Inverting shear-wave polarizations for anisotropy using three-component offset VSPs: synthetic seismograms

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Accepted 1991 July 23. Received 1991 July 10; in original form 1991 February 21

SUMMARY
A numerical algorithm is presented for interpreting shear-wave polarization directions, from offset VSP data, as subsurface anisotropy. The technique compares observed (synthetic) horizontal polarization angles with expected values for a range of physically plausible anisotropic models with different orientations. The comparison is implemented in a database inversion scheme. The anisotropic models are formed by combining the effective anisotropy of parallel, aligned, vertical cracks with matrix anisotropy due to thin-layering or lithology. Different models are defined by varying the crack and matrix properties. The technique is applied to synthetic offset VSP data for a multilayered anisotropic structure, generated by multi-azimuth sources. Analysis of the results indicates that polarization measurements from non-zero offsets with at least two and preferably three different azimuths are required in order to reasonably constrain the final inversion solutions. The parameters which are resolved depend upon the depth of the subsurface zone of interest. Crack strike is well resolved in the deeper layers for which directions of propagation are sub-vertical, whereas other model parameters such as aspect ratio and crack content can only be resolved for wider ranges of incidence angle. The key to a successful inversion appears to lie in the selection of propagation paths through the relatively unique patterns of disruption created by singularities.

Key words: anisotropy, inversion, shear-wave polarizations, shear-wave splitting.

1 INTRODUCTION
The analysis of three-component recordings of shear-wave splitting, acquired using vertical seismic profiles (VSPs), provides a good way of examining details of seismic anisotropy in the crust. The interpretation of the shear wavefield in this acquisition geometry is potentially simpler and easier to interpret than in other seismic surveys, eliminating problems associated with surface or near-surface phenomena (MacBeth 1991). The optimum technique would be to perform a waveform inversion of three-component recordings. Although forward modelling packages such as ANISEIS (Taylor 1991) can construct complete waveforms for a variety of anisotropic models, and hence can be used to simulate the expected waveforms from a variety of exploration scenarios, this is a time-consuming and expensive procedure, which has not yet been optimized. For example, Bush & Crampin (1987) were successful in modelling a multi-offset, multi-azimuth, VSP data set in the Paris Basin, France, using trial-and-error matching of the horizontal polarization diagrams (particle motion in the horizontal plane). This procedure was tedious, as waveform variations for all the parameters involved in the fitting had to be studied, and the non-uniqueness inherent in the problem delineated. The alternative strategy used here is to derive estimates for the parameters of shear-wave splitting for particular arrivals, and relate these to anisotropic models for different regions of the subsurface, which may then be used to build a reliable starting structure for the full-wave modelling, as well as provide estimates of resolution and reliability. Refinements to this model can then proceed using waveform matching to achieve the final solution. The approach saves considerable time in approaching the full-wave solution, and also may ensure the convergence of waveform matching algorithms. This type of forward modelling approach is particularly appropriate for this problem, as the variations of the wave properties with changes in anisotropy are extremely non-linear.

The approach consists of three stages: first, estimation of the parameters of shear-wave splitting from observations; secondly, assigning the parameter estimates to particular layers; and thirdly, inverting for the elastic constants of each layer. Although processing procedures to estimate the parameters of shear-wave splitting from shear-wave data are
still in their formative stages [for a review of processing techniques, see MacBeth & Crampin (1991a)], it is possible to obtain measurements from shear waves by a variety of techniques under certain circumstances. Interpreting these estimates in terms of anisotropic contributions from different rock layers is a formidable problem. For example, the time delay between two split shear waves is particularly prone to misinterpretation if the recorded shear waves have passed through several layers of differently oriented anisotropic material with abrupt changes in the polarization directions of the media. In this case there will be multiple shear-wave splitting. Consequently, the geophone will record a complicated sequence of split shear waves, with the relative spacing of the individual arrivals depending on the proportion of each particular anisotropic material through which the waves have propagated, and their ray directions. (Note however, that because the polarizations of the split shear wave are determined by the anisotropy near the geophone, the sequence of multiply split shear waves usually display repetitions of the two polarizations along the appropriate ray paths.) In this situation, it may be difficult to pursue a direct interpretation of the estimated time delays, and relate the time delays to a particular subsurface region of known depth and size. Further difficulties may arise if some of the arrivals in this multiply split shear wavetrain have energies too low to be discerned by the processing algorithm. The polarizations recorded at each geophone level in the VSP can usually be directly related to the rock structure within several wavelengths or more of the borehole for different depths. The exact volume of the medium for which this representation is valid depends on the percentage of differential anisotropy. Consequently, estimates of polarization direction for each geophone may give a profile as a function of depth into the Earth, which may be representative of subsurface rocks surrounding the borehole. It is therefore possible, in principle, to relate these estimates directly to the polarization patterns for different anisotropic materials. This assumption is used as the basis for the approach used in this study, which concentrates on the polarization of the leading shear wave. Here, a method is developed for completing the final stage in the analysis of shear-wave splitting, by inverting polarizations in terms of a generally anisotropic subsurface. Note that although specific terms referring to anisotropy are usually defined when they occur, a consistent terminology for anisotropy is given by Crampin (1989).

