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Inversion for subsurface anisotropy using estimates of shear-wave splitting

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SUMMARY

Inversion of the parameters characterizing shear-wave splitting (polarizations, time delays, and differential attenuation) provides a way of obtaining a good starting model for full-wave anisotropic modelling, and improves the convergence to a final waveform match. This is necessary as the shear waves behave in a highly non-linear fashion with respect to anisotropy. Here we discuss the extent to which polarization and time-delay information, estimated from split shear waveforms, may supply details of this starting model. 3-D patterns of these parameters for different anisotropic models are considered, to investigate the potential of the inversion scheme. Guidelines are formed for the best arrangements of sources and geophones, to provide optimum resolution of an anisotropic model. The study indicates that in most cases different anisotropic models cannot be adequately distinguished using vertical seismic profiles (VSPs), reflection profiles, and cross-hole surveys in which separate polarization or time-delay measurements at one azimuth are made. Better results are obtained by combining polarization and time-delays estimates, preferably from more than one azimuth. The type of model parameters which can be resolved depends crucially upon the geometry of the acquisition system, and to a lesser extent on the background velocity structure. Individual model parameters are more difficult to extract, but some details appear (in principle) to be resolvable for certain types of survey. VSPs are most suitable to monitor crack strike and density of parallel, vertical, aligned cracks, and the dip of the fine layering contributing to layer-induced anisotropy. Cross-hole experiments are best for measuring the dip of the cracks. The resolution of the crack content and aspect ratio are determined principally by the position of line singularities, and either an offset VSP, or an adapted cross-hole survey may be adequate for resolving these properties if the line is shot at an appropriate azimuth with respect to the strike of the cracks. The ratio of crack-induced anisotropy to matrix anisotropy in sedimentary basins can be evaluated by measuring the positions of the point singularities in recordings of polarization and time-delay patterns using a near-offset VSP.

Key words: anisotropy, inversion, shear-wave splitting.

1 INTRODUCTION

As the vector analysis of exploration data becomes more widespread, there is increasing need for multicomponent, multisource recording, and the consequent necessity to include anisotropy in processing, and interpretation steps. Analysing seismic waves for anisotropy provides the analyst with an opportunity to improve upon existing seismic methods for imaging the subsurface, in which one may hope to obtain extra details about the internal structure of the rockmass. This information is coded into the waves predominantly through the phenomenon of shear-wave splitting, an effect which has been observed in shear-wave data recorded from a wide variety of experiments, ranging from surface recordings of local earthquakes, and reflection profiles, to borehole measurements using vertical seismic profiles (VSPs), and cross-hole surveys (Crampin 1987). In this phenomenon, a single shear wave incident upon a region of effective anisotropy usually splits into two waves, with different polarization directions (fixed by the direction of propagation, and the medium properties), and travelling at different velocities. Information on the effective anisotropic system, through which the shear waves propagate, may be obtained by interpreting the morphology of this distinctive phenomenon; a subject currently exciting much interest (MacBeth & Crampin 1991). The most satisfactory way in which to analyse this phenomenon, to estimate subsurface details, involves matching field seismograms with full-wave synthetic seismograms. Unfortunately, such forward modelling is time-consuming, and not wholly satisfactory. An alternative approach to this problem is to relate estimates of features of the seismic waves to the elastic properties of the medium in an inversion scheme, from which a preliminary set of models can be obtained, and used as a starting point for the full-wave modelling. To arrive at an anisotropic solution for each region of the subsurface, a two stage process is involved. First, estimates for the parameters which characterize split shear waves must be obtained; and secondly, these values are interpreted in terms of a range of physically plausible anisotropic models. The polarizations, attenuation, velocities, differential traveltime, and attenuation of the shear waves, are in theory all candidates for this inversion, and may be related to the elastic constants of the media through which they propagate.

An example of this type of approach is the inversion of velocity variations for either, or both, *P*- and shear waves, which has been a popular problem in recent years. These studies usually rely on the simplifications inherent in the assumption of azimuthal isotropy (hexagonal anisotropic systems, with a vertical axis of symmetry). The five elastic constants describing this type of anisotropy have been estimated by techniques such as those of White, Martineau-Nicoletis & Monash (1983), for *P* and *SV* phase velocities between two closely spaced boreholes, or Byun & Corrigan (1990), for a multilayered medium. Pratt & Chapman (1990) have investigated the problem in the context of P-wave tomography. Velocity studies have also been carried out for media which contain distributions of parallel, vertical cracks (Crampin 1978), leading to azimuthal anisotropy (hexagonal anisotropic system with a horizontal axis of symmetry). Crack densities, and crack strike (with corresponding elastic constants) have been estimated using the azimuthal variation of P-wave velocities (Crampin, McGonigle & Bamford 1980; Crampin, McGonigle & Ando 1986a). Including a further series of complexities to this problem, Sayers (1988) derived, from ultrasonic rock measurements, a way of relating wave velocity to a microcrack orientation function for an orthorhombic material (due to for example, two intersecting orthogonal crack sets). Helbig (1990) has introduced a technique for relating *P*, and shear-wave velocities to the elastic constants for a material with orthorhombic symmetry, however the equations are only tractable in symmetry planes. For most of these investigations, the only reliable way of determining the anisotropy is to use measurements at more than one azimuth, and for several symmetry planes. For this reason, the most successful studies so far have been restricted to azimuthal isotropy.

