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SUGGESTIONS FOR A CONSISTENT TERMINOLOGY FOR SEISMIC ANISOTROPY[1]

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ABSTRACT

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Seismic anisotropy is an unfamiliar concept to many geophysicists and the use of misleading and ambiguous terminology has made it more difficult to understand. I suggest here a consistent terminology in which simple expressions have specific meanings similar to their colloquial meanings. It is hoped that use of such language will help to make the increasing number of papers reporting seismic anisotropy more readily comprehensible to the non-specialist. This not a manual of anisotropy, and it is not intended for theoreticians. It is a list of terms which may make anisotropy a little easier to understand for those more familiar with wave propagation in isotropic solids.

INTRODUCTION

In seismological literature the dominant characteristic of seismic anisotropy is often taken to be the variation of wave velocities with direction. However, an equally important feature of wave propagation in anisotropic solids is the three-dimensional (3D) coupling that links radial, vertical and transverse particle displacements, and leads to shear-wave splitting or birefringence. The structure of the Earth is complicated and it is seldom possible to obtain accurate measurements of velocity over a range of directions in one plane of a material. This means that it is difficult to observe velocity anisotropy.

Shear-wave splitting is distinctive and easily observable, and there are now an increasing number of reports of shear-wave splitting along raypaths in the crust. At the SEG Convention in Houston, November 1986, there were reports of shear-wave splitting in 12 out of 14 shear-wave reflection surveys across North America (Lynn and Thomsen 1986; Alford 1986; Willis, Rethford and Bielanski 1986); in VSPs in the Austin Chalk, Texas (Johnston 1986; Becker and Perelberg 1986), and the Paris Sedimentary Basin, France (Crampin and Bush 1986; Crampin *et al.* 1986); and

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above small earthquakes in many different geological regimes. These many observations were reviewed by Crampin (1987). Shear-wave splitting is now recognized along almost all suitable raypaths in the crust. At the recent Chapman Conference (the Third International Workshop on Seismic Anisotropy), in Berkeley, May 1988, there were over 80 papers on various aspects of seismic anisotropy.

Anisotropy is a multifaceted 3D phenomenon that may be described in different and confusing ways. Such ambiguity adds unnecessary difficulties to understanding anisotropy. At present, whenever any anisotropic phenomenon is mentioned it needs to be qualified to identify the exact meaning. These qualifications are tedious, and authors do not always bother to make them, which can lead to needlessly inaccurate or ambiguous statements and conflicts of meaning.

This paper defines some of the terms used to describe anisotropy in an attempt to unify the terminology so that the increasing number of publications on anisotropy can be more easily read and understood, and results and descriptions by different authors more easily compared. The following list gives some definitions and useful expressions, and suggests discarding a few usages which are misleading. This list presents a consistent terminology which, if adopted, would remove some of the ambiguities from the exploration literature.

I have tried to use expressions which have simple descriptive meanings as these are easier to remember and present. Preferred expressions are in ***italics.*** Alternative expressions that may be less convenient or even misleading are <u>underlined</u>. For example, I suggest ***azimuthal isotropy*** as an alternative to <u>transverse isotropy with a vertical axis of symmetry,</u> so that two words replace seven. Previously, 'transverse isotropy' without any qualification has often been used (ambiguously) to define this spatial orientation, whereas in the original definition (Love 1944) transverse isotropy was synonymous with hexagonal symmetry, a class of anisotropic symmetry, and had no particular spatial orientation.

I also suggest that it is useful to adopt a slightly different notation from that used for discussing propagation in isotropic rocks. If the anisotropy is strong enough to have observable effects, the behaviour of seismic waves in fundamentally different from their behaviour in isotropic rocks, although many of the effects are subtle and may be easily overlooked. A different notation reminds us of these subtleties.

GENERAL TERMINOLOGY

Any uniform material which contains an internal structure (such as a crystal or a distribution of aligned cracks), so that the elastic properties vary with direction, is elastically anisotropic. The variation of properties for purely elastic solids, such as crystals, can be fully described by a fourth-order tensor of anisotropic elastic constants. Similarly, the variation of properties of a compound material containing an internal structure, such as a solid containing aligned cracks or one made up of periodic thin-layers, can be simulated by anisotropic elastic constants. Such simulations of two-phase solids are valid, subject to constraints such that, for example, the dimensions of the cracks or the thickness of the layers must be several times smaller than the wavelength of the seismic waves propagating through the material. In general, such approximations usually begin to break down for a variation of velocity with direction greater than about 10% – see for example the differences between the Garbin and Knopoff (1975) and Hudson (1980, 1981) formulations for aligned cracks illustrated by Crampin (1984b).

Note that an anisotropic solid is essentially a 3D phenomenon and the behaviour of seismic waves in such a solid cannot be completely described in terms of plane sections. For example, Musgrave (1970) presents a concise, elegant treatment of anisotropy, but it is based on algebraic solutions. The algebra of anisotropy is usually only easily resolvable in symmetry planes, and, as a consequence, Musgrave's treatment is heavily biased towards symmetry planes. Symmetry planes are a special case of general anisotropy, and it is usually impossible to extrapolate from the special to the general. Consequently, Musgrave's analysis is correct, but the interpretation is oversimplified and, because the 3D behaviour is barely mentioned, an understanding of wave propagation in anisotropic rocks based only on Musgrave's text could be seriously misleading. Much of the recent understanding of anisotropy has been derived from numerical experimentation with computers and graphical output, where tensor rotation by computer absolves one of the need for extensive algebra (see e.g. Crampin 198 1).

Anisotropic symmetry systems

There are eight physically realizable systems of anisotropic or crystalline symmetry (including isotropic symmetry) and two subsystems, which can be specified by patterns of elastic constants. Figure 1a shows the preferred notation for the fourth-order tensor of elastic constants (c_{ijmn}). The alternative two-suffix notation (c_{ij}) of Fig. lb is not (usually) a tensor, and is thus not strictly consistent with (c_{ijmn}). This might not matter in the literature of exploration geophysics, where mathematical tensors are not often required, except that the two-suffix notation ***can*** be written as a tensor by using modified elastic constants, as in Hudson (198 1). This ambiguity can cause problems unless it is recognized. The preferred four-suffix notation is unambiguous, and in particular is the natural input for any numerical process in a computer involving matrix manipulation. Another alternative, the capital letter notation (A, B, C, . ..) of Fig. 1c (Love 1944) is not easy to memorize, and again does not easily lend itself to numerical computations.