2 POLARIZATION INVERSION

The scheme for inverting the polarization directions of the leading split shear wave is summarized in the flowchart of Fig. 1. The first stage in the procedure involves estimating the polarization direction of the leading split shear wave at each geophone level using an appropriate processing technique. The polarization is specified by the angle, $\psi_0^i$, which the polarization vector makes with the radial direction in the horizontal plane. The index $i$ relates to the geophone level, $i = 1, m$. For the purposes of this description, it is assumed $\psi_0^i$ have been estimated from the data, with a small associated error. Only direct arrivals are considered at present, although the information provided by the reflected arrivals may be incorporated into the analysis as a future extension. The next stage in the procedure involves dividing these polarization values into $n$ groups ($\psi_0^i; i = 1, m; j = 1, n$), each group corresponding to a particular layer or region of the earth model where it is judged that the physical properties of the rocks (in terms of the internal structure) may be reasonably uniform. This sub-division may be guided by the results of conventional seismic and geophysical processing procedures (well logs, traveltime inversions), or by geological data (core samples, BHTV images), suggesting regions or 'layers' of uniform fracture intensity, and alignment. $m_j$ is the number of polarization estimates available for the $j$th layer. In order to relate each group of polarization measurements to the material anisotropy in each region, it is necessary to obtain an estimate for the ray direction through the region to the geophone, described by the angle of incidence $\theta_j\parallel$, and azimuth $\phi_j\parallel$. $\theta_j\parallel$ is estimated, for the present investigation, by a ray tracing procedure using the background isotropic structure determined from traveltime inversion and well logs. The errors involved in using the angles from this procedure are discussed in Appendix A, and are taken into account in the final expression for non-uniqueness in the inversion solution. The azimuth $\phi_j\parallel$ is determined directly from the recorded azimuth of the source offset, assuming there are no significant deviations from the source–receiver plane. The output from this stage of analysis is several groups of observed polarization values as a function of incident angle, and azimuth, $\psi_j^o (\theta_j, \phi_j)$, for each region of uniform anisotropy.

We use the anisotropic modelling package ANISEIS (Taylor 1991) to construct a database of theoretical polarization angles, $\psi^o$, corresponding to each discrete set of $\theta_j$ and $\phi_j$ values specified by the acquisition geometry, for a particular anisotropic model. This group of polarizations may be visualized as one segment in a database, with similar segments provided by a wide variety of feasible anisotropic models. Anisotropic systems with hexagonal, monoclinic, triclinic, and orthorhombic symmetry are supported in the study. The polarization angles $\psi^o$, corresponding to each particular combination of anisotropies, are computed using the microcrack formulations of Hudson (1980, 1981) as adapted by Crampin (1984), and the results of Postma (1955), and Backus (1962) for the matrix (fine-layering) anisotropy. Combining these anisotropies simulates the expected complexities in the sediments (Bush & Crampin 1987; Crampin 1991a; Wild & Crampin 1991). The theory of Hudson (1986) is used to introduce microcracks into the matrix anisotropy, and is implemented within the ANISEIS package. Anisotropy due solely to microcracks, or to matrix anisotropy, may also be accommodated.

For the present scheme, the database consists of sets of polarizations for anisotropic models corresponding to a system of aligned vertical, parallel microcracks, oriented within $\pm 5^\circ$ of the vertical, and fine-layering, orientated within $\pm 5^\circ$ of the horizontal. This appears a reasonable variation, as a depth it is suggested that the maximum stress components are typically vertical, so that cracks and microcracks are mainly aligned sub-vertical (Crampin 1990), and layers are arranged sub-horizontal. The aspect ratio of the cracks is 0-001, 0-05, or 0.2, crack density varies from no cracks to 0.2 in seven uneven steps, crack content is either
Figure 1. Flowchart showing the scheme for inverting polarizations of the leading split shear wave, derived from VSP recordings, to yield details of subsurface anisotropy. The boxed region refers to the construction of the database. The same database is then used repeatedly for each layer. The asterisk indicates the path which may be taken for layers in which the rays propagate sub-vertically.

water or dry, the orientation angle of the model is between 0° and 360° in steps of 4.5°, the strength of the component of matrix (fine-layering) anisotropy is 0 per cent, 1 per cent, and then 5 to 40 per cent in steps of 5 per cent. This gives a database corresponding to 43,740 different models, 80 different azimuths (0° to 360°, in 4.5° increments), and 20 angles of incidence (0° to 90°, in increments of 4.5°). A maximum of 0.2 for the aspect ratio represents the limit of this parameter, for which the Hudson model is valid (Douma 1988; Cramp 1991b). It is possible to extend this limit further using the formulations of Nishizawa (1982), and this is the subject of a future extension of the present technique. A crack density of 0.2 represents a reasonable upper limit on the expected amount of crack-induced anisotropy present in the upper crust (Cramp 1985).