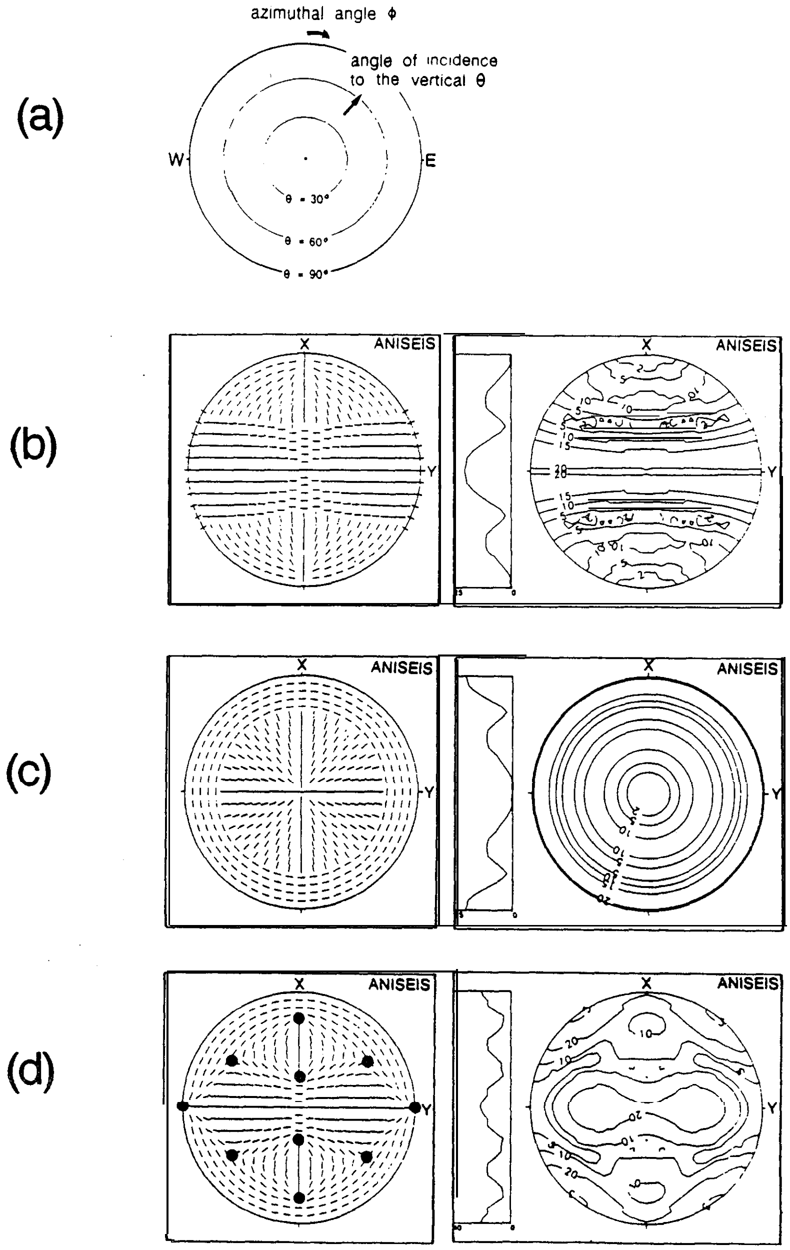
In recent years, a direct relation has been established between the polarizations and time delay of split shear waves, and a description of the anisotropy, which can be effectively considered an inversion process (Crampin 1987). If one assumes that the medium under investigation is azimuthally anisotropic, as a result of thin parallel, vertical aligned cracks, then the polarization direction of the fast shear wave is approximately parallel to the strike of the cracks, for measurements within the shear-wave window (Booth & Crampin 1985). Additionally, the time delay can be directly related to the density of the cracks. These relations between the polarization and fracture orientation, and degree of splitting and density are expected from theoretical formulations (Crampin 1978, 1984), and have been confirmed by laboratory experiments (Tatham et *al.* 1987; Yale & Sprunt 1989). These relationships have been used to estimate fracture orientation and density in subsurface reservoir rocks. This has been used extensively in the interpretation of field data from reflection profiles (Alford 1986), numerous VSPs (Crampin et *al.* 1986b; Squires, Kim & Kim 1989; Campden & Crampin 1991; Queen & Rizer 1990; Winterstein & Meadows 1990), and cross-hole data (Liu, Crampin & Booth 1989).

The anisotropy in many sedimentary basins is also due to horizontal fine layering and lithology leading to matrix anisotropy possessing transverse isotropy with a vertical axis of symmetry, as well as anisotropy due to parallel, vertical cracks possessing transverse isotropy with a horizontal symmetry axis. The combination of the two anisotropies leads to anisotropy with an orthorhombic symmetry (Bush & Crampin 1987). In this case, the simple inversion procedure above will not be valid, as the polarizations and time delays are perturbed from the expected behaviour of the thin parallel crack model (Crampin 1991a; Wild & Crampin 1991). A similar conclusion may be reached if the medium contains a dipping crack set, or there are more than one set of cracks (Crampin & McGonigle 1981; Winterstein & Meadows 1990). In this situation, one must attempt to solve a more general problem of inversion, and match the recorded pattern of polarizations or time-delay measurements, with known distributions from particular anisotropic models. In this paper, the feasibility of this approach is investigated, by examining the extent to which different anisotropic models may be distinguished from one another in different exploration situations, and the degree to which individual model parameters may be resolved. The study concentrates on the polarization direction of the leading shear wave, and the time delay between the fast and the corresponding slower split shear wave. Note that a consistent terminology for seismic anisotropy is given by Crampin (1989).

2 SHEAR-WAVE BEHAVIOUR FOR RELEVANT ANISOTROPIC MODELS

2.1 Polarization and time-delay distributions

Different anisotropic models may be distinguished by a particular 3-D pattern of behaviour of the polarizations and time delays through the medium, given in terms of incidence angle and azimuth. These distributions are conveniently displayed as equal area plots, which project the values over a hemisphere of directions onto a plane horizontal surface. In this representation, vertical propagation is given by the centre of the plot, and horizontal propagation by points around the edge of the plot (Fig. la). A description of this



Fiie **1.** Equal-area polar projections for the horizontal projections of the polarizations of leading split shear wave (left-hand side), and time delay between faster and slower waves normalized to ms km^{-1} (right-hand side), for different types of anisotropy. (a) Interpretation of the polar plot in terms of directions of wave propagation in the medium; (b) polar projections for EDA model; (c) PTL model; (d) CLA model. Model parameters are detailed in Table 2. Black dots represent approximate directions of point singularities.

mapping is given by Wild & Crampin (1991). This representation will be used to describe the patterns for the polarization of the leading split shear wave, and the corresponding time delay between the two split shear waves in the analysis below.