Figure 2 lists the tensors of elastic constants that specify the various classes of anisotropic symmetry. It can be shown that the eight systems and two minor subsystems contain all possible systems of elastic symmetry (Crampin 1984a). Crystals in all these systems exist, but symmetry in the Earth is almost always aligned directly or indirectly by stress. Since stress has essentially orthorhombic symmetry, in the sense of possessing three mutually orthogonal symmetry planes, all symmetries commonly present in the Earth also contain orthorhombic or higher symmetry systems, or combinations of orthorhombic symmetries resulting in lower

(a) PREFERRED FOUR-SUFFIX NOTATION

c_{1111}	c_{1122}	c_{1133}	c_{1123}	c_{1131}	c_{1112}
c_{2211}	c_{2222}	c_{2233}	c_{2223}	c_{2231}	c_{2212}
c_{3311}	c_{3322}	c_{3333}	c_{3323}	c_{3331}	c_{3312}
c_{2311}	c_{2322}	c_{2333}	c_{2323}	c_{2331}	c_{2312}
c_{3111}	c_{3122}	c_{3133}	c_{3123}	c_{3131}	c_{3112}
c_{1211}	c_{1222}	c_{1233}	c_{1223}	c_{1231}	c_{1212}

ALTERNATIVE NOTATIONS

(b) TWO-SUFFIX

c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}
c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}
c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}
c_{41}	c_{42}	c_{43}	c_{44}	c_{45}	c_{46}
c_{51}	c_{52}	c_{53}	c_{54}	c_{55}	c_{56}
c_{61}	c_{62}	c_{63}	c_{64}	c_{65}	c_{66}

(c) CAPITAL LETTER

A	*H*	*G*	*P*	*Q*	*R*
H	*B*	*F*	*S*	*T*	*U*
G	*F*	*C*	*V*	*W*	*X*
P	*S*	*V*	*L*	*K*	*J*
Q	*T*	*W*	*K*	*M*	*I*
R	*U*	*X*	*J*	*I*	*N*

FIG. 1. (a) The preferred ***four-suffix representation*** for the symmetrical fourth-order tensor of elastic constants, where the constants are subject to the symmetries $c_{jkmn} = c_{kjmn} = c_{mnjk}$. Alternative notations are: (b) the two-suffix notation, and (c) the capital letter notation of Love (1944).

ANISOTROPIC SYMMETRY SYSTEMS

Triclinic (21)

c_{1111}	c_{1122}	c_{1133}	c_{1123}	c_{1131}	c_{1112}
c_{1122}	c_{2222}	c_{2233}	c_{2223}	c_{2231}	c_{2212}
c_{1133}	c_{2233}	c_{3333}	c_{3323}	c_{3331}	c_{3312}
c_{1123}	c_{2223}	c_{3323}	c_{2323}	c_{2331}	c_{2312}
c_{1131}	c_{2231}	c_{3331}	c_{2331}	c_{3131}	c_{3112}
c_{1112}	c_{2212}	c_{3312}	c_{2312}	c_{3112}	c_{1212}

Monoclinic (13)

c_{1111}	c_{1122}	c_{1133}	0	0	c_{1112}
c_{1122}	c_{2222}	c_{2233}	0	0	c_{2212}
c_{1133}	c_{2233}	c_{3333}	0	0	c_{3312}
0	0	0	c_{2323}	c_{2331}	0
0	0	0	c_{2331}	c_{3131}	0
c_{1112}	c_{2212}	c_{3312}	0	0	c_{1212}

Orthorhombic (9)

c_{1111}	c_{1122}	c_{1133}	0	0	0
c_{1122}	c_{2222}	c_{2233}	0	0	0
c_{1133}	c_{2233}	c_{3333}	0	0	0
0	0	0	c_{2323}	0	0
0	0	0	0	c_{3131}	0
0	0	0	0	0	c_{1212}

Trigonal I (6)

c_{1111}	c_{1122}	c_{1133}	c_{1123}	0	0
c_{1122}	c_{1111}	c_{1133}	$-c_{1123}$	0	0
c_{1133}	c_{1133}	c_{3333}	0	0	0
c_{1123}	$-c_{1123}$	0	c_{2323}	0	0
0	0	0	0	c_{2323}	c_{1123}
0	0	0	0	c_{1123}	c_{1212}

where $c_{1212} = (c_{1111} - c_{1122})/2$

FIG. 2. The arrangement of elastic constants for all possible systems of anisotropic symmetry with x, y, z (1, 2, 3) as the principal axes. The number in brackets after the system names is the maximum number of independent elastic constants possible for the particular symmetry system. The c_{ijkm} are independent elastic constants, except where stated otherwise. If some of these constants are zero, the solid may be placed in a system of higher order symmetry. (After **Nye** 1969.)

Trigonal II (7)

$$\begin{matrix} c_{1111} & c_{1122} & c_{1133} & c_{1123} & c_{1131} & 0 \\ c_{1122} & c_{1111} & c_{1133} & -c_{1123} & -c_{1131} & 0 \\ c_{1133} & c_{1133} & c_{3333} & 0 & 0 & 0 \\ c_{1123} & -c_{1123} & 0 & c_{2323} & 0 & -c_{1131} \\ c_{1131} & -c_{1131} & 0 & 0 & c_{2323} & c_{1123} \\ 0 & 0 & 0 & -c_{1131} & c_{1123} & c_{1212} \end{matrix}$$

where $c_{1212} = (c_{1111} - c_{1122})/2$

Tetragonal I (6)

$$\begin{matrix} c_{1111} & c_{1122} & c_{1133} & 0 & 0 & 0 \\ c_{1122} & c_{1111} & c_{1133} & 0 & 0 & 0 \\ c_{1133} & c_{1133} & c_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{2323} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{2323} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{1212} \end{matrix}$$

Tetragonal II (7)

$$\begin{matrix} c_{1111} & c_{1122} & c_{1133} & 0 & 0 & c_{1112} \\ c_{1122} & c_{1111} & c_{1133} & 0 & 0 & -c_{1112} \\ c_{1133} & c_{1133} & c_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{2323} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{2323} & 0 \\ c_{1112} & -c_{1112} & 0 & 0 & 0 & c_{1212} \end{matrix}$$