Once the database is established, it can be used to compute the difference between estimated, and expected polarization angles at any geophone position in a material with a particular anisotropy. In order to make an overall comparison of the degree of fit for a particular subsurface layer, a misfit function is computed:

\[ F_{\text{mis}} = S \sum_{i=1}^{n} (\psi_{\text{est}}^{(i)} - \psi_{\text{true}}^{(i)})^2, \]  

which is evaluated for each anisotropic region \( j \), where \( S \) is a constant scaling factor; \( F_{\text{mis}} \) is a function of the index \( k \), representing a particular anisotropic model, and model orientation index \( l \) which indicates a horizontal rotation angle \( \theta_{\text{H}} \) between 0° and 360°. The model orientation is defined clockwise relative to the horizontal \( X \)-axis (north). The geographical azimuth of each VSP used in the inversion corresponds to the axis. The misfit function is also evaluated for the polarization distribution expected for angles of incidence \( \theta_{\text{inc}} \pm \delta \theta \), representing the degree of expected error, \( \delta \theta \), in the isotropic ray tracing (Appendix A). Thus, for a particular subsurface region, consultation with the database yields three misfit functions \( F_{\text{mis}}^{(1)}, F_{\text{mis}}^{(2)}, \) and \( F_{\text{mis}}^{(3)} \), corresponding to \( \theta_{\text{inc}} \), \( \theta_{\text{inc}} - \delta \theta \), and \( \theta_{\text{inc}} + \delta \theta \), for different combinations of model anisotropy and \( \varphi_{\text{H}} \). This set of values is inspected to find models and values of \( \varphi_{\text{H}} \) which correspond to a minimum value of \( F_{\text{mis}}^{(3)} \). All models and orientations which possess misfit functions \( F_{\text{mis}}^{(1)}, F_{\text{mis}}^{(2)}, \) or \( F_{\text{mis}}^{(3)} \) less than a threshold value \( F_{\text{th}} \), are then accepted as viable solutions. To further constrain the model search, only those solutions corresponding to non-outlying events are accepted. An outlying event is a polarization distribution, which has one or more polarization angles which display a large deviation from the expected value (but for which the overall
misfit function is still acceptably low). For our present purposes, an event is considered outlying if $\psi_{\text{a}} > \psi_{\text{a}} + 5^\circ$.
A further qualifier for the solutions is that they are only acceptable if each is a minimum with respect to the model parameters. Two-valued parameters, such as crack content, or solutions at the limit of a parameter range, are not considered with this restriction. The final result of the database search is the best solution, which has the lowest misfit function, and a number of solutions acceptable under the above conditions. The range of acceptable solutions represents the degree of non-uniqueness, and consequent resolution of the model parameter.

The inversion was performed on a Convex C210 computer, with code compiled to make use of the Convex vectorization facility. It took 90 s CPU time to perform one search of the database for one angle of incidence.

This inversion procedure can, under the appropriate circumstances, be circumvented by establishing a direct relationship between the crack strike and the polarization direction of the leading split shear wave (Fig. 1). This interpretation is not justified in a sedimentary basin, however, where the anisotropy due to aligned cracks combines with the matrix anisotropy producing an anisotropic system with orthorhombic, or perhaps triclinic symmetry. Our present inversion scheme treats this general case, of anisotropy caused by cracks, and matrix layering, both with various orientations and dips.

3 APPLICATION OF INVERSION PROCEDURE

3.1 Inherent non-uniqueness

It is essential to test the degree of non-uniqueness, inherent in the database of polarization values, before the inversion can be applied to estimates extracted from shear-wave data. The extent to which different anisotropic models with various orientations fit the selection criteria, will depend primarily on the size, and position of the data aperture (MacBeth 1991). The aperture is the range of directions of propagation though the anisotropic medium, for which estimates of polarizations are available for analysis. This aperture usually includes the azimuth of the vertical plane containing the source and receiver, and angles of incidence from the source to each geophone. The data aperture thus depends on the acquisition geometry and the subsurface velocity structure.

To investigate the circumstances under which a narrow range of anisotropic models can be resolved using measurements of polarization, three sets of polarization values are extracted directly from the database. The polarizations correspond to hypothetical VSPs illuminating a uniform anisotropic model with water-filled vertical cracks with crack density 0.02 (differential shear-wave anisotropy of 2 per cent), combined with 10 per cent matrix anisotropy. The cracks have an aspect ratio of 0.001, and strike at 90° to the X-axis. The 3-D variation of the polarizations of the leading split shear wave for this chosen model are shown in Fig. 2. This pattern displays a number of point singularities (discussed by Wild & Crampin 1991). Each set of polarization values is extracted from the database for the range of incidence angles and azimuths corresponding to the Apertures 1, 2 and 3 in Fig. 2. Aperture 1, at an azimuth of 40.5° lies close to two point singularities in the polarization pattern, and does not intersect the line singularity. It covers a wide range of angles of incidence from 4.5° to 45°. Apertures 2, and 3 lie at azimuths of 112.5°, and 153°, respectively, covering angles of incidence from 36° to 76°. Consequently, they intersect the line singularity in the polarization plot.

Table 1 shows details of the criteria used to accept the inversion solutions, together with the number of estimates, and the CPU time. Figs. 3(a) to (d) show the results of performing an inversion using the data from (a) Aperture 1 alone, (b) Aperture 2 alone, (c) the results of combining the information from the two data sets, and (d) the improvement when combining data from all three apertures. For the inversions performed using data taken from a single azimuth (Fig. 3a and b), the best solution corresponds to the correct database model, but there are many anisotropic models which also satisfy the inversion criteria. The results for Aperture 1 display marginally better resolution than Aperture 2, presumably relating to the uniqueness of the pattern of disruptive behaviour around the point singularities, whilst the intersection of a line singularity appears

<table>
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<tr>
<th>Table 1. Details of inversion of polarization directions extracted directly from database, for testing the inherent non-uniqueness.</th>
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<tbody>
<tr>
<td>Aperture</td>
</tr>
<tr>
<td>Minimum $F_{\text{a}}$</td>
</tr>
<tr>
<td>Threshold, $F_{\text{a}}$ for acceptance</td>
</tr>
<tr>
<td>Number of outlying points</td>
</tr>
<tr>
<td>Number of local minima</td>
</tr>
<tr>
<td>Number of estimates used in inversion</td>
</tr>
<tr>
<td>Approximate CPU time (min)</td>
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to be a less unique occurrence. When the data from these apertures are combined, the resolution is improved except for the dip parameters—there are now three possible solutions (Fig. 3c). It is necessary to include the additional data from Aperture 3 before the correct solution is resolved without confusion (Fig. 3d). Note that there are three solutions for the horizontal orientation angle, $\phi_R$ of the anisotropic system: $0^\circ$, $180^\circ$, and $360^\circ$. This is a consequence of the symmetric pattern of polarizations for the solution, and may not occur in the general case. It is concluded that the solution can be resolved only if data from apertures at several azimuths are used. The resolution is immediately improved if the apertures intersect, or pass near to, an unusual feature of the polarization patterns such as a point singularity.

3.2 Multi-azimuth synthetic seismogram data

We now test the procedure by analysing synthetic seismogram data computed for three offset VSPs, shot around the same borehole, with identical geophone positions, but at different azimuths. The geophone string consists of 50 levels, lying between 50 and 1275 m, with a spacing of 25 m (Fig. 4). Data are computed for three cross-line sources, all at 200 m offset, exciting the subsurface at azimuths of $0^\circ$, $50^\circ$, and $70^\circ$. Details of the three-layered anisotropic structure used for the computations are given in Table 2, and the size of the data apertures are given in Table 3. Each layer possesses crack and matrix anisotropy, with a different crack-strike direction in each layer, representing a complex sequence of discrete layering. The shallowest layer of 200 m thickness, consists of dry cracks, with a crack density of 0.04 (4 per cent differential shear-wave anisotropy), permeating a medium with matrix anisotropy of 5 per cent. The second layer at 500 m consists

![Figure 3. Results of inverting polarization values taken from database. The inversion uses polarizations from 10 geophones. The solid triangles correspond to the inversion solution with the smallest value of the fit-function. The open triangles correspond to the actual parameters used in the synthetic seismogram model. The shaded areas (and the dots on the $\phi_R$ scale) correspond to the solutions which also produce an acceptable value of the fit-function. Results for (a) Aperture 1 alone; (b) Aperture 2 alone; (c) Apertures 1 and 2; (d) Apertures 1, 2, and 3. Several solutions are possible for the orientation angle due to the symmetry of the polarization patterns for the solutions. The matrix anisotropy is specified by the strength of the anisotropy (%PTL), and dip values about the $X$- and $Y$-axes, $\delta^x$ and $\delta^y$. The cracks are specified by the crack density (CD), aspect ratio (AR), two dip values, $\delta^x$ and $\delta^y$ content of the cracks (dry, D, or water-filled, W), and the strike value, $\phi_R$ (and hence the orientation of the crack system).](image)

![Figure 4. Geometry of acquisition system used to generate synthetic seismograms for a VSP shot in a three-layered anisotropic medium. Only data generated with a cross-line source are considered in this study. Details of the anisotropy for each layer are shown in Table 2.](image)
Table 2. Three-layered anisotropic structure for generating the synthetic VSP data of Fig. 5, which is used to test the inversion procedure. The model is set up by firstly introducing matrix anisotropy into each layer, and then introducing the cracks using Hudson (1986). (a) Isotropic model is specified by layer thickness (t), P-waves velocity (a), S-waves velocity (½), density (ρ), with Poisson’s ratio (ν) shown for reference. HS specifies layer which is the terminating half-space of the model. (b) Anisotropic model specified by crack parameters CD (crack density), AR (aspect ratio), δc and δs (dips of cracks along X- and Y-axes), CT (content), and φc (model orientation about Z-axis). Matrix anisotropy is specified by %PTL (maximum differential anisotropy), and δc and δs (dips of layering along X- and Y-axes).