Rock containing a set of parallel, vertical aligned inclusions is effectively anisotropic, corresponding to a hexagonal system of symmetry, in which the axis of symmetry is horizontal. The anisotropy attributed to this type of model is known as extensive dilatancy anisotropy or EDA (Crampin 1985). The behaviour of the shear waves varies with both the angle of incidence in a vertical plane, and with azimuth. A typical pattern of the polarizations of the leading shear wave, and differential time delays resulting from shear-wave propagation through this type of material is shown Fig. 1(b). A characteristic feature of this model is a broad band of parallel polarizations in the direction of the crack strike, leading to a maximum in the time-delay pattern at vertical incidence. Time delays drop to zero at the edges of the band (line singularities), before increasing for more horizontally propagating ray paths away from the crack strike. The band is delimited by these two line singularities, point at which the velocity surfaces of the two split shear waves intersect, and at which the waves experience an abrupt transition in polarization (Crampin & Yedlin 1981). This occurs at an incidence angle of about 30° at an azimuth of 0° (perpendicular to the crack-strike) for the model shown. There are six crack parameters of interest in this model, each of which can change these patterns, and consequently, in theory, may be extracted from analysis of shear-wave splitting. These parameters are the crack density, crack strike, two dip angles, aspect ratio, and the content of the cracks (which is considered here as a one-valued function of content material type, but may be dependent on several descriptive parameters). The absolute values of the time delays are also affected by the properties of the isotropic background matrix.

The effective matrix anisotropy of repeated thin layers of different isotropic materials to seismic waves, or lithological anisotropy, gives rise to an anisotropic medium with hexagonal symmetry, the axis of symmetry being vertical (see many papers by Levin, and Helbig , and their colleagues). The anisotropy attributed to this type of medium is known for convenience as PTL anisotropy. The elastic properties of the medium (and hence the behaviour of the shear waves) vary only with angle of incidence in a vertical plane (Fig. lc). The polarizations of the leading shear wave for a cone of directions around vertical propagation exhibit pure ***SV*** motion. As the propagation direction becomes progressively horizontal, the polarizations undergo a transition into a pure ***SH*** motion. The time delays are circularly symmetric about the vertical, for which there is zero time delay, increasing to a maximum in the horizontal direction. The patterns of polarization and time delay may be affected by the dip angles of the layering, the strength of the anisotropy, and the material consituents for the layers, and their relative thicknesses.

In sedimentary basins, the two types of anisotropy, EDA and PTL, may combine to form an anisotropic system with orthorhombic symmetry (Bush & Crampin 1987; Crampin 1991a). Unlike the hexagonal systems, the corresponding equal area plots for this system exhibit a number of distinctive point singularities (Crampin 1991a; Wild & Crampin 1991); directions near which shear-wave behaviour changes rapidly with small changes in direction. By combining EDA and PTL anisotropy in different proportions, a wide variety of patterns of behaviour is possible. This behaviour is catalogued in Wild & Crampin (1991). In such materials, the polarization of the faster split shear wave may not be wholly parallel to the strike of the cracks, even near to vertical directions of propagation. For small ratios of EDA to PTL anisotropy (2 per cent EDA anisotropy to 10 per cent PTL anisotropy), point singularities may occur for ray directions close to vertical propagation. The number of parameters specifying this model is a combination of those for the two separate models. For convenience in further discussion, this model is referred to as CLA anisotropy (crack-layer anisotropy).

2.2 Sensitivity of polarization and time-delay distributions

The resolution of a particular model parameter in any scheme of inversion depends on the sensitivity of the shear waves to this parameter, and on the nature and geometry of the acquisition system. Here, we discuss the overall effect of each model parameter on the pattern of polarizations and time delays, and in a later section consider the restrictive effects of individual acquisition geometries.

The effect of changing crack strike in EDA anisotropy is to rotate the patterns of polarization or time delay in Fig. l(a) about the vertical axis, in the horizontal plane. This procedure clearly has a significant effect on the recorded polarizations, which are strongly azimuthally dependent. Sloping the cracks to the vertical also produces significant changes in the general pattern of these distributions. Figs 2(a)-(d) show the effect of changing the slope of the crack set by rotating about the Y-axis in the X-Z plane, by 0", 30°, 60°, and 90" respectively. As the slope increases, the band of parallel polarizations (and hence the line singularities associated with the edge of this band, highlighted by the solid lines on the figure) moves outwards from the centre of the diagram creating asymmetry. This is eventually replaced, for horizontal cracks, by radial (SV) pattern with a small region of tangential ***(SH)*** polarizations for shallow angles of incidence, the typical feature of PTL anisotropy. Other patterns may also be produced by sloping the cracks around the X-axis, or combining the two types of rotation. By introducing changes of strike and slope, a number of interesting features relating to the non-

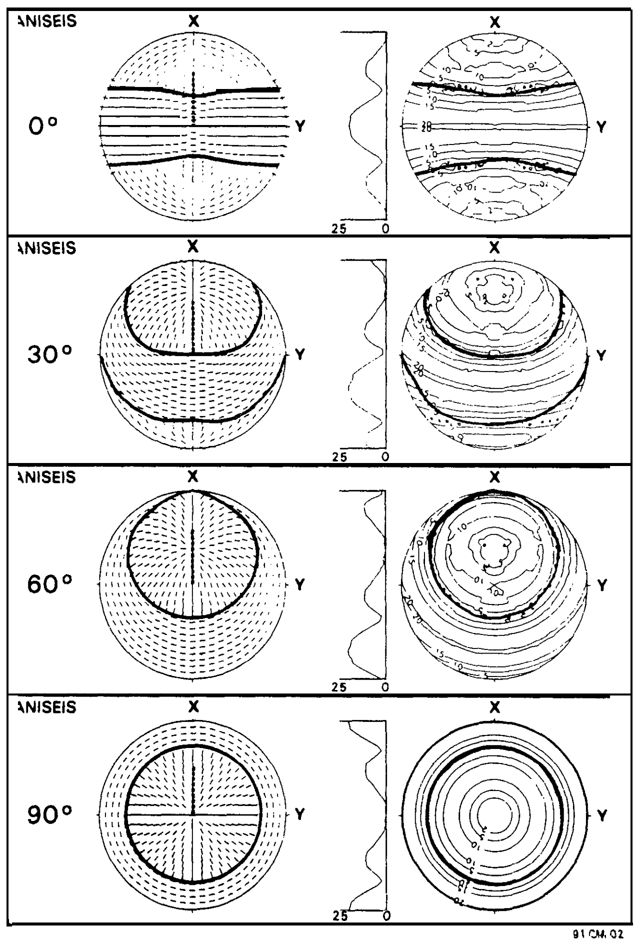


Figure 2. The effect of slope of the cracks on the behaviour of shear-wave polarizations and time delays for EDA anisotropy. Slopes range from 0° to 90°. The positions of the line singularities are highlighted for reference. The dotted line corresponds to the ray directions through the medium for an offset VSP experiment. The variation with dip for PTL anisotropy may be visualized by reversing the order of the diagrams.

