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Effects of point singularities on shear-wave propagation in sedimentary basins

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SUMMARY

In most directions of propagation in anisotropic solids, seismic shear waves split in regular and predictable ways that, in principle, can be directly related to the degree of anisotropy and the anisotropic symmetry of the rockmass. In all anisotropic solids, however, there are directions of propagation, known as shear-wave singularities, where the split shear-waves have the same phase-velocities. For directions of propagation near the commonest type of singularity, the point singularity, the relationship between the phase and group-velocities may undergo rapid variations for small changes in direction. This results in shear-waves along rays (propagating at the *group*-velocity) behaving anomalously, with irregular polarizations and amplitude changes as if they were propagating near cusps, although the degree of anisotropy may be too small to cause conventional cusps on the group-velocity wave surfaces.

The effects of propagation near such point singularities have been identified in sedimentary basins where they are features of the well-established phenomenon of ***azimuthal isotropy*** (transverse isotropy with a vertical axis of symmetry) caused by horizontal lithology, or by fine layering (PTL anisotropy), combining with the more recently recognized ***azimuthal anisotropy,*** caused by distributions of near-parallel near-vertical fluid-filled inclusions (EDA anisotropy). This paper demonstrates these irregular effects by calculating synthetic shear waves in directions near a point singularity in a material simulating a possible sedimentary basin. Such anomalies may be important in exploration seismology as point singularities can occur along nearly vertical ray paths in sedimentary basins. If not identified correctly, the effects of such point singularities could be mistakenly attributed to structural irregularities, and if correctly identified, the directions of such singularities can place tight constraints on possible combinations of PTL and EDA anisotropy in sedimentary basins.

Key words: anisotropy, point singularities, sedimentary basins, shear-wave splitting.

1 INTRODUCTION

Shear-wave splitting, diagnostic of some form of effective seismic anisotropy, is a common feature along most shear-wave ray paths in the Earth's crust (Crampin 1987a). Except in sedimentary basins, shear-wave splitting usually behaves in a comparatively orderly way with the polarizations of the leading split shear wave generally parallel or subparallel to the direction of maximum horizontal stress. This appears to be the result of distributions of stress-aligned fluid-filled inclusions pervading most rocks in at least the upper half of the crust (Crampin 1987a). These distributions of stress-aligned inclusions are known as *extensive-dilatancy* ***anisotropy*** or *EDA*. Since many of the seismic effects can be simulated by distributions of flat parallel cracks (Crampin 1991), the inclusions are conveniently referred to as ***EDA cracks,*** although it is recognized that individual inclusions may have a wide variety of shapes (Crampin 1991).

Shear-wave splitting in sedimentary basins, however, may show more complicated behaviour with non-parallel polarizations of the leading split shear waves (Bush & Crampin 1987; Bush 1990). This is a result of the azimuthal isotropy (transverse isotropy, with a vertical symmetry axis, due to lithology, as in shales, or to fine layering known as PTL anisotropy) common in many rocks in sedimentary basins combining with the azimuthal anisotropy of the EDA cracks. This leads to orthorhombic anisotropic symmetry, which implies the presence of directions of propagation, called ***point singularities,*** where the shear waves display irregular behaviour (Crampin 1981; Wild & Crampin 1991).

Shear waves along ray paths near point singularities display rapid variations in polarization and amplitude, quite unlike the regular behaviour of shear-wave splitting in anisotropic media in directions away from such singularities. In the particular, the solid angles of directions of irregular behaviour may be quite large. For the material examined in this paper, the anomalous behaviour extends more than 10" either side of the singularity. Since these point singularities may occur along nearly vertical ray paths in sedimentary basins (Crampin 1988; Wild & Crampin 1991), it has become important to recognize their effects.