Cubic (3)

$$\begin{matrix} c_{1111} & c_{1122} & c_{1122} & 0 & 0 & 0 \\ c_{1122} & c_{1111} & c_{1122} & 0 & 0 & 0 \\ c_{1122} & c_{1122} & c_{1111} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{2323} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{2323} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{2323} \end{matrix}$$

Hexagonal (5)

$$\begin{matrix} c_{1111} & c_{1122} & c_{1133} & 0 & 0 & 0 \\ c_{1122} & c_{1111} & c_{1133} & 0 & 0 & 0 \\ c_{1133} & c_{1133} & c_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{2323} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{2323} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{1212} \end{matrix}$$

where $c_{1212} = (c_{1111} - c_{1122})/2$

Isotropic (2)

$$\begin{matrix} c_{1111} & c_{1122} & c_{1122} & 0 & 0 & 0 \\ c_{1222} & c_{1111} & c_{1122} & 0 & 0 & 0 \\ c_{1122} & c_{1122} & c_{1111} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{2323} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{2323} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{2323} \end{matrix}$$

where $c_{1212} = (c_{1111} - c_{1122})/2$

FIG. 2 *(continued).*

order symmetry systems such as in monoclinic systems. [A higher order symmetry system is one which contains more symmetry planes. This is only strictly applicable to the sequence: triclinic (no symmetry planes); monoclinic (one symmetry plane); orthorhombic (3); tetragonal (5); cubic (9); hexagonal (cylindrical symmetry); and isotropy (complete symmetry). Trigonal symmetry, with three symmetry planes, is outside this sequence, and except for 'individual crystals is not commonly present in the Earth.]

A physically meaningful classification of anisotropic symmetry for wave propagation is by the arrangement of planes of mirror symmetry shown in Fig. 3. (Note that Trigonal II in Fig. 2 has no symmetry planes but has rotational symmetry every 120" about the x,-axis, and Tetragonal II has one symmetry plane $x_3 = 0$ and rotational symmetry every 90" about the x_3-axis. Both these subsystems are not common in the Earth or in crystallography, but are included for completeness.) In general, the names can be visualized in terms of the arrangement of symmetry planes in Fig. 3. Thus: monoclinic – one symmetry plane; orthorhombic – three mutually orthogonal symmetry planes; trigonal – three symmetry planes; tetragonal, trigonal, and cubic – respectively, tetragonal, trigonal and cubic arrangements of symmetry planes (see Fig. 3). The two apparent exceptions are:

(1) Triclinic symmetry, where there are no symmetry planes. It is so called because the three axes have different lengths and are obliquely inclined.
(2) Hexagonal symmetry, which has transverse isotropy. It is called hexagonal because elastic constants cannot distinguish between transverse isotropy (with cylindrical symmetry) and hexagonal prismatic crystals. For crystallography the crystalline analogy takes precedence.

The best reason for retaining these names is that they are firmly established in a wide variety of literature.

Symmetry plane .or ***plane*** of ***mirror symmetry*** is a plane in which the elastic properties have reflection symmetry. Planes of symmetry are arranged in patterns in space that are characteristic of the class of anisotropic symmetry (Fig. 3). Mathematically, the plane $x_p = 0$ is a plane of symmetry, if and only if $c_{ijmn} = 0$ whenever one or three of i, j, m, n, are equal to p, and we see why the spatial pattern of symmetry planes in Fig. 3 is related to the pattern of zeros and symmetrical constants in Fig. 2.

Sagittal plane is the vertical plane through source and receiver.

Sagittal symmetry. A sagittal plane is said to have sagittal symmetry when it is also a symmetry plane.

I suggest that the term *principal axes of anisotropy* should be restricted to meaning the three mutually orthogonal Cartesian axes to which the anisotropic elastic constants are usually referred (see Fig. 3). The expression has been used in other ways which may be misleading and ambiguous: see entry in the section of alternative expressions.

Symmetry axis. The only (unambiguous) symmetry axis in anisotropy is the axis of cylindrical symmetry in systems of hexagonal symmetry. Also see ***principal axes of anisotropy*** in the section on alternative expressions.

