) \end{bmatrix}. \quad (13)
\]

An objective function \(F(\alpha, \tau)\) may be defined as,

\[
F(\alpha, \tau) = \sum_t R_{1r}(\alpha, t)R_{1t}(\alpha, t + \tau). \quad (14)
\]

Thus, the optimization process is to search for a rotation angle \(\alpha = \phi\) and time shift \(\tau = \Delta t\), which maximize \(F(\alpha, \tau)\) (Figure 4). MacBeth and Crampin (1991) gave a good review of this kind of techniques. For processing land converted waves, Donati and Brown (1995) introduced the cross-correlation sum of the radial and transverse components as the objective function.

\[\text{Figure 3. Convered shear-waves in 3C sea-floor seismic acquisition over fractured media. The survey line is at angle } \phi \text{ to the fracture strike, and a conversion at the reflection point is assumed.}\]

\[\text{2D Rotation analysis for P-S wave} \quad \text{Consider a 2D acquisition over horizontally stratified media with uniform azimuthal anisotropy. Further assume a P-S raypath with the conversion point at the reflector, and a displacement vector confined to the horizontal plane, } \phi \text{ representing the polarization azimuth of the fast shear wave (Figure 3). The recorded radial and transverse components can be written as,}\]

\[\begin{bmatrix} V_r(t) \\ V_t(t) \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} S_r(t) \\ S_t(t) \end{bmatrix} = R(\phi) \begin{bmatrix} S_r(t) \\ S_t(t) \end{bmatrix}, \quad (12)\]

\[\text{where } R(\phi) \text{ is the 2D rotation matrix, and } S_r(t) \text{ and } S_t(t) \text{ represent the amplitudes of the fast and slow waves, respectively.}\]

\[\text{Assuming that the fast and slow shear-wave have similar waveforms with only a time delay } \Delta t, S_r(t) = S(t), S_t(t) = S(t - \Delta t), \text{ equation (12) can be solved by rotation analysis based on an objective function which measures the similarity of the two waveform. Rotating the radial and transverse components with an angle } \alpha \text{ gives the rotated components } R_{1r}(\alpha, t) \text{ and } R_{1t}(\alpha, t) \text{ as,}\]

\[\begin{bmatrix} R_{1r}(\alpha, t) \\ R_{1t}(\alpha, t) \end{bmatrix} = R^T(\alpha) \begin{bmatrix} V_r(t) \\ V_t(t) \end{bmatrix}. \quad (13)\]

\[\text{An objective function } F(\alpha, \tau) \text{ may be defined as,}\]

\[F(\alpha, \tau) = \sum_t R_{1r}(\alpha, t)R_{1t}(\alpha, t + \tau). \quad (14)\]

\[\text{Thus, the optimization process is to search for a rotation angle } \alpha = \phi \text{ and time shift } \tau = \Delta t, \text{ which maximize } F(\alpha, \tau) \text{ (Figure 4). MacBeth and Crampin (1991) gave a good review of this kind of techniques. For processing land converted waves, Donati and Brown (1995) introduced the cross-correlation sum of the radial and transverse components as the objective function.}\]

\[\text{Figure 4. Determining the polarization azimuth based on the similarity of the fast and slow waves. (a) from top to bottom, rotation angles between 0 and 75°; (b) the corresponding angles between 90° and 165°. The increment is set to 15°. Note that the two waveforms are most similar at rotation angle } \alpha = 60°, \text{ indicating the polarization of the fast shear-wave.}\]
Assume a 3D cross geometry where the source boat sails across the receiver cable. An orthogonal pair of source-receiver azimuths is selected for analysis. Note that the fracture strike (polarization azimuth) is measured from the boat direction x.