This paper presents an initial study, in an idealized sedimentary basin, of what are probably the first calculations of synthetic shear waves along ray paths near point singularities. Wave propagation is calculated in a uniform material so as to isolate the effects of the singularity from the effects of inhomogeneities. In an attempt to understand the principles causing the phenomena, the paper interprets the behaviour in terms of the polarizations and time delays between the split shear waves. The paper begins with a brief review of what is known about shear-wave singularities.

[Note that shear-wave behaviour in anisotropic structures is fundamentally different from the behaviour in isotropy (Crampin 1981). In order to increase our physical understanding of the behaviour of shear-wave propagation in sedimentary basins, the paper attempts to interpret the behaviour in terms of more conventional shear-wave propagation. Note also that discussions of seismic anisotropy require a rather different notation from discussions of isotropy, and a more comprehensive explanation of the terminology can be found in Crampin (1989).]

2 SHEAR-WAVE SINGULARITIES

The locus of shear-wave phase-velocity propagation in anisotropic rock traces out two analytically continuous surfaces, the phase-velocity surfaces or phase-velocity sheets (Crampin 1981). These sheets have different particle-displacement polarizations so that, in most directions of propagation, an incident plane shear wave (propagating at the phase velocity) splits and excites two phases with strictly orthogonal polarizations and different velocities. However, all anisotropic solids have at least two directions, known as singular points, where the two shear-wave phase-velocity sheets touch and are analytically continuous (Crampin & Yedlin 1981). There are three types of singularity, kiss, point and line singularities. Table 1 lists the numbers of singularities on planes of mirror symmetry in the various systems of anisotropic symmetry. In some circumstances, kiss and point singularities may also occur in directions that do not lie in planes of mirror symmetry (as in Fig. 4, below).

Kiss and line singularities generally cause little disturbance to rays of shear waves along neighbouring ray paths (propagating at the group velocity), but the presence of the most frequency occurring singularity, the point singularity, may significantly disturb the behaviour of shear waves along neighbouring ray paths at the group velocity (Crampin 1981). The directions of phase- and group-velocity propagation of body waves in anisotropic solids usually diverge so that the ray (group-velocity) direction is, in general, not normal to the surface of constant phase (Crampin 1981). For directions near point singularities the divergence of phase and group velocity may undergo particularly rapid variations. This means that, in directions near point singularities, the behaviour of rays of shear waves radiating from point sources at the group velocity may be very different from the behaviour of plane shear waves, propagating at the phase velocity. [As an approximate guide, the largest deviation of phase and group velocity (in degrees) in any particular anisotropic structure is usually similar in value to the differential shear-wave anisotropy (in per cent).

2.1 Kiss singularity

Kiss singularities are directions where two shear-wave phase-velocity sheets touch tangentially at isolated points (Fig. 1a). There is always a kiss singularity in the direction of the symmetry axis of solids with hexagonal symmetry (transverse isotropy) as in Fig. 1(b), but kiss singularities may also occur in trigonal, orthorhombic, monoclinic, and triclinic symmetries for particular values of the elastic constants.

Except for strong anisotropy (differential shear-wave anisotropy greater than 10 per cent, say), the behaviour of group-velocity sheets at kiss singularities is similar to the behaviour of phase-velocity sheets, as illustrated in Fig. 1(a), and shear-wave ray paths near such singularities display few anomalies. The shear-wave sheets touch tangentially, and there is little difference in the velocities of the two split shear waves, so that the waveforms of the initial shear-wave pulse will propagate with little distortion. The delays between the split shear waves will increase for directions away from the singularity, but by the time the

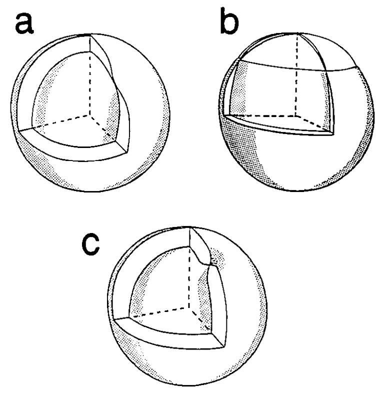


Figure 1. Schematic illustration of the three types of singularity on the two shear-wave phase-velocity sheets: (a) kiss singularity, where the two sheets touch tangentially at a point; (b) line singularity, where the two sheets may be considered as intersecting (only possible in hexagonal symmetry, as illustrated, when there is also a kiss singularity along the symmetry axis); and (c) point singularity, where the two sheets touch at the vertices of shallow cones (here much exaggerated). [After Crampin & Yedlin (1981) with modifications.]