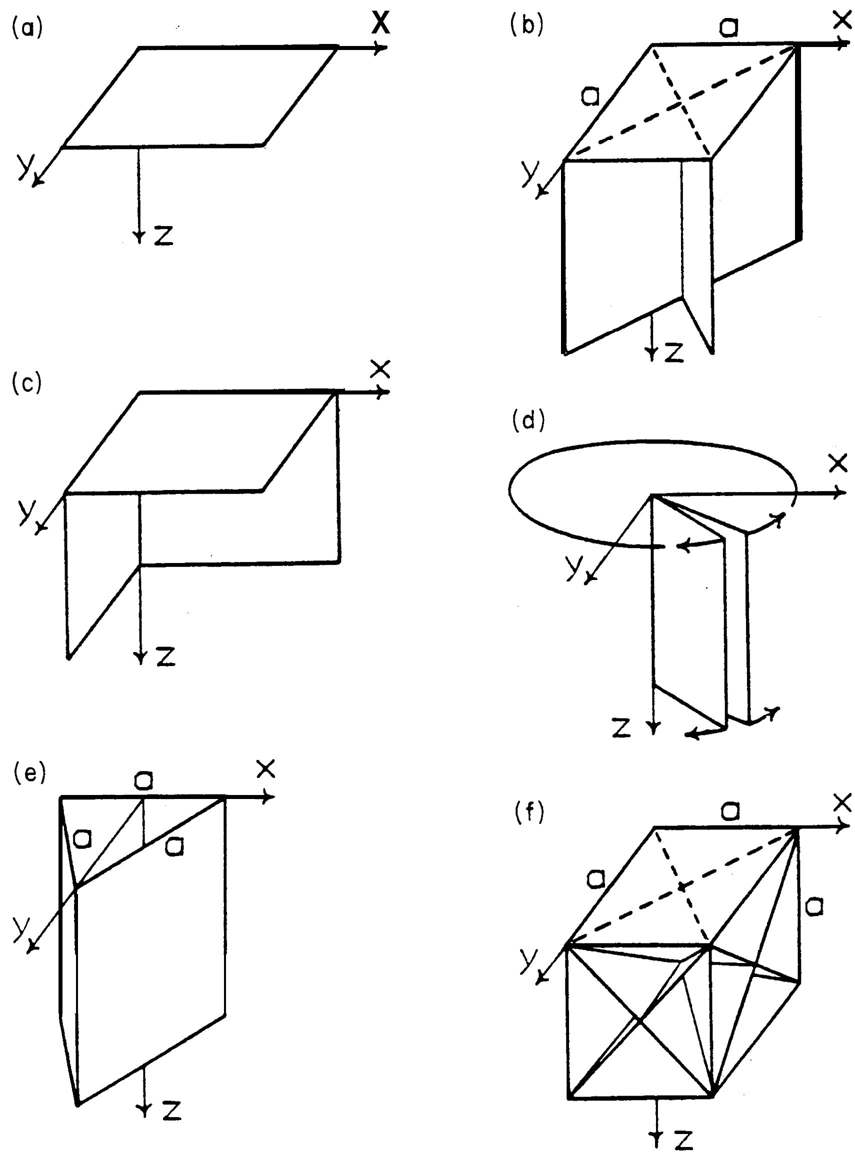

Fig. 3. Spatial orientations of symmetry planes of the more common symmetry systems: (a) monoclinic, (b) tetragonal, (c) orthorhombic, (d) hexagonal, (e) trigonal, and (f) cubic. The triclinic system (Fig. 2) has no symmetry planes, and every plane of an isotropic system is a symmetry plane. (After Crampin 1984a.)

Velocity-anisotropy is the percentage measure of the maximum variation in velocities usually specified as $(V^{max} - V^{min})/V^{max}$ x 100, where V^{max}, and V^{min} are the maximum and minimum velocities, respectively. (The velocity-anisotropy in a range of observations is also sometimes known as the *degree of anisotropy.)* V^{max} and V^{min} may range either over all directions or only over directions in a plane (the meaning is usually clear from the context) and do not necessarily refer to directions along crystallographic axes of the anisotropy. Often assumed to refer to P-wave anisotropy, the wave-type of the velocity-anisotropy should be specified if there is any chance of confusion. The P-wave velocity-anisotropy has also sometimes been called, misleadingly, the coeffkient of anisotropy. When describing the velocity variations of shear waves, I suggest that the use of velocity-anisotropy should be confined to the velocity variation of a single specified wave-type (e.g. the faster split shear-wave), and ***differential shear-wave anisotropy*** used for the percentage difference in velocity of the two split shear-waves. Note that parameterizing a 3D vector variation by a single scalar quantity, such as velocity-anisotropy (coefficient of anisotropy) is a crude measure which can be misleading. Such terms should be avoided whenever possible. However, it is sometimes convenient to use the **velocity-**anisotropy as a measure of the overall velocity variation.

Phase velocity is the velocity in the direction of the phase propagation vector, normal to the surface of constant phase. This is the velocity of propagation of a plane wave, and is the velocity which appears (naturally) in most analytical and numerical expressions.

Group velocity, replacing ray velocity, is the velocity of energy transport and the velocity of a signal along a seismic ray. The group velocity is almost always the velocity determined from the ratio of the source distance and the traveltime in field observations. The directions of phase-velocity- and group-velocity-propagation diverge for most directions of propagation in anisotropic rocks, so that the seismic ray is not generally normal to the surface of constant phase.

Phase-velocity surface or ***velocity surface*** is the locus of phase propagation vectors through a point. There is usually no physical arrival corresponding to the phase-velocity **surface,** but it is a useful concept theoretically, as the more important group-velocity surface is derived by differentiation from the phase-velocity surface. Note that in mathematical literature 'surfaces' are often referred to as 'sheets'. In general, there are three phase-velocity surfaces for any anisotropic solid: an outer P-wave surface, and two inner shear-wave surfaces, which are analytically continuous and necessarily touch in ***shear-wave singularities*** **(Crampin** 1981). Note that the polarizations of the three plane waves, a P-wave and two shear-waves, propagating in the same direction of phase-velocity propagation are strictly orthogonal.

Group-velocity surface or ***wave surface*** is the locus traced out by all waves radiating from a point source along seismic rays at the group velocity. The wave surfaces of P-waves vary smoothly and cannot contain cusps, but the group-velocity surfaces of shear waves may have cusps (cuspoidal **fins** and ridges) when the **shear-**wave velocity-anisotropy is greater than about 10%. The group-velocities of shear waves have severe complications (due to the relation between group and phase velocities) in directions close to where the two velocity surfaces touch in ***shear-wave singularities,*** although the surface of the group-velocity surfaces may still vary smoothly.

Polarization diagrams, replacing hodograms, are projections on specified planes of the particle displacements for successive time intervals along the three-component seismograms. It is convenient sometimes to take cross-sections fixed in space, Vertical, North-South, East-West, and sometimes fixed in the sagittal plane, Vertical, horizontal Radial, and horizontal Transverse. It is seldom useful to plot, and frequently difficult to interpret, polarization diagrams oriented with respect to the dynamic axes along the **raypath,** Vertical (normal to the ray), **Radial** (parallel to the ray), horizontal Transverse to the ray. Polarization diagrams of shear waves have characteristic patterns which are effective ways of displaying the subtle phase and amplitude variations between three-component seismograms, and are crucial to recognizing and understanding the behaviour of shear waves in anisotropic rocks.

COMMON SYMMETRY SYSTEMS IN THE EARTH

Isotropy. In a uniform isotropic rock all planes are planes of symmetry and the elastic properties are the same in all directions. Isotropy is usually a result of uncracked intrinsic isotropy, randomly cracked rock, or rock with random crystal or grain orientations. In uniform isotropic rock, the velocity of shear waves is constant and is independent of the polarization of the particle displacements.

Hexagonal symmetry or *transverse isotropy* has an axis of cylindrical symmetry (see Fig. 3). *This is usually the result of parallel cracks (Crampin 1978, 1984b), cracks with co-planar normals (Crampin and Radovich 1982), aligned grains (e.g. in shales, see Kaarsberg 1968), or repeated sequences of fine layers (PTL-anisotropy, see below).