uniqueness of interpretation of the recorded polarizations may be introduced. These are also illustrated in Fig. 2, which shows, superimposed upon the polarization patterns, a dotted line corresponding to the directions of propagation of shear waves recorded in an offset VSP. If one observes parallel polarizations which are striking along the radial (in-line) direction, then this may be interpreted as an azimuthal rotation of the EDA model in Fig. 2 by 90°, or by a slope of 30". In this case, accurate estimates of the time delay are required to resolve this ambiguity. Non-uniquenesses may also arise with the distribution of time delay. These parameters determining the orientation of the anisotropy are clearly important in interpreting the recorded patterns of polarization and time delay. Some problems in such an interpretation may be avoided by constraining these parameters on physical grounds. For example, large changes of crack slope at depth may be excluded on the grounds of predominant vertical stresses (Crampin 1990). For shallower surveys, geological evidence must be used. The effect of sloping cracks has been observed by Douma, den Rooijen & Schokking (1990), for shear-wave measurements taken from a shallow (14Om) VSP in clay. In this experiment, the recorded polarizations could be explained by geological evidence of fissures dipping at 60°. If the order of Figs 2(a)-(c) is reversed, the sequence also illustrates how the patterns related to PTL anisotropy change with dip angle of the bedding. The line singularities, corresponding to the transition between ***SV*** and ***SH*** polarizations, now move upwards through the diagram. Similar conclusions hold regarding the ambiguities in interpretation for this case.

The behaviour of shear waves is also sensitive to changes in the geometry of the inclusions in the EDA model. The results of Douma & Crampin (1990) and Crampin (1991b) show that increasing the aspect ratio (ratio of thickness and the diameter) of the inclusions, increase the width of the parallel band of polarizations in Fig. l(a). This implies that the angle the line singularity makes with the direction of the symmetry axes (horizontal) decreases with increasing aspect ratio. At an aspect ratio of 0.2, for a crack density of 0.05, these studies conclude that the line singularity coalesces with the kiss singularity, where the two shear-wave velocity surfaces touch tangentially (Crampin & Yedlin 1981), and the entire plot is filled with parallel polarizations. For this particular model, the time delays have only one maximum at vertical incidence, the delays decreasing slowly with increasing incidence. For large aspect ratios, it may thus be difficult to obtain the exact position of the singular point, especially if scattering plays a dominant role in the seismogram. An alternative way to extract information about the cracks in this case, is to utilize the significant Z-'-wave anisotropy which results with a maximum at aspect ratio of 0.2 (Crampin 1991b). For smaller aspect ratios, the line singularity may be detected more easily. This particular result has been used to interpret temporal changes in anisotropy preceding and after an earthquake (Peacock *et* al. 1988; Crampin *et al.* 1990; Booth *et al.* 1990), the inclusions changing geometry in response to the changing stress field.

Changing the content of the cracks also has an effect on the position of the line singularity. In particular, a decrease in the fluid velocity of the contents in the inclusions shifts the shear-wave line singularities towards the horizontal

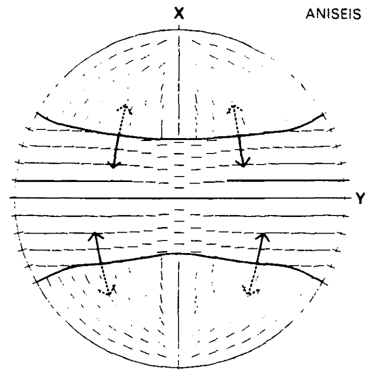


Figure 3. Changing crack aspect ratio and content for an EDA model varies the directions of the line singularities, which increase (dotted arrows), or reduce (solid arrows) the width of the parallel band of polarizations. Increasing aspect ratio moves the directions for the line singularity closer to the horizontal (dotted lines). Decreasing aspect ratio moves the line singularity directions closer to vertical, until a limiting width (minimum incidence angle of about 30°, at an azimuth of 0°) is encountered at an aspect ratio of 0.001 or smaller. As the crack content changes from water-filled to dry, the band of parallel polarization becomes wider, and fills the entire plot for dry cracks, except for very small aspect ratios and large crack densitites.

(Crampin *et al.* 1990). Fig. 3 summarizes the effects of aspect ratio and velocity of crack contents. Consequently, for most dry cracks, the behaviour is similar to those with aspect ratios of 0.3 or greater. This result represents an inherent non-uniqueness which may confuse a prospective inversion scheme. Material permeated by cracks containing fluids with small acoustic velocities, or dry cracks, will have the same influence on the shear-wave polarizations as water-saturated cracks with large aspect ratios. An increase in crack density, caused by either an increase in the number of cracks, or by an increase in the size of the cracks, increases the delays between the split shear waves along almost all ray paths. The effect is to scale the time-delay patterns, whilst maintaining the overall pattern of polarizations.

The behaviour of shear waves in sedimentary basins, where there is a combination of both EDA and PTL anisotropy, has been shown to be complicated (Crampin 1991a; Wild & Crampin 1991). Specific directions exist called point singularities (Crampin & Yedlin 1981), at which the two shear-wave phase velocity surfaces touch at the vertices of convex and concave cones. In directions near these point singularities, there may be severe disturbance to the amplitudes, and polarizations of shear waves radiating from a point source propagating along ray paths at the group velocity (Crampin 1991a). These singularities are associated with rapid variations in the polarization directions, and cause patterns of distruptive behaviour in cones of directions around them. The time delays may display rapid changes in sign, but usually remain finite (Crampin 1991a). The directions of these singularities is strongly influenced by the ratio of EDA to PTL anisotropy in the sedimentary rock. When this ratio is sufficiently small, point singularities may occur in directions of propagation very close to vertical directions. The aspect ratio of the cracks has some effect on the patterns of polarization and time delay, but significant effects are mainly confined to small (<6 per cent maximum differential anisotropy) values of PTL anisotropy. Clearly a wide range of patterns of polarizations and time delays can be produced by varying various parameters.