**Azimuthal rotation for 3D data.** Assume a 3D cross geometry where the source boat sails across the receiver cable. Figure 5 shows a plan view of the geometry. Denote the boat direction as x, and the cable direction as y. Consider any orthogonal pairs of source-receiver azimuths 1 & 2. Azimuth 1 is at angle $\varphi$ from the boat direction, and azimuth 2 at angle $\pi/2 - \varphi$ to the boat direction (Figure 5). For these two azimuths, after correcting the ray-path difference by moveout correction, the recording components $V_{1x}$ and $V_{1y}$ for azimuth 1 can be combined with $V_{2x}$ and $V_{2y}$ for azimuth 2 as,

$$
\begin{pmatrix}
V_{1x}(t_0) & V_{2x}(t_0) \\
V_{1y}(t_0) & V_{2y}(t_0)
\end{pmatrix} = \mathbf{R}^\varphi \left( \begin{array}{cc}
\lambda_1(t_0) & 0 \\
0 & \lambda_2(t_0)
\end{array} \right)
\times \mathbf{R}(\phi - \varphi) \left( \begin{array}{c}
PS_1(t_0) \\
0 \quad PS_2(t_0)
\end{array} \right)
$$

(15)

where $t_0$ is the moveout-corrected two-way time, $\lambda_1$ and $\lambda_2$ are propagating functions for the fast and slow wave, respectively, and $PS_1(t)$ and $PS_2(t)$ are the effective shear-wave sources at azimuths 1 and 2, respectively.

Rotating the two horizontal components with angle $\varphi$, the ratio of the effective sources $PS_1(t_0)/PS_2(t_0)$ can be estimated from the off-diagonal elements of the rotated data matrix. This ratio can then be used as a scaling factor to compensate for the difference between the effective sources. The rotated and amplitude-corrected data matrix $\mathbf{D}_c(t_0)$ can be written as:

$$
\mathbf{D}_c(t_0) = \mathbf{R}^\varphi(\phi - \varphi) \left( \begin{array}{cc}
S_1(t_0) & 0 \\
0 & S_2(t_0)
\end{array} \right) \mathbf{R}(\phi - \varphi).
$$

(16)

Thus, the polarization azimuth $\phi$ can be solved by minimizing the off-diagonal elements of $\mathbf{D}_c(t_0)$.

To sum up, the following procedures can be used for determining the polarization azimuth for a 3D cross geometry:

1) sorting data into pairs of orthogonal azimuths;

2) applying moveout correction for the azimuthal gather, and forming a 2x2 data matrix using the horizontal components;

3) rotating the horizontal components by $\varphi$ into the azimuthal direction, and estimating the scaling factor and correcting for the amplitude difference;

4) determining the polarization azimuth by minimizing the off-diagonal elements of $\mathbf{D}_c(t_0)$ using the linear-transform technique of Li and Crampin (1993).

**Discussion and conclusions**

I have examined the effects of azimuthal anisotropy on both P-P and P-S waves using analytical expressions for weak anisotropy. This enables some understanding of various dependencies, and allows the development of processing algorithms to extract these effects from multicomponent seismic data.

The effects of azimuthal anisotropy on P-waves are relatively well known. The azimuthal variations in P-P amplitude, velocity, and interval moveout show elliptical variations in azimuthally anisotropic media. This can be used to determine the fracture strike of the medium and has been verified from real data. The use of azimuthal interval moveout has some distinct features. The method, based on a four-line configuration, utilizes cross-plot analysis, shows good flexibility in handling irregular acquisition conditions, and reveals good potential for effective compensation for the overburden anisotropy through the alignment of the top target reflections.
The effect of azimuthal anisotropy on P-S wave amplitudes is less well known. The P-S AVO and its azimuthal variations appear to be more sensitive to fractures than P-P AVO. For near-vertical propagations, the polarization of the fast shear-wave is parallel to the fracture strike, and the time delay between the two split shear-waves is proportional to the fracture intensity. For a 3D cross geometry where the source boat sails across the receiver cable, horizontal components of P-S waves within orthogonal pairs of source-receiver azimuths may be sorted into 2x2 data matrices. After correcting for moveout and amplitude differences, the polarization azimuth can then be determined by minimizing the off-diagonal elements of these data matrices.

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Reference