Orthorhombic symmetry has three mutually orthogonal planes of symmetry. The orthorhombic symmetry of the upper mantle is believed to be caused by orthorhombic crystals of olivine aligned relative to the spreading centres (Hess 1964). Orthorhombic symmetry is also expected to occur in sedimentary basins as a result of combinations of vertical cracks with a horizontal axis of symmetry, and periodic thin-layer anisotropy *(PTL-anisotropy)* with a vertical symmetry axis (Bush and Crampin 1987). Note that the stress tensor has three mutually orthogonal symmetry planes, and any stress-aligned phenomenon is also likely to have orthorhombic symmetry.

Monoclinic symmetry has only one plane of symmetry. This is the symmetry of two sets of non-orthogonal parallel cracks, where the plane of symmetry is perpendicular to the line of intersection of the two sets of crack faces (Crampin, McGonigle and Bamford 1980). Monoclinic symmetry of systems of cracks is likely only very near the surface, where lithostatic pressures have not closed cracks perpendicular to the maximum compressional stress.

SPATIAL ORIENTATIONS IN THE EARTH

Azimuthal anisotropy is anisotropy where there is a variation of properties with azimuth in the horizontal plane.

Azimuthal isotropy, replacing transverse isotropy used as a spatial orientation, is anisotropy where there is no variation of properties with azimuth in the horizontal plane. This is the result of hexagonal symmetry with a vertical axis of symmetry.

Orthotropic orientation has three mutually orthogonal planes of symmetry and one of the planes is horizontal (this may be a new usage, but is a convenient terminology which would remove some ambiguity). Note that, with this usage, orthotropic is a spatial orientation and is not a class of anisotropic symmetry: cubic, hexagonal, tetragonal and orthorhombic symmetry systems may all have orthotropic orientation. Since the orientation of stress in the crust of the Earth is usually orthotropic, most symmetry systems in the Earth's crust also have orthotropic orientation. The Earth's upper mantle displays azimuthal anisotropy (with orthotropic orientation) in P_n-wave velocity-anisotropy in oceanic basins, shear-wave splitting above deep earthquakes, and particle trajectories of higher mode seismic surface waves. This is caused by aligned crystals (usually considered to be principally olivine) in the upper mantle (Hess 1964).

Extensive-dilatancy anisotropy (EDA). The Earth's crust also displays azimuthal anisotropy, usually with orthotropic orientation (Crampin 1987). The crust appears to be pervaded by distributions of parallel vertical fluid-filled cracks, microcracks, or preferentially oriented pore-space. They are effectively anisotropic to seismic waves and known as ***extensive-dilatancy anisotropy (EDA)***: extensive because it exists almost everywhere in the crust; dilatancy because it consists of distributions of open cracks; and anisotropy because it is effectively anisotropic (Crampin, Evans and Atkinson 1984; Crampin 1987). ***EDA-cracks*** is, I suggest, a convenient term to describe something which appears to exist almost everywhere in the crust. The azimuthal anisotropy in the crust reported in the introduction appears to be the result of propagation through EDA-cracks.

EDA-cracks are usually aligned vertically, perpendicular to the minimum horizontal compression and have a horizontal axis of cylindrical symmetry and orthotropic orientation, similar to the orientations of large hydraulic fractures (Hubbert and Willis 1957). These alignments are a result of the stress relation, below the topmost layers of the crust. This is usually $S_H, S_v > S_h$, where S_h is the minimum horizontal stress, S_H the maximum, and S_v the vertical stress. This leads to stress-induced EDA-anisotropy with hexagonal symmetry and an axis of symmetry parallel to the direction of minimum horizontal stress. This stress relation often persists to near the surface so that all EDA-cracks are essentially vertical, but in some cases the zero vertical stress at the surface may result in horizontal EDA-cracks. There is also evidence in some areas that in the topmost layers the stress relation may be $S_H > S_v, S_h$ (Herget 1987), which would lead to cracks with coplanar normals: a system which also leads to stress-induced anisotropy of hexagonal symmetry, but with the axis of symmetry parallel to the direction of maximum horizontal stress (Crampin and Radovich 1982). Note that stress anomalies near the free surface may modify the orientations of EDA-cracks.

PTL-anisotropy is the name suggested for the Periodic Thin-Layer anisotropy described by Postma (1955). (Note that periodicity is not strictly necessary (Backus 1962) but to produce a rock with uniform properties some repetition is required) Fine layers in the crust, particularly in sedimentary basins, lead to PTL-anisotropy with a nearly vertical axis of symmetry. The combination of PTL-anisotropy, with a vertical axis of symmetry, and EDA-anisotropy, with a horizontal axis of symmetry, leads to orthorhombic symmetry with orthotropic orientation (Bush and Crampin 1987), and appears to be a common phenomenon in sedimentary basins.

SHEAR-WAVE PROPAGATION

Shear-waves. It is suggested that ***shear-waves*** should replace S-waves in anisotropic rocks. The 'S' of S-waves originally referred to the secondary arrival. In isotropic rock there is usually no conflict with identifying the secondary arrival with transverse shear-wave motion. In anisotropic rock, however, there are two shear-wave arrivals and it is not strictly correct to refer to them as S(secondary)-waves. As conflicts of meaning can sometimes arise, it is useful to refer to shear-waves in anisotropic rock as ***shear-waves*** not S-waves. This is a small point, but the use of 'shear-waves' rather than S-waves would indicate that the fundamental distinction between propagation in anisotropic and propagation in isotropic solids has been recognized by the author.

The ***shear-wave window,*** for a geophone at the surface, is the range of angles of incidence (the solid angle) within which the waveforms of shear-waves recorded by the geophone will be similar to those of the incident wave (Booth and Crampin 1985). Shear-waves observed outside the window suffer severe distortions of phase and amplitude so that it is difficult to reconstruct the waveforms of the incident shear-wave. The window is (strictly) defined for plane waves incident on the surface of a uniform isotropic half-space as incidence less than the critical angle arcsin (Vs/Vp), which is approximately 35" for a Poisson's ratio of 0.25. The appropriate Vp and Vs for an anisotropic half-space would be the horizontal P- and (either of two) shear-wave velocities in that particular azimuth of propagation. The effective critical angle for a curved wavefront may be increased by about 5°. An alternative, but consistent, terminology in earthquake seismology, is that the ***shear-wave window*** is the area above the (subsurface) source where the angles of incidence are less than the critical angle.