2.3 Inversion for anisotropy using polarizations and time delays

The objective of a scheme for inverting the parameters which characterize shear-wave splitting is to find a suitable anisotropy which models the observed polarization directions and time delays. The discussion of sensitivity shows that a wide variety of polarization and time-delay patterns can be produced by varying the crack and layer parameters in the models, particularly for sedimentary rocks. The discussion also highlights areas where some ambiguity in interpretation may be expected for some variations of model parameters. The overall appearance of the patterns does appear unique, and it is anticipated that inversion can be achieved. Unfortunately, in practice, the problem of selecting a particular representative anisotropic model to match the observed values is constricted by the type and geometry of the exploration survey. This will necessitate the comparison between estimates of the anisotropic parameters from shear-wave observations, and the distributions of these values corresponding to theoretical anisotropic models, being made for a limited range of 3-D directions, so that only a small area of the equal area plot is illuminated. This region, which is determined by the acquisition geometry, type of exploration survey, also the layered isotropic background model (discussed in Section 3.6), is defined here as the data aperture. The inverse problem for polarizations or time delays thus becomes: given this discrete sample of values as a function of limited direction (defining the aperture or apertures), what is the statistical probability of these being related to a distribution associated with a particular anisotropic model. This comparison must also be viewed with consideration to the errors which arise in the estimation process and the extent to which a random set of values may fit the observations. In the next section we address the form which this data aperture takes for different exploration surveys, and then examine the extent to which different anisotropic parameters can be resolved with these systems.

3 COMPARING OBSERVED AND PREDICTED DISTRIBUTIONS TO EXTRACT MODEL PARAMETERS

3.1 Vertical seismic profile

The vertical seismic profile provides a controlled way of examining the subsurface anisotropy, without the problems associated with surface recordings of shear waves. The special geometry of the VSP offers many advantages over other acquisition geometries for detecting and analysing the effects of anisotropy. The data aperture which is available to a typical offset VSP is shown in Fig. 4(a), superimposed upon a pattern of polarizations for reference. Here a uniform medium is assumed for simplicity. The variation of the aperture due to the refraction of the rays through the earth structure is discussed in Section 3.6. Depending on the offset, this survey samples the subsurface at ray angles ranging from the vertical (deepest geophone level) to near horizontal (shallowest geophone level). If the aperture is small (zone of interest is deep, or zero offset VSP), it may not extend beyond the band of parallel polarizations corresponding to azimuthal anisotropic models. In this case, however, the crack strike and density may always be resolved, assuming that the dip of the anisotropy has been constrained. The dip is difficult to identify from these nearly vertical ray paths, unless observations are available from a range of azimuths and angles of incidence at the appropriate range of directions. EDA and PTL anisotropy can be distinguished unambiguously for a zero-offset VSP, using the time delays, as PTL anisotropy implies no shear-wave splitting for vertical incidence, with an increase in time delay for more horizontal rays. Depending on the azimuth of the survey, EDA anisotropy may result in a decreasing time delay or constant time delay as the rays become more horizontal. Out-of-plane dips in the bedding planes associated with PTL anisotropy may also be measured in this system. For this position of aperture, it may also be possible to resolve small amounts of EDA anisotropy, combined with strong PTL anisotropy, by locating the positions of point singularities.

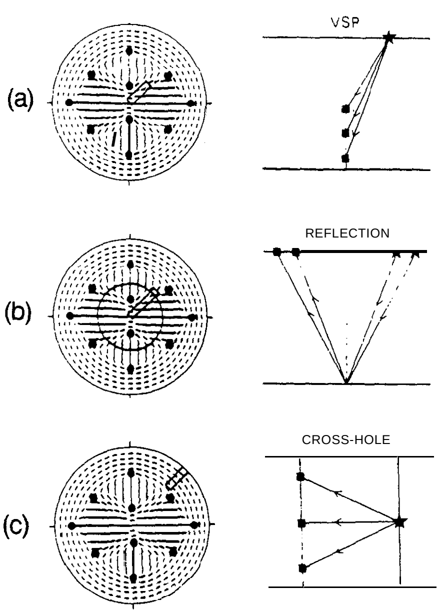
Multi-offset techniques at the same azimuth increase the length of the aperture, and consequently sample a larger range of directions along a particular azimuth. The apertures can now cover the line singularity, and hence resolution of aspect ratio and crack content may be possible. Fig. 5 shows data apertures for three specific geometries. The apertures in Fig. 5(a) cover the transition region outside the band of parallel polarizations for appropriate azimuths in the EDA model, and the region of transition for the PTL model. The first region may be observed with a sequence of geophone positions, with the shallowest at 1025 m, and a source offset of 500 m, the second aperture is obtained for the shallowest geophone now at 265 m, and the same source offset. Unfortunately, the results for the shallower geophones will be degraded, as the estimates of the shear-wave splitting decrease in accuracy with increasing incidence angle. This is due to increased scattering, in addition to the effect of the internal shear-wave window, for which the particle motions become disturbed and unreliable if the ray direction lies outside the internal shear-wave window of an interface (Liu & Crampin 1990). Consequently, it may be more difficult to obtain good estimates of the aspect ratio and crack content.