<table>
<thead>
<tr>
<th>LAYER</th>
<th>t (m)</th>
<th>a (km/s)</th>
<th>ρ (g/cm³)</th>
<th>σ (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>3.000</td>
<td>1.604</td>
<td>2.300</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>4.000</td>
<td>2.139</td>
<td>2.450</td>
</tr>
<tr>
<td>3</td>
<td>HS</td>
<td>5.000</td>
<td>2.674</td>
<td>2.600</td>
</tr>
</tbody>
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(b) Anisotropic parameters

<table>
<thead>
<tr>
<th>CRACK ANISOTROPY</th>
<th>MATRIX ANISOTROPY</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD AR δc, ½ C ½ CT φc</td>
<td>XPTL ½ L ½ L</td>
</tr>
<tr>
<td>1.00 0.001 0° 0° W 0°</td>
<td>5 0° 0°</td>
</tr>
<tr>
<td>2.00 0.050 0° 0° W 10°</td>
<td>20 0° 0°</td>
</tr>
<tr>
<td>3.00 0.001 0° 0° W 70°</td>
<td>5 0° 0°</td>
</tr>
</tbody>
</table>

of a medium with 20 per cent matrix anisotropy, permeated by water-filled cracks with a crack density of 0.02 (2 per cent anisotropy). The deepest layer in the sequence at 700 m depth has 5 per cent matrix anisotropy, combined with water-filled cracks with a crack density of 0.1 (10 per cent anisotropy). Both the cracks and matrix anisotropy have 0° dip (vertical and horizontal, respectively).

Figure 5 shows the three-component synthetic seismograms generated for each of the three azimuths. Only the direct shear-wave arrivals are modelled for these data. Also present in this data, especially noticeable on the vertical component, are two branches of shear to P-converted waves. These are not analysed in the present study. Sections for identical components, but different azimuths, look similar on the basis of traveltimes, with minor variations of amplitude with offset being the only obvious visual difference between the seismograms. The polarization diagrams (not shown) do, however, indicate that there is a considerable difference between the sections. Equal area plots of the horizontal projections of the leading polarization corresponding to each layer are shown in Fig. 6 with the appropriate data apertures. The largest apertures are for Layer 2, with those for Layer 3 being the smallest.

Also shown in this diagram are the expected polarization patterns for each of the three layers. Two of the apertures for Layer 1 lie close to a region of disruption surrounding a point singularity. Aperture 1 for Layer 2 includes a point singularity (due to the acquisition geometry, there are no synthetic seismograms corresponding to the cone of directions associated with this singularity). All the apertures for Layer 3 lie within the band of parallel polarizations associated with the crack-induced anisotropy and sample a very narrow range of angles of incidence.

Polarization angles for the leading split shear wave, corresponding to each direct shear-wave arrival in the VSP, are estimated using an algorithm which determines the polarization angle of the initial linear onset (MacBeth 1990). The technique works effectively for the noise-free synthetic seismograms used in this study, but for noisy field data it will have similar difficulties to the visual analysis of polarization diagrams (MacBeth & Crampin 1991a). Fig. 7 plots the resultant polarization estimates, ψ, as a function of geophone level for each VSP, together with the actual profile obtained directly from the database. The accuracy of the estimated polarization values is dependent on the orientations of the source polarization, and the polarization directions of the leading split shear wave in each layer (this point is discussed further in Appendix B), the accuracy of the incidence angles, and the inherent resolution of the processing technique. The polarizations in Layer 1 match the expected values well for azimuths of 0° and 50°, but

Table 3. Source offset, and directional coverage of subsurface layers for earth model used to generate synthetic seismograms. Range of incident angles corresponds to data apertures of Fig. 6.

<table>
<thead>
<tr>
<th>OFFSET (m)</th>
<th>INCIDENT ANGLES (°)</th>
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<tbody>
<tr>
<td></td>
<td>LAYER 1</td>
</tr>
<tr>
<td>200</td>
<td>45 - 76</td>
</tr>
<tr>
<td>400</td>
<td>45 - 76</td>
</tr>
<tr>
<td>600</td>
<td>45 - 76</td>
</tr>
</tbody>
</table>
overestimate the values for an azimuth of 70°. This is due to a drop in the energy of the leading split shear wave at this aperture, and the inability of the processing technique to distinguish between this arrival, and the small background of numerical noise. This situation would be exacerbated for noisy field data. The estimates for the polarization directions of Layer 2 agree within 10° for all of the azimuths, with the exception of measurements at the topmost six geophones of the VSP at 0° azimuth. The sharp drop, and then rise of the polarization angle in the expected profile at 0° corresponds to crossing through the point singularity. The lack of resolution here is due to the small time delays (although still finite) associated with the geophone positions (Crampin 1991a). The polarization values of Layer 3, a quite strongly cracked layer, match the expected values well, after a small transition interval of several geophone levels, in which sufficient time delay (larger than the sampling interval of 2 ms) is built up for the processing technique to measure the polarization.

Note that Fig. 7 also demonstrates that polarization directions (and time delays, not shown) can vary quite considerably within a uniform anisotropic material, and a recorded feature such as a gradient in polarization direction needed not be indicative of a change in anisotropic properties, but reflects the changes in the direction of wave propagation with each geophone level. It is these variations in the polarization direction which the inversion algorithm utilizes in identifying suitable anisotropic models to satisfy the observations. Note that the rays to each geophone pass inside shear-wave windows for each of the structural interfaces (Liu & Crampin 1990), so the effects of these windows are minimized, and all the polarization estimates can be used in the inversion procedure.

The polarization estimates for the VSPs at different azimuths are inverted, and the results are shown in Figs 8(a) and (b). Fig. 8(a) shows the estimates from inverting apertures at azimuths of 50° and 70°, and Fig. 8(b) shows the estimates for all three apertures. Details of the parameters used to select the solutions for the inverse shown in Table 4.

The results for the two apertures (Fig. 8a) show the trend of predicted resolution in the model parameters noted by MacBeth (1991). In the top layer of the model, polarization values from a range of shallow incident angles, and from

Figure 6. Equal-area projections of polarizations of leading split shear wave onto horizontal plane, with superimposed data apertures available for the inversion of the synthetic offset VSP data. Solid circles correspond to directions of point singularities. Apertures 1, 2, and 3, correspond to azimuths of 0°, 50°, and 70°, and (a) refers to Layer 1, (b) to Layer 2, and (c) to Layer 3.

Figure 7. Results of processing seismograms from synthetic VSP. The estimated polarizations are shown as open triangles, the expected (actual) polarizations are solid dots. Polarization profiles are compared for three azimuths, 0°, 50°, and 70°. Polarization angles are measured clockwise from the X-axis.
Figure 8. Results of inverting polarization values from Fig. 7. The inversion uses polarizations from seven geophones for Layer 1, 20 geophones for layer 2, and 23 geophones for Layer 3. The solid triangles correspond to the inversion solution with the smallest value of the fit-function. The open triangles correspond to the actual parameters used in the synthetic seismogram model. The shaded areas (and the dots on the polarization scale) correspond to the solutions which also produce an acceptable value of the fit-function. Results for (a) Apertures 1 and 2, (b) Apertures 1, 2, and 3. Several solutions are possible for the orientation angle due to the symmetry of the polarization patterns for the solutions. The matrix anisotropy is specified by the strength of the anisotropy (%PTL), and the two dip values for the system, $\delta_L^x$ and $\delta_L^y$. The cracks are specified by the crack density (CD), aspect ratio (AR), two dip values, $\delta_C^x$ and $\delta_C^y$, content of the cracks (dry, D, or water-filled, W), and the strike value, $\phi_R$ (and hence the orientation of the crack system).
Figure 8. (continued)
apertures at both azimuths, provide inaccurate solutions for the model parameters. This may be expected from Fig. 7, as only the aperture with 0° azimuth supplies accurate estimates of the variation in polarization direction with depth for this layer. The orientation angle of the model (and hence crack strike) is not found. These results are expected as the data apertures are far away from the central band of parallel polarizations (supplying crack-strike information), and do not intersect or pass close to a singularity [supplying information about aspect ratio, crack content (if a line singularity), or ratio of crack density to strength of matrix anisotropy (if a point singularity)]. In Layer 2 the ray angles cover a range of directions close to, or intersecting, point singularities in the anisotropic model. Most model parameters are well resolved, and the orientation angle of the model is well defined, although there is an overestimate of the aspect ratio. For Layer 3, as expected, the rotation angle of the anisotropic material (and hence the strike of crack-induced anisotropic component) is accurately defined. Unusually, the crack content and aspect ratio are also well defined. For many of the other models studies (not shown), for which the data apertures correspond to sub-vertical propagation, the crack content and aspect ratio were not well resolved. It is suggested that detection of the limiting width of the band of parallel polarizations was sufficient information with which to determine these parameters on this occasion.

By including the results from the aperture at an azimuth of 0° in Fig. 8(b), the resolution in Layer 1 is improved for the aspect ratio of the cracks and the crack content, together with an increased accuracy in the solution of aspect ratio and the percentage of matrix anisotropy. However, the accuracy of the solutions for Layer 2 decreases. This is related to the six poor estimates at the top of Layer 2 at this azimuth, which were unable to follow the variation in polarization direction across the point singularity. The solutions for Layer 3, on using data from three azimuths, are quite close to the expected model parameter values.

The dip values are not well resolved for any of the layers, or solutions. This may be attributed to the relative non-uniqueness that exists between dip and strike (MacBeth 1991). This emphasizes that proper constraint of the crack dip is an important aspect in any successful inversion.

4 DISCUSSION AND CONCLUSIONS

The pattern of polarization directions for the leading split shear wave, described as a function of azimuth, and angle of incidence, is dependent on the parameters which define the internal structure of the rockmass. The nature of this dependence is highly non-linear, but is predictable from particular anisotropic models. An inversion procedure is constructed, which relates, in a numerical fashion, polarization angles estimated from offset VSPs to the polarization values predicted for a variety of theoretical anisotropic models. The method used is defined as a database inversion, because a search is performed over a large quantity of polarization information stored in an organized way. Each segment of the database, corresponding to an anisotropic model, is consulted sequentially to determine the polarization profiles which provide the best fit.

As a prelude to the inversion procedure, an analysis of the inherent non-uniqueness in the database inversion was performed. Here it was concluded that data from source offsets at several azimuths are required to provide adequate resolution of the model parameters. It was essential that the data apertures passed through or near the pattern of disruption caused by a point singularity. Line singularities appeared to be less deterministic of model parameters, although they did constrain the solutions to some degree. The inversion procedure was then applied to synthetic seismograms for three VSP experiments with a common borehole and offset, but different source azimuths. Each layer of the Earth structure used in these computations was anisotropic with orthorhombic symmetry. Estimates of the polarization directions of the leading split shear wave were obtained to a reasonable accuracy using a processing technique which detected the initial linear onset. These polarization values compared well with the expected profiles obtained directly from the ANISEIS package. Disparities between the two profiles were related to the relative orientations between the polarization directions in each layer, the direction of the source polarization, and the inherent resolution of the processing technique. These polarizations were inverted for data from the first two azimuths, and then for all three together. The model parameters were poorly resolved in the topmost layer of the Earth model (thickness 20 m) for which the directions of propagation are sub-horizontal, and well defined for the next layer in the model (between 200 and 700 m depth) for which the directions of propagation cover a large range of incidence. In the deepest layer (below 700 m), the horizontal orientation of the model is well resolved. Other model parameters also appear to be well constrained for the deepest layer in this particular model, as they appear sensitive to the width of the band of parallel polarizations.

This result is unusual and is not a generality. The horizontal orientation of the anisotropic model appeared well resolved for all but the shallowest layer. Dip variations larger than 5° produced many possible inversion solutions with different horizontal orientations. These parameters need to be tightly constrained to limit the degree of non-uniqueness.

From this synthetic seismogram study, a number of guidelines can be drawn, relating to the inversion and interpretation of polarizations estimated from shear-wave
VSP data:
(i) data from at least three azimuths should be available;
(ii) data apertures should pass near to, or across, a pattern of disruption created by a point singularity;
(iii) dip values need to be well constrained by a priori geological or geophysical studies; and
(iv) crack strike/model orientation is usually defined for the deeper layers in the Earth model.

The next stage of this work will involve inversion of appropriate field VSP data. This will require an adjustment of the technique to accommodate errors accumulating from the isotropic background medium, which must be estimated using traveltime inversion (for P and S arrivals), and well logs (if available). This increases the incidence angle errors, and consequently increases the number of acceptable solutions. With real data, the dynamic range of the instrumentation and the signal-to-noise ratio decrease the resolution and increase the uncertainty in the polarization estimates. Under these circumstances, it is important to use an estimation algorithm which is more robust to noise than the present one. Methods which use data acquired from several horizontal, orthogonal, source polarizations (such as Alford 1986; MacBeth & Crampin 1991b; Li & Crampin 1991) are suitable for this purpose. Multi-azimuth, multisource data sets have recently been acquired, and are at present being analysed.

The inversion of polarization directions provides the first step in obtaining numerical solutions to the inverse problem for shear-wave splitting. From the analysis above, it can be seen that before the inversion results can be used with any degree of confidence, and used within a statistical framework, further constraints need to be placed on the solutions. One proposed additional constraint is to include differential time-delay information from the split shear waves. Another proposed development of this work is to extend the crack density and aspect ratio to their full range, and to vary the acoustic velocities of the contents within the cracks. The database may also be reduced in size by taking advantage of symmetries between patterns arising from different models. The flexibility of the database scheme also allows the user to incorporate additional constraints, such as information from other arrivals, or a priori constraints from other measurements.

ACKNOWLEDGMENTS
I thank Phil Wild for help with the computing aspects of this work, and also Applied Geophysical Software Inc. and Macroc Ltd for permission to use the ANISEIS software. I thank Stuart Crampin and Gareth Yardley for comments on the manuscript. This work was supported by the sponsors of the Edinburgh Anisotropy Project, and the Natural Environment Research Council, and is published with the approval of the Director of the British Geological Survey (NERC).