Shear-wave splitting is suggested to replace birefringence or double refraction. A shear-wave entering a region of anisotropy generally splits into the two (or more) quasi-shear-waves with different velocities and with different nearly-orthogonal polarizations fixed for the particular direction of propagation (Crampin 1981). The only exceptions are when the incident wave is parallel to one of the fixed polarizations, so that only one split shear-wave is transmitted. Figure 4 illustrates shear-

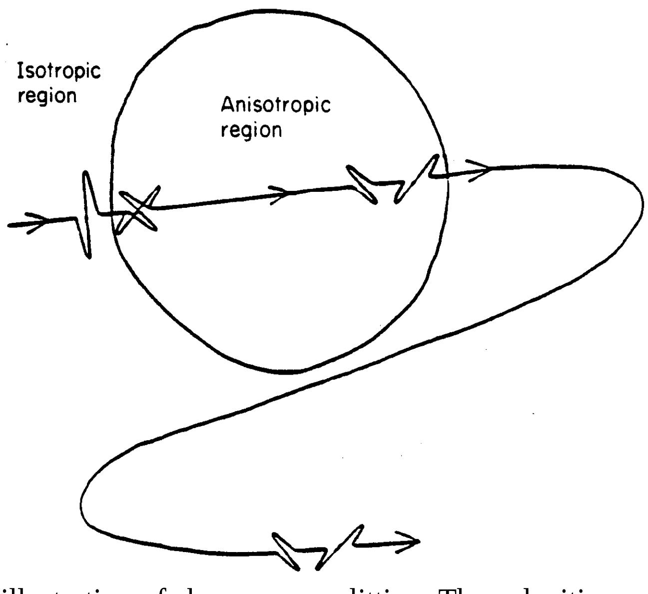

FIG. 4. Schematic illustration of shear-wave splitting. The velocities and orientations of the split shear-waves are fixed by the particular direction of propagation through the anisotropic symmetry.

wave splitting schematically. Shear-wave splitting is the most diagnostic evidence of anisotropy in complicated Earth structures.

Split shear-waves are the result of shear-wave splitting.

qP-, qS1-, and ***qS2-waves,*** replacing P- and S-waves, are the three quasi-body-waves propagating in every direction of phase propagation in an anisotropic solid. ***Quasi-,*** replacing pseudo-, refers to when the polarization is not exactly radial or transverse, and the prefix '*q*' is sometimes dropped when no confusion can arise. ***qS1-*** is the faster and ***qS2-*** the slower split shear-wave. The three waves propagating with the same direction of phase propagation have mutually orthogonal polarizations. Note that the displacement of ***qP*-waves** is usually within one or two degrees of the direction of the raypath for anisotropy less than about 10% (Crampin, Stephen and McGonigle 1982).

qSP- and ***qSR-waves*** are the two quasi-shear-waves propagating in a symmetry plane which necessarily have polarizations (strictly) parallel (***qSP***) and at right angles (***qSR***) to the symmetry plane. The ***qSR*-wave** is strictly transverse, and the qualifying '*q*' for quasi could be omitted if no confusion is likely to occur.

qSH- and ***qSV-waves*** (or sometimes ***SH-*** and W-waves, where no confusion can arise) are the names given to shear-waves with motion (strictly) in the horizontal transverse direction and the sagittal plane, respectively. This notation agrees with the conventional notation for propagation in isotropic rocks. In anisotropy, such motion is only possible in horizontal or vertical symmetry planes, so that in azimuthal isotropy all shear-waves are either polarized ***qSH*** or ***qSV***. Note that the difficulty of using ***qSH*** and ***qSV*** in non-symmetry directions, by defining them as having for example SH-like and W-like particle motion; is that although the terminology may be unambiguous in any given direction, as the direction of propagation changes, the polarization also changes. Thus, no matter how weak the anisotropy, any particular wave may change continuously (and analytically) and vary from ***SH*-like** motion, say, to W-like motion. I have found this gives rise to complications in nomenclature, which are unnecessary, and can be avoided by using ***qSP*** and ***qSR*** for in a symmetry plane, ***qSH*** and ***qSV*** for azimuthal isotropy, and ***qS1 and qS2 for the faster and slower split shear-waves, respectively, in all other anisotropic symmetry conditions.***

Time delay or ***delay*** is the difference in arrival time of the split shear-waves. It is a direct indication of the amount of velocity-anisotropy along a particular raypath and can be measured directly from the three-component seismograms, or, more accurately, by counting samples in polarization diagrams.

Differential (or ***relative) shear-wave anisotropy*** is the percentage measure of the difference in velocity anisotropy of the two shear-waves. It is defined as $(V_{qS1}^{max} - V_{qS2}^{min})/V_{qS1}^{max}$ x 100, where ***qS1*** is the faster and ***qS2*** the slower split shear-wave, 'and V_{qS1} and V_{qS2} may range over all directions, or over directions confined to a plane. The exact meaning should be specified unless it is clear from the context. An estimate of the differential shear-wave anisotropy in a particular direction can often be made from the time delay between the split shear-waves on a single three-component seismogram. As with velocity-anisotropy, this single quantity is a crude measure of 3D variations and should be used with discretion.