3.2 Reflection survey

The reflection survey has in general four problems associated with its use for distinguishing between anisotropic models. First, for a given offset (length of geophone spread), the data aperture is smaller than the corresponding one for the VSP. Secondly, the aperture over which the shear-wave estimates is restricted to incidence angles less than about 35” by the shear-wave window (Booth & Crampin 1985), outside which the surface recordings are distorted and unreliable. Thirdly, because the shear-wave polarizations may vary with angle of incidence, conventional CDP gathers degrade the shear-wave data. Fourthly, shear-waves may be severely disturbed on reflection from an interface between isotropic or anisotropic material (Yardley & Crampin 1991). Fig. 5(b) shows the data aperture for a typical acquisition geometry, and demonstrates the restriction of the shear-wave window (circle in centre of diagram). Consequently, for a reflector at 500 m, increasing the geophone spread from 1430 m (limit of shear-wave window), to 2000 m (beyond shear-wave window) (Fig. 5b) does not alter the information content for resolving the subsurface anisotropy. Note that a similar conclusion is reached for a reverse VSP. As the band of parallel polarizations diagnostic of the EDA model is of a similar size to the shear-wave window, it may be difficuh to identify the position of the line singularity, and hence resolve the content or aspect ratio of the cracks using this type of acquisition geometry. The same conclusions for crack strike and density hold for this survey as for the VSP, especially for near-vertical angles.

3.3 Cross-hole survey

Cross-hole surveys give sub-horizontal coverage of the earth, between two boreholes. A typical data aperture for this geometry is shown in Fig. 4(c). It should be noted that the apertures corresponding to up- and downgoing waves superimpose each other on the polar projection plot. This does not affect the interpretation of the results for patterns where there is a horizontal plane of symmetry. If an asymmetry is created, due to dipping cracks for example, then the up- and downgoing waves do not have similar patterns of shear-wave splitting. A more convenient representation for cross-hole surveys, in this case, is the cylindrical projection (Liu *et al.* 1989). In principle, as this acquisition geometry records waves from a subsurface source to subsurface receiver, the difficulties arising due to near-surface irregularities are avoided. From a synthetic seismogram analysis of cross-hole surveying, Liu *et al.* (1989) conclude that information about the EDA model, taken from shear-wave splitting, is unlikely to be estimated or interpreted easily from cross-hole surveys unless sufficient observations can be made at a range of azimuths. The azimuth of the survey, relative to the crack strike is crucial to the amount of information which can be extracted on this parameter. This result can be understood by comparing the aperture in Fig. 4(c) with the patterns of polarizations, at different azimuths, for the anisotropic models in Figs l(a), and l(c). Unless the range of angles of incidence is sufficiently large, then the aperture will sample similar polarization variations. The strike may only be determined if the data aperture lies, by chance, in the band of parallel polarizations. As the width of this band depends on the aspect ratio or content of the cracks, then dry cracks, or large aspect ratios, are more likely to be hit upon by chance. For a large range of angles of incidence (larger data aperture), and several azimuths, it may be possible to identify the strike using the symmetry in the patterns of



Fire 4. Three types of acquisition geometry considered in this study, with apertures (ranges of direction sampled by geometry), superimposed upon equal-area polarization patterns for an orthorhombic material. (a) VSP; (b) reflection survey; and (c) cross-hole survey.

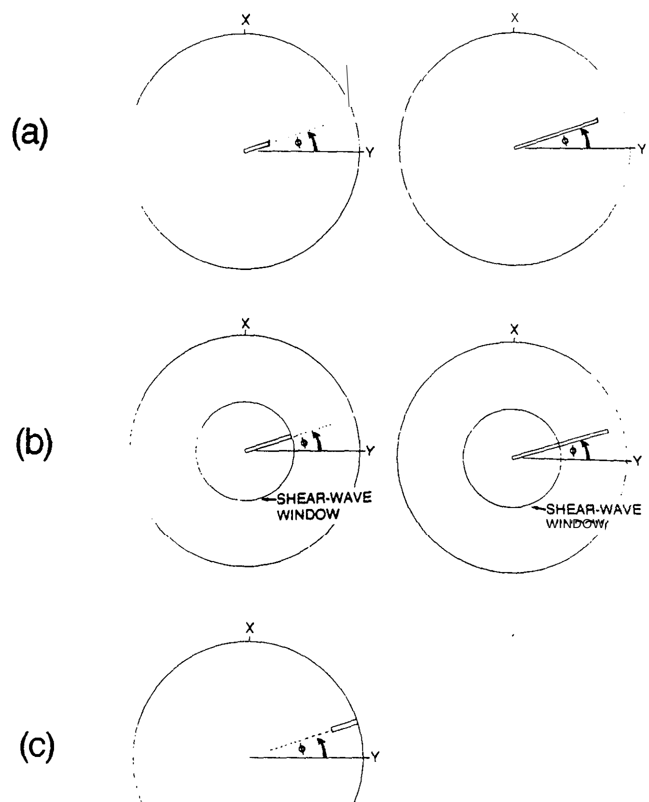


Figure 5. Specific apertures for particular types of exploration survey; (a) VSP with shallowest geophone at 1025 m, and source offset at 500 m (left), and shallowest geophone at 265 m for the same offset (right), covering transition zones for EDA and PTL models; (b) reflection profile with reflector depth at -500 m, for geophone spread ranging over 1430 m (left) (up to the shear-wave window), and 2000m (right) (beyond the shear-wave window); and (c) cross-hole experiment for borehole separation of 500 m, and a maximum geophone/source spread of 265 m, covering the line singularity for PTL model.

polarizations and time delays. Cross-hole surveys display the effects of dip as asymmetries in polarization patterns. For azimuths parallel to the crack strike, dip can be read directly from dip of the polarization (Liu et al. 1989). It appears that the effect of the dip, noted in Fig. 2, will have a significant effect on patterns of polarizations and time delays from crack-induced and matrix anisotropies. A small area of uncertainty in dip evaluation is present when estimating polarizations with a small aperture, for the PTL model. Information about the aspect ratio or content of the cracks, may be extracted in a cross-hole survey, except for cracks with large aspect ratios or dry cracks. The aperture corresponding to a single azimuth survey, covering the line singularity, is shown in Fig. 5(c). This aperture corresponds to two boreholes, 500 m apart, geophone and source levels covering a range of 265 m in the boreholes. Although this geometry is sufficient to distinguish between EDA and PTL models, it is difficult to distinguish between the CLA and PTL models.