REFERENCES

APPENDIX A
Treatment of errors for incidence angle

For the purposes of the inversion, a ray tracing procedure is involved in computing the angles of incidence at each geophone, using as input the isotropic background medium. In field data this background may be determined from traveltimes inversion and well-logs, but for this present work it is assumed known. It is also assumed that ray path deviations in and out of the source--receiver plane are not significant for weak anisotropy, and that isotropic raytracing can be used to obtain the incidence angles, with a small correction error (δθ). By the same token it is assumed that the azimuths of the rays can be determined by the azimuth of vertical plane containing source and receiver. To test these assertions, estimates of incidence angles are made using isotropic ray tracing through a uniform anisotropic medium, with the same elastic constants as Layer 2 of Table 2. The material has orthorhombic symmetry. The same VSP geometry was used as in Fig. 4, with Layers 1, 2 and 3 consisting of the same material. Synthetic seismograms for an in-line source at offset azimuth of 0° are computed. For comparison purposes, as the actual incidence angle cannot be determined (no anisotropic ray tracing algorithm was available at the time of this work), a second set of estimates is obtained from the synthetic seismograms by assuming orthogonality between the ray and polarization directions. The resultant angles obtained using the two methods are shown in Fig. A1 for the first 700 m of the model. As an interpretational aid, in-plane deviations for the ray (group velocity) and polarization vectors, with respect to the phase propagation direction are also computed using ANISEIS (but not shown). These indicate that for this particular anisotropic model, in-plane deviations in the sagittal plane between these vectors amount to no more than 4.5°. The error in making the assumption of orthogonality between the ray direction, polarization direction, and the phase propagation direction is thus 9°. This is the maximum possible error in making a direct inference of incidence angle (ray direction) from polarization direction. It is assumed that there is an error of similar magnitude between estimates from isotropic ray tracing and polarization directions, and that estimates from the former may be closer to actual ray directions than estimates from the latter. The two sets of values in Fig. A1 deviate by an average of 10°. This error is taken into account in the inversion by evaluating misfit functions for three different theoretical polarization profiles with angles of incidence adjusted by ±10°, giving $F_{K'}(θ)$, $F_{K'}(θ+10°)$, and $F_{K'}(θ-10°)$. The way in which these values are treated in the subsequent inversion procedure is described in Section 2. Further development is planned to include anisotropic ray tracing, and errors in the azimuth.

APPENDIX B
Effect of source and medium orientations on resolution of polarizations

The procedure for comparing estimates of the polarization profiles, obtained from the processing technique for

Figure A1. Estimates of angle of incidence for rays propagating through an anisotropic material (Layer 2 in Table 1) with orthorhombic anisotropy. Solid circles correspond to values inferred from measurement of the direction of polarization of synthetic seismograms in the sagittal plane, from an in-line source. Open circles correspond to values from isotropic ray tracing, using the isotropic matrix velocities.

Figure B1. Equal-area projections of polarizations of leading split shear wave showing the data apertures available for the inversion of the synthetic seismograms for offset VSP data. Solid circles correspond to directions of point singularities. Details of anisotropic structure are in Table B1. Apertures 1, 2, and 3, correspond to azimuths of -20°, 0°, and 20°. (a) Layer 1, (b) Layer 2, (c) Layer 3.
Table B1. Anisotropic parameters for a model in which polarization estimates diverge from expected values. Parameters are defined in Table 2.

<table>
<thead>
<tr>
<th>Crack Anisotropy</th>
<th>Matrix Anisotropy</th>
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</thead>
<tbody>
<tr>
<td>CD</td>
<td>AR</td>
</tr>
<tr>
<td>1</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>0.02</td>
</tr>
<tr>
<td>3</td>
<td>0.10</td>
</tr>
</tbody>
</table>

detecting the initial linear onset of the shear waves, with the expected values from the ANISEIS package, demonstrates an interesting feature of model structures with more than one type of anisotropy. Fig. B1 shows the equal area plots for three VSPs shot around a common borehole, with the same dimensions as in Fig. 4, but with the relative orientations of the media being slightly different (see Table B1) (the anisotropy in each layer is the same as in Table 2). Azimuths of 0°, and ±20° are chosen for the source offsets. The expected patterns of the polarization distributions, and data apertures are shown in Fig. B1. Fig. B2 shows a comparison between expected polarizations and those estimated from synthetic seismograms. Although the polarization directions for Layers 1 and 3 appear reasonably well resolved, to about the same accuracy as for example in Section 3.2, Layer 2 is not resolved for azimuths of 0° and −20°, and the lower geophone levels in this layer for an azimuth of 20° is also unable to fully resolve the polarizations. This problem is related to the relative orientations for the fast shear-wave polarization directions in Layers 1 and 2. For the first two azimuths (Figs B2a and B2b), the fast polarization directions in the two layers are approximately orthogonal. Consequently, the faster split shear wave propagating in Layer 1 excites only the slower split shear wave in Layer 2, and vice versa. Thus, the technique for detecting the polarization direction of the initial onset does not detect the fast split shear wave travelling in Layer 2, but the slower split shear wave excited by the faster split shear wave in Layer 1, with a different polarization direction. The threshold for the detection of the faster split shear wave in Layer 2, determines the range of orientation angles over which this phenomenon occurs. A similar effect may arise due to the relative orientation of the polarization directions in Layer 1, relative to the source polarization, which determines the amount of energy exciting each split shear wave. Depending upon the relative orientations of the polarization directions in each layer, it is possible that the energy in the fast arrival may not be resolved using either this or probably any other processing technique. This may present difficulties in field data, where there are no ways of validating the polarization estimates unless a priori geological data are available. The extent of this problem will depend upon the area which is being surveyed. If discrete changes in polarization direction are suspected [for example, Winterstein & Meadows (1990a, b) and Squires, Kim & Kim (1989) for the Lost Hills Field, California] then this phenomenon is important. If smooth changes are observed [for example, Alford (1986) observes a progressive build in time delays with depth in Dilley, Texas], the phenomenon probably does not occur.

Figure B2. Estimates of polarization (open circles) derived from seismograms generated by a cross-line source exciting model in Table B1. Expected values are shown as solid circles.