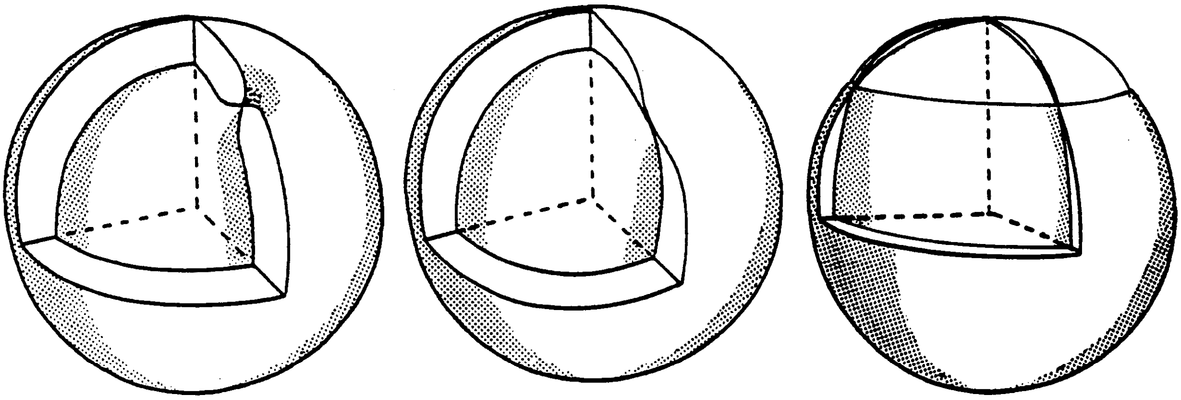

FIG. 5. Schematic illustration of the three types of shear-wave singularity, where the two shear-wave velocity surfaces touch. (a) Point singularity, where the two surfaces touch at the vertices of (usually shallow) conical projections; (b) kiss singularity, where the two surfaces touch tangentially; and (c) line singularity, where the two surfaces may be considered as intersecting (possible only in systems of hexagonal symmetry, where the line singularity is a concentric circle about the symmetry axis and where there is a kiss singularity in the direction of the symmetry axis, as illustrated). (After Crampin and Yedlin 1981.)

Shear-wave singularities. The two shear-wave phase-velocity surfaces are necessarily analytically continuous through a limited number of singular points arranged in spatial patterns typical of the class of anisotropic symmetry (Crampin and Yedlin 1981). These *singularities* have complicated effects on the group velocity and group-velocity surface (the wave surface) which may be severely disturbed in directions near such points (Crampin 1981). There are three types of singularity, which are illustrated schematically in Fig. 5.

Point singularities, the commonest type of shear-wave singularity, are where the two phase-velocity surfaces intersect in a number of isolated points at the vertices of (usually shallow) cones projecting from the surface (Crampin and Yedlin 1981). For this reason, in optics, point singularities are known as conical points. The only way for a seismic ray to pass from one surface to the other is by following a range of phase-velocity directions which pass exactly through such a point singularity. Figure 5a shows a point singularity for seismic shear-waves. Point singularities occur in all symmetry classes except hexagonal and isotropy, and lead to complicated behaviour, particularly in shear-wave polarizations, in group-velocity surfaces. Near a singularity, the wave surface of the faster shear-wave, ***qS1***, has a hole; and the wave surface of the slower shear-wave, ***qS2,*** has a flat cuspoidal lid or disc which fits exactly into the hole of the outer sheet (Crampin 1981). The direction of the point singularity in the phase-velocity surface transforms to the edge of the hole in the outer group-velocity surface, and the rim of the lid on the inner surface. The two surfaces join smoothly, unless there are cusps due to the high curvature of the phase-velocity surfaces. The combined ***qS1*** and ***qS2*** group-velocity surfaces may still vary smoothly, and the complications arise because of the rapid variations of phase-velocity for small changes in direction of the seismic ray (or vice versa). This causes particular anomalies in the polarization of the ray, and may lead to amplitude variations similar to those associated with conventional caustics. Point singularities may disturb shear-waves propagating along nearly vertical raypaths in sedimentary basins (Crampin 1989) if there are appropriate combinations of EDA- and PTL-anisotropy (Bush and Crampin 1987).

Kiss singularities are points where the two phase-velocity surfaces touch tangentially (Fig. 5b). There is always a kiss singularity in the direction of the symmetry axis in hexagonal systems (in the vertical direction in PTL-anisotropy), and 'they may also occur for specific combinations of elastic constants in other symmetry systems. Kiss singularities are unlikely to seriously disturb rays of shear-waves.

Line singularities, replacing intersection singularities, occur where the two-phase-velocity surfaces touch along a (circular) line, and are only possible in systems of hexagonal symmetry (Fig. 5c). The line singularity in hexagonal symmetry systems is the only occasion that the two shear-wave phase-velocity surfaces may be considered as a simple intersection of two surfaces (Crampin and Yedlin 1981). Line singularities in group-velocity surfaces may also be considered as a simple intersection of two sheets, although the intersections of the phase- and group-velocity surfaces will be at slightly different directions. There are line singularities at about 60° from the symmetry axes in both EDA- and PTL-anisotropy, where the circular intersections are in vertical and horizontal planes, respectively. Line singularities cause little disturbance to the polarization of rays of shear-waves.

ALTERNATIVE (AND POSSIBLY MISLEADING) EXPRESSIONS

Slowness, where S_v is the reciprocal of the phase velocity, and S_w is the reciprocal of the group velocity, is a useful concept for some geometrical relations between the velocities in a uniform anisotropic solid (Musgrave 1970), but has little application in computational and observational seismology.

Slowness surface is the polar reciprocal of the group-velocity surface. After the phase- and group-velocity surfaces, it is the third characteristic surface of wave propagation in anisotropic solids. It has some simple geometrical relations with the other surfaces, but has little application to practical seismology.

Transverse isotropy when it is used as a spatial orientation.

Orthotropic symmetry when it is used as a synonym for orthorhombic symmetry.

Principal axes of anisotropy or fast direction are terms which have sometimes been used about shear-wave splitting in the upper mantle, referring to the two shear-wave polarization directions at the surface, and the direction of the faster split shear-wave, respectively. These terms suggest that the directions of shear-wave polarization are the same for all propagation directions to the surface. There is no anisotropic solid which has parallel shear-wave polarizations over a hemisphere of directions, although many have parallel polarizations within a solid angle large enough to fill the shear-wave window at the surface when there is an appropriate orientation of the anisotropy.

Elliptical anisotropy or elliptic symmetry is used in various ways, but usually refers to an ellipsoidal slowness surface, and correspondingly, an ellipsoidal group-velocity surface. Although a few solids do have approximately elliptical variations of P-wave slowness and group-velocity surfaces, most do not. Some common systems, such as liquid-filled cracks, have P-wave velocities with a cos (4θ)-variation with direction so that a plane section of the slowness surface has ' bulges' in four directions and is clearly not ellipsoidal, and most P-wave slowness and group-velocity surfaces are a long way from elliptical anisotropy. The only slowness surface which has a strictly ellipsoidal section is the *qSR*-wave in transverse isotropy (the *qSH*-wave in azimuthal isotropy).