3.4 Statistical comparison of polarization and time-delay patterns to resolve different types of anisotropy

Here the resolution between EDA, PTL and CLA models is analysed more quantitatively by comparing the theoretical patterns of polarization and time delay for three different anisotropic models. The ANISEIS package (Taylor 1991) is used to produce the expected polarization and time-delay values for shear-wave propagation through one of these three particular anisotropic models, as recorded on a particular acquisition system at one specific azimuth. This profile of values is then compared with the expected polarization and time-delay distributions for each azimuth of the EDA, PTL, and CLA models, at corresponding angles of incidence. The comparisons are made for azimuths between 0” and 360°, in steps of 4.5”. The degree of agreement between ‘recorded’ and model profiles is monitored using the misfit function:

$$\chi^2 = \sum (p_i^{\mathrm{o}} - p_i^{\mathrm{m}})^2 / \Delta p_i^2, \tag{1}$$

where p_i^{o} and p_i^{m} are the ith ‘recorded’, and model polarization angles respectively, and Δp_i is the estimation error. If the fit function is at an acceptable level for any of the model azimuths, then the ‘recorded’ profile at that azimuth is flagged as acceptable. Another ‘recorded’ profile at a different azimuth is then chosen and the comparison procedure repeated. The process continues until all azimuths of the ‘recorded’ model have been analysed. The procedure gives the percentage of ‘recorded’ profiles which agree with at least one model profile.

Tables l(a), (b) and (c) give details of the comparison between EDA, PTL, and CLA models. (The model parameters are given in Table 2.) Three single azimuth acquisition geometries are considered, these corresponding to a VSP, reflection profile, and cross-hole survey. Two tables are given for each acquisition geometry, corresponding to comparisons for the polarization patterns of the leading split shear wave, and the time delay between split shear waves. Each entry in the table represents the percentage of ‘recorded’ profiles, for which the criterion: $\chi^2 < \varepsilon(n)$ is satisfied for at least one model profile. n is the number of estimates used in the computation, and ε is the threshold value for a 99 per cent confidence level. For this computation, polarization error is assumed to be 10°, and the time delay error is 5 ms.

Table 1. Statistical comparison between the polarization and time-delay patterns generated by EDA, PTL, and CLA models. Details of these models are given in Table 2. Comparisons are made for three separate field experiments: (a) VSP with source offset at 500 m, and 20 geophone levels between 1000 and 2000 m; (b) reflection profile, with principal reflector at a depth of 1000 m, source offset at 700 m (to stay within shear-wave window), and 20 geophones distributed over this distance; and (c) cross-hole experiment, with boreholes 300 m apart, and 20 geophones spaced between 1200 and 1400 m. Errors in polarization estimates are assumed to be 10°, and for time delays 5 ms. The comparison is made for an experiment shot at one azimuth only.

(a)

RECORDED	POLARIZATIONS MODEL EDA	PTL	CL4	TIME-DELAYS MODEL EDA	PTL	CLA
EDA	100.0	9.8	23.0	100.0	19.7	75.4
PTL	100.0	100.0	16.3	0.0	100.0	0.0
CLA	49.2	9.8	100.0	0.0	0.0	100.0

(b)

RECORDED	POLARIZATIONS MODEL EDA	PTL	CLA	TIME-DELAYS MODEL EDA	PTL	CLA
EDA	100.0	9.8	16.4	100.0	23.0	100.0
PTL	100.0	100.0	100.0	0.0	loo.0	0.0
CLA	23.0	9.8	100.0	0.0	0.0	100.0

(c)

RECORDED	POLARIZATIONS MODEL EDA	PTL	CLA	TIME-DELAYS MODEL EDA	PTL	CLA
EDA	100.0	0.0	6.5	100.0	29.5	32.8
PTL	0.0	100.0	100.0	0.0	100.0	0.0
a” 4	13.0	29.5	100.0	0.0	0.0	100.0

The analysis results tor a VSP experiment with a 500 m source offset, and 20 geophones between 1000 and 2000 m are shown in Table l(a). The data aperture ranges from 14” to 27”. The results show that the EDA model (‘recorded’) can be distinguished from the PTL model on the basis of polarizations, although for a small range of azimuths, the band of parallel polarizations may be confused with the SV polarizations in the PTL model. Interestingly, a comparison between a PTL model (‘recorded’) and EDA model suggests that the two models cannot be distinguished on the basis of polarizations alone. Given a set of aligned polarizations, from the PTL model (‘recorded’) each one can fit the band of parallel polarization for the EDA model. The chance of the PTL model (‘recorded’) fitting the CLA model is much smaller. There is a high probability of a CLA model (‘recorded’) fitting an EDA model, but a smaller chance of fitting a PTL model. The results of the intercomparison for the time delays are much more

Table 2. EDA, PTL, and CLA anisotropic structures used for generating patterns of splitting analysed in Table 1. Isotropic matrix material is specified by compressional wave velocity (α), shear-wave velocity (β), and density (p). Anisotropy due to matrix fine-layering is specified by maximum differential anisotropy (%PTL), and dips around X and Y horizontal axes (δ_L^x and δ_L^y). Cracks are specified by crack density (CD), aspect ratio (AR), dips around X and Y horizontal axes, relative to the vertical (δ_C^x and δ_C^y), and the content.

	ISOTROPIC MATRIX MATERIAL			CRACK AND MATRIX PROPERTIES							
	α (km/s)	β (km/s)	ρ (g/cm^3)	CD	AR	δ_C^x	6,'	CONTENT	δ_L^x	δ_L^y	% PTL
EDA	4.00	2.31	2.00	0.04	0.001	0	0	waler	00		0
PTL	2.15	0.84	2.13	0.00	0.000	0	0	---	0	0	15
CLA	2.15	0.84	2.13	0.04	0.001	0	0	water	0	0	15

conslusive. Each model appears to be well resolved from the others, except for the comparison between the EDA model ('recorded') and the CLA model, for which the measurements are similar. One would expect the EDA to become more distinguishable from the CLA model as the ratio of EDA to PTL anisotropy decreases, and the point singularities in the CLA pattern move towards the vertical direction (Wild & Crampin 1991). Table l(b) shows the results of an intercomparison of models for a reflection profile with a reflector depth of 2000 m, and a geophone spread of 1400 m. These dimensions are chosen so that the recorded rays have propagation directions within the shear-wave window at the free surface. Similar conclusions are reached as for the VSP experiment, except that the PTL model ('recorded') appears indistinguishable from the EDA, or CLA model, on the basis of polarizations. The EDA model ('recorded) cannot be distinguished from the CLA model on the basis of time delays.