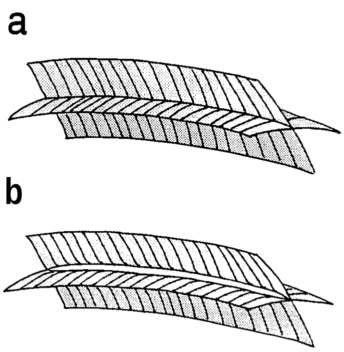


Figure 2. Schematic illustration of: (a) a line singularity in hexagonal symmetry, where the two phase-velocity sheets intersect; and (b) the pulling apart of the inner and outer phase-velocity sheets into a ring pinch when there is a small perturbation to the hexagonal symmetry. Note point singularities at both ends of the section illustrated.

Table 1. Number of shear-wave singularities on planes of mirror symmetry in different systems of anisotropic symmetry (after Crampin & Kirkwood 1981).

	Type of singularity		
Symmetry system	Kiss	Line	Point
Cubic	6	0	8
Hexagonal	2	2,0[1]	0
Trigonal[2]	0	0	8,20[1]
Tetragonal	2	0	8
Orthorhombic[2]	0	0	4,12[1]
Monoclinic*	0	0	8
Triclinic[2,3]	0	0	0[3]

[1]More than one configuration possible.
[2]Systems with the possibility of kiss and point singularities in off-symmetry directions for particular values of elastic constants.
[3]Systems with triclinic symmetry necessarily have at least two kiss or point singularities in off-symmetry directions.

angular distance is large enough to cause a significant delay, the rapid variation of polarizations near the kiss singularity will also be reduced, and the shear waves will split in a regular manner without any particular anomaly.

2.2 Line singularity

Line singularities, where the two shear-wave phase-velocity sheets may be considered as intersecting, only occur in systems of hexagonal symmetry (transverse isotropy), as in Fig. l(b), where the two sheets cut each other in a circle concentric about the axis of cylindrical symmetry. Line singularities do not cause anomalies in group-velocity sheets as the two sheets can be considered as simply intersecting, in the same way as that illustrated for the phase-velocity sheets in Fig. 2(a). The shear waves will then split into two nearly orthogonal components with a time delay between the split shear waves depending on the degree of differential shear-wave velocity anisotropy along the particular ray path, which will diminish as the ray passes closer to the intersection.

The difficulty of considering a line singularity as the intersection of two sheets as in Fig. 2(a) is that as soon as any, even small, perturbations are introduced into the hexagonal symmetry (see Section 3, below), the inner (slower) and outer (faster) phase-velocity sheets pull apart along the line of the singularity, as in Fig. 2(b), forming a ***ring pinch*** (Crampin & Yedlin 1981). This ring pinch introduces strong curvatures into the phase-velocity sheets which generates a cuspoidal fin on the surface of the inner sheet of the associated group-velocity sheets, and a corresponding opening (hole) in the outer group-velocity sheets. The two phase-velocity sheets do not pull apart completely, however, and for small perturbations the sheets touch at eight point singularities along the line of the original line singularity, as in the point singularities shown in Fig. 2(b). Four of these eight singularities are in symmetry planes, but alternate with singularities in off-symmetry directions [as in Figs 4(b) and (c), below]. [Note that however weak the anisotropy, if the perturbation is sufficient small, the curvatures near the pinch will be large and a cuspoidal fin will exist, although it may be small in angular dimensions and velocity differences, and consequently difficult to identify in observations either in the field or the laboratory.]

The sheets can no longer be considered as simply intersecting when there is a ring pinch, although they are still analytically continous at the point singularities. An unambiguous way to identify the shear waves is to refer to the outer (faster) velocity sheet as $qS1$, say, and the inner (slower) sheet as $qS2$. The 'qS' refers to ***quasi-shear** waves,* where the polarizations are no longer strictly pure shear (Crampin 1981, 1989). Identification of faster and slower shear-wave sheets is wholly consistent, and it is often easier to avoid changes in nomenclature, and corresponding difficulties in numerical identification, by identifying the sheets as faster and slower sheets ($qS1$, and $qS2$, respectively), even in purely hexagonal symmetry where two sheets may be otherwise considered as intersecting (Crampin 1989).

2.3 Point singularities

Point singularities are directions where two shear-wave phase-velocity sheets touch (coincide) at the vertices of, usually very flat, convex and concave cones on the inner and outer velocity sheets, respectively, as shown in Fig. l(c). Such singularities are called conical points in crystallography literature. Symmetry considerations show that point singularities cannot occur in hexagonal symmetry, but all other systems of anisotropic symmetry must contain some, often many, point singularities (Table 1). Although point singularities appear to have a comparatively simple representation in phase-velocity sheets, there is high curvature of the phase-velocity sheet near the singularity, and the behaviour of shear-wave polarizations of shear waves radiated from a point source may be complicated, with the polarizations of rays passing near point singularities varying by up to 180". The corresponding behaviour in the group-velocity sheets may be very complicated, and we need to refer to a specific example to show the effects. The example shown in the next section simulates the behaviour in a possible sedimentary basin.

Table 2. Elastic constants of solids in Pa x 10' with density ρ = 2450 kg m^{-3}. Note symmetry about the main diagonal. North is taken as the x direction and south as the y direction.

PO7: Solid made up of periodic thin-layers (PTL) with differential shear-wave anisotropy of about 7%.

	(1111)	(1122)	(1133)	(1123)	(1131)	(1112)
(1111)	40.125					
(2211)	10.883	40.125				
(3311)	8.887	8.887	33.324			
(2311)	0.000	0.000	0.000	12.626		
(3111)	0.000	0.000	0.000	0.000	12.626	
(1211)	0.000	0.000	0.000	0.000	0.000	14.621

C10: Isotropic matrix (V_P, V_S = 4.0. 2.3 km/sec) pervaded by thin parallel EDA-cracks with a crack density of 0.1 and differential shear-wave anisotropy of about 10%.

	(1111)	(1122)	(1133)	(1123)	(1131)	(1112)
(1111)	39.917					
(2211)	13.184	39.167				
(3311)	13.184	13.247	39.167			
(2311)	0.000	0.000	0.000	12.960		
(3111)	0.000	0.000	0.000	0.000	10.334	
(1211)	0.000	0.000	0.000	0.000	0.000	10.334

P07C10: Structure PO7 pervaded by EDA-cracks C10 giving combination of PTL- and EDA-anisotropy.

	(1111)	(1122)	(1133)	(1123)	(1131)	(1112)
(1111)	39.841					
(2211)	10.809	40.095				
(3311)	8.835	8.886	33.347			
(2311)	0.000	0.000	0.000	12.627		
(3111)	0.000	0.000	0.000	0.000	10.052	
(1211)	0.000	0.000	0.000	0.000	0.000	11.594

3 ANISOTROPY OF SEDIMENTARY BASINS

Bush & Crampin (1987) and Bush (1990) have shown that many features of shear-wave propagation in the Paris Basin can be simulated by propagation through uniform rock with combinations of (P)eriodic (T)hin (L)ayer anisotropy (*PTL anisotropy)* and parallel vertical inclusions (*EDA anisotropy).* PTL anisotropy has hexagonal symmetry (transverse isotropy) with a ***vertical*** axis of cylindrical symmetry, whereas *EDA* anisotropy has hexagonal symmetry with a ***horizontal*** axis of symmetry, parallel to the minimum horizontal compressional stress. This combination of the two hexagonal symmetries with perpendicular symmetry axes yields a rock with orthorhombic symmetry. [Note that an alternative source of transverse isotropy with a vertical axis of symmetry is lithologic anisotropy due to aligned grains, as in shales. Such lithologic anisotropy has similar patterns of velocity variations to PTL, and PTL anisotropy will be used to model the phenomena as structures with given amounts of velocity anisotropy can be conveniently derived from the formula for creating PTL anisotropy (Postma 1955).]

The specific example here, P07C10, is made up of a possible sedimentary rock, P07, possessing typical PTL anisotropy with a differental shear-wave velocity anisotropy of about 7 per cent, containing a distribution of thin parallel vertical EDA cracks, C10, with a differential shear-wave anisotropy of about 10 per cent. {Differential shear-wave anisotropy is defined by $[\max(V_{qS1}) - \min(V_{qS2})]/[\max(V_{qS1}) \times 100]$ (Crampin 1989).} The cracks strike east-west, and the elastic constants of the anisotropic structures used in this paper are given in Table 2. The cracked structure is calculated with the formulations of Hudson (1980, 1981, 1986) and Crampin (1984).

Figure 3 shows the properties of PO7 and Cl0 separately as phase- and group-velocity variations in three mutually orthogonal symmetry planes. Note that although the sections of the group-velocity surface in the plots of velocity variations generally lie with (are slower than) the phase-velocity surfaces, the group velocity in any particular direction is always greater in value than the phase velocity (except in directions of turning points where the two velocities coincide).

Figure 3 also displays the 3-D variations as equal-area projections of the horizontal polarizations of the faster split shear wave, $qS1$, and the contoured delays between the split shear waves. The equal-area projections are polar maps of the behaviour of shear waves over a (lower) hemisphere of directions, so that the centre point represents vertical propagation, and the edge represents horizontal propagation at the appropriate azimuth. The hexagonal symmetries in Fig. 3 contain kiss and line singularities.

The properties of the resulting orthorhombic solid, P07C10, are shown in Fig. 4, and the elastic constants in Table 2. Point singularities in the phase-velocity sheets have been indicated by solid circles. Since there is no single direction in the group-velocity sheets corresponding to the unique direction of a point singularity on the phase-velocity sheets, point singularities in the group-velocity sheets in Fig. 4(c) have been indicated schematically by open circles. [The reason for the irregularities in the contoured projection in Fig. 4(c) is discussed in the next section]. Orthorhombic P07C10 contains 20 point singularities. Eight of the 20 point singularities of P07C10 are in off-symmetry directions as a result of the pulling apart of a line singularity, as illustrated in Fig. 2(b).

Since P-waves in sedimentary basins may have velocity anisotropy from PTL anisotropy of up to 30 per cent (corresponding differential shear-wave anisotropies are likely to be similar), and differential shear-wave anisotropy from EDA anisotropy of up to 10 per cent, a very wide range of combinations of PTL and EDA anisotropy are possible in the field (Wild & Crampin 1991), and no single example can be considered as typical. The particular combination P07C10 was chosen to illustrate behaviour near a point singularity and near a conventional (high-curvature) cusp.

4 GROUP-VELOCITY SHEETS

Various segments of the inner (slower), and outer (faster), shear-wave group-velocity sheets (wave surfaces) of P07C10 are shown in Fig. 5. The locus of lines of group-velocity direction are shown for the corresponding phase-velocity directions tracing out an equally spaced grid of lines of latitude and longitude on the phase-velocity sheets. Note that the lines in the sections Fig. 5 represent the stretching and contraction of the comparatively smooth wave surfaces, and are not perspective drawings. The contractions