Conclusions

The investigation of effective anisotropy in the Earth has only just begun; there will certainly be new developments and priorities will change. The statements here are true for anisotropy, however weak, as long as the anisotropy is strong enough to have some effect on seismic waves. If it has no effect, it is seismically isotropic. I have tried to suggest terms which are unlikely to be out-dated too soon, but we must expect some modifications to be required in the future. I welcome comments and suggestions so that a revised notation can be published in due course.

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References

ALFORD, R.M. 1986. Shear data in the presence of azimuthal anisotropy; Dilley, Texas. 56th SEG meeting, Houston, Expanded Abstracts, 476-479.

BACKUS, G.E. 1962. Long-wave elastic anisotropy produced by horizontal layering. *Journal of Geophysical Research* **67**, 4427–4440.

BECKER, D.F. and PERELBERG, A.I. 1986. Seismic detection of subsurface fractures. 56th SEG meeting, Houston, Expanded Abstracts, 466468.

BOOTH, D.C. and CRAMPIN, S. 1985. Shear-wave polarizations on a curved wavefront at an anisotropic free-surface. *Geophysical Journal of the Royal Astronomical Society* **83**, 31-45.

BUSH, I. and CRAMPIN, S. 1987. Observations of EDA and PTL anisotropy in shear-wave VSPs. 57th SEG meeting, New Orleans, Expanded Abstracts, 646649.

CRAMPIN, S. 1978. Seismic wave propagation through a cracked solid: polarization as a possible dilatancy diagnostic. Geophysical Journal *of the* Royal *Astronomical* Society 53, 467-496.

CRAMPIN, S. 1981. A review of wave motion in anisotropic and cracked elastic-media. ***Wave Motion 3, 343-391.***

CRAMPIN, S. 1984a. An introduction to wave propagation in anisotropic media. Proceedings of the First International Workshop on Seismic Anisotropy, Suzdal, USSR, S. Crampin, R. G. Hipkin and E. M. Chesnokov (eds). ***Geophysical Journal of the Royal Astronomical Society 76, 135-145.***

CRAMPIN, S. 1984b. Effective elastic-constants for wave propagation through cracked solids. Proceedings of the First International Workshop on Seismic Anisotropy, Suzdal, USSR, S. Crampin, R. G. Hipkin and E. M. Chesnokov (eds). ***Geophysical Journal of the Royal Astronomical*** Society 76, 17-28.

CRAMPIN, S. 1987. Geological and industrial implications of extensivedilatancy anisotropy. ***Nature* 328, 491–496.**

CRAMPIN, S. 1989. Effects of singularities on shear-wave propagation in sedimentary basins. ***Journal of Geophysical Research*** (in press).

CRAMPIN, S. and BUSH, I. 1986. Shear-waves revealed: extensive-dilatancy anisotropy confirmed. 56th SEG meeting, Houston, Expanded Abstracts, 481-484.

CRAMPIN, S., BUSH, I., NAVILLE, C. and TAYLOR, D.B. 1986. Estimating the internal structure of reservoirs with shear-wave VSPs. ***The Leading Edge* 5, 35–39.**

CRAMPIN, S., EVANS, R. and ATKINSON, B.K. 1984. Earthquake prediction: a new physical basis. Proceedings of the First International Workshop on Seismic Anisotropy, Suzdal, USSR, S. Crampin, R. G. Hipkin and E. M. Chesnokov (eds). ***Geophysical*** Journal ***of*** the ***Royal Astronomical Society 76, 147-I 56.***

CRAMPIN, S., MCGONIGLE, R. and BAMFORD, D. 1980. Estimating crack parameters from observations of P-wave velocity anisotropy. ***Geophysics* 45, 345–360.**

CRAMPIN, S. and RADOVICH, B.J. 1982. Interpretation of synthetic CDP gathers for a single anisotropic layer. ***Geophysics* 47, 323–335.**

CRAMPIN, S., STEPHEN, R.A. and MCGONIGLE, R. 1982. The polarization of P-waves in anisotropic media. ***Geophysical Journal of the Royal Astronomical Society* 68, 477–485.**

CRAMPIN, S. and YEDLIN, M. 1981. Shear-wave singularities of wave propagation in anisotropic media. ***Journal of Geophysics* 49, 43–46.**

GARBIN, H.D. and KNOPOFF, L. 1975. Elastic moduli of a medium with liquid-filled cracks. Quarterly Journal ***of*** Applied Mathematics **33, 296–300.**

HERGET, G. 1987. Stress assumptions for underground excavations in the Canadian Shield. ***International Journal of Rock Mechanics and Mining Sciences* 24, 95–97.**

HESS, H. 1964. Seismic anisotropy of the uppermost mantle under oceans. ***Nature 203, 629-*** 631.

HUBBERT, M.K. and WILLIS, D.G. 1957. Mechanics of hydraulic fracturing. ***Transactions American Institute of Mining and Metallurgical Engineers*** 210, 153–170.

HUDSON, J.A. 1980. Overall properties of a cracked solid. ***Mathematical Proceedings of the Cambridge Philosophical Society 88, 371-384.***

HUDSON, J.A. 1981. Wave speeds and attenuation of elastic waves in material containing cracks. ***Geophysical Journal of the Royal Astronomical Society* 64, 133–150.**

JOHNSTON, D.H. 1986. VSP detection of fracture-induced velocity anisotropy. 56th SEG meeting, Houston, Expanded Abstracts, 464-466.

KAARSBERG, E.A. 1968. Elasticity studies of isotropic and anisotropic rock samples. ***Transactions of the Society of Mining Engineering* 241, 470–475.**

LOVE, A.E.H. 1944. ***A Treatise on the Mathematical Theory of Elasticity,*** 4th edn. Dover Publications Inc.

LYNN, H.B. and THOMSEN, L.A. 1986. Reflection shear-wave data along the principal axes of azimuthal anisotropy. 56th SEG meeting, Houston, Expanded Abstracts, 473-476.

MUSGRAVE, M.J.P. 1970. ***Crystal Acoustics.*** Holden-Day, Inc.

NYE, J.F. 1969. ***Physical Properties of Crystals.*** Oxford University Press.

POSTMA, G.W. 1955. Wave propagation in a stratified medium. ***Geophysics 20, 780-806.***

WILLIS, H.A., RETHFORD, G.L. and BIELANSKI, E. 1986. Azimuthal anisotropy: occurrence and effect on shear-wave data quality. 56th SEG meeting, Houston, Expanded Abstracts, 479-481.