The results for a cross-hole survey are shown in Table l(c). The survey consists of two boreholes 300 m apart, with geophone and source levels clustered over 200 m. Here, on the basis of polarizations, it appears that the PTL model ('recorded') is indistinguishable from the CLA model, although the other models can be distinguished. Similarly, on the basis of time delays, the EDA model ('recorded') has some similarities with the PTL and CLA models. It appears that the comparison between models on the basis of time-delays provides a better separation than polarization distribution. Each model can be distinguished on the basis of combining the polarization and time-delay results, from a VSP, reflection profile, or cross-hole survey.

These comparisons were intended to show the basic similarities and difficulties which may arise in distinguishing polarization and time-delay patterns from three different anisotropic models. The question of specific resolution of the model parameters from a particular set of data values (inherent non-uniqueness), together with the decision of which anisotropic model fits the data best, within the limits of experimental error, is discussed in a separate study (MacBeth 1991).

3.6 Multilayered media

The above discussions have been restricted to a uniform anisotropic half-space, for simplicity of interpretation. When the problem of finding, or distinguishing between anisotropic models is extended to multilayered media, it is naturally more complex, and the resolving power of the anisotropic parameters is reduced. This is especially true if the earth structure consists of layers of different types and orientation of anisotropy, in which the phenomenon of multiple shear-wave splitting occurs. Apart from the obvious difficulties associated with estimating polarizations and time delays of multiply-split shear waves, the range of incidence angles passing through each layer, and hence the size of the aperture for each layer, is smaller than for a uniform half-space. Refraction through each layer also alters the positions of the data apertures. An example of these effects is shown in Fig. 6, which shows the data apertures for a 200 m offset-VSP shot in a three-layered model with 200 m of low-velocity material ($V_S = 1.6$ km s^{-1}), overlying a 500 m layer of higher velocity material ($V_S = 2.3$ km s^{-1}), and a half-space (3.0 km s^{-1}). The shallowest layer has a data aperture positioned close to the horizontal, so that the resolution of the crack strike and density will not be good. The aperture for the second layer, covering a wide range of angles, partially overlaps that of the first due to refraction, The aperture for the half-space is small, with most of the rays propagating along the same direction. The crack strike

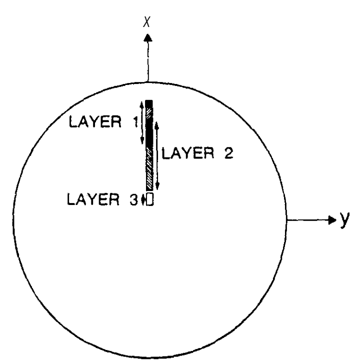


Figure 6. Apertures corresponding to each layer of a multilayered medium. The upper layer has a shear-wave velocity of 1.6 km s^{-1} and a thickness of 200 m, overlying two layers with velocities of 2.3 and 3 km s^{-1}, these being 500 m thick and a half-space. The experiment here is an offset VSP with 200 m source offset, with geophones placed at 25 m intervals down a 1275 m borehole. Shallowest geophone is at 50 m depth.

and density are well resolved for this case as the aperture is still positioned sub-vertically. There is less information about aspect ratio and crack content in this layer.

4 DISCUSSION AND CONCLUSIONS

Patterns of polarizations for the leading shear wave, and time delays between the faster and slower split shear waves, have been analysed to examine the feasibility of determining subsurface anisotropy using an inversion procedure. The finite number of directional recordings available in an exploration survey places limitations on the recognition of different types of anisotropy and their parameters, and creates ambiguities which may invalidate a prospective inversion scheme.

On the gross scale, EDA, PTL or CLA models cannot be totally distinguished from separate polarization or time-delay measurements at one azimuth, for any acquisition geometry. Combined measurements, preferably at several azimuths, are required to distinguish the different anisotropic models with any degree of success. On a more detailed scale, a qualitative statement can be made about the resolution of specific parameters of the three types of anisotropies. It appears that only certain acquisition geometries have the potential to resolve particular anisotropic parameters, these being summarized in Table 3. The entries in this table are not intended to suggest that certain parameters cannot be resolved using a particular experimental configuration. The table is only intended to act as a guideline to the selection of a suitable experiment for optimum resolution of these details using shear-wave splitting. Estimates of shear-wave splitting along the nearly vertical raypaths in a zero- or near-offset VSP are most suitable for estimating the strike and crack density of vertical cracks, and the dip of matrix anisotropy due to fine layering. Cross-hole experiments are best for measuring the slope of the cracks. The resolution of the crack content and aspect ratio are determined principally by positions of line singularities, and either an offset VSP, or a cross-hole survey may (in principle) be adequate for resolving this property. The ratio of EDA to PTL anisotropy in sedimentary basins may be resolved using a near-offset VSP. Reflection surveys supply similar information to an offset VSP, but are limited by the shear-wave window. Velocity contrasts and depth variations in anisotropy limit the size of the data apertures available for the inversion, and reduce the accuracy of the splitting estimates.

Although these results are valid for the particular model parameters used in this study, the general conclusions of this comparison would not be significantly affected by a different choice of values. From the basis of this study it may be concluded that the inversion for the polarization of the leading shear wave, and the time delay between the faster and slower shear wave appears possible, provided care is taken in the selection of the acquisition geometry for the model parameters of interest. Each anisotropic experiment must be designed according to the velocity structure in the survey region, and the zone of interest. In a subsequent study (MacBeth 1991, this issue), the inversion of shear-wave splitting, and the associated interpretational non-uniqueness, will be put on a more quantitative footing.

Table 3. Summary of resolving power of different acquisition systems for estimating parameters of anisotropy from shear-wave splitting. RP and CH are reflection profile, and cross-hole respectively. A tick indicates that the parameter can be adequately resolved, and a cross indicates a potentially low resolution. This table is intended as a guide to the inversion of shear-wave splitting. The results may vary depending upon the relative dimensions of different acquisition geometries.

MODEL PARAMETER	NEAR-OFFSET VSP	FAR-OFFSET VSP	RP	CH
STRIKE	✓			
CRACK DIP				
CRACK DENSITY				
ASPECT RATIO				
CONTENT				
% PTL				
LAYER DIP				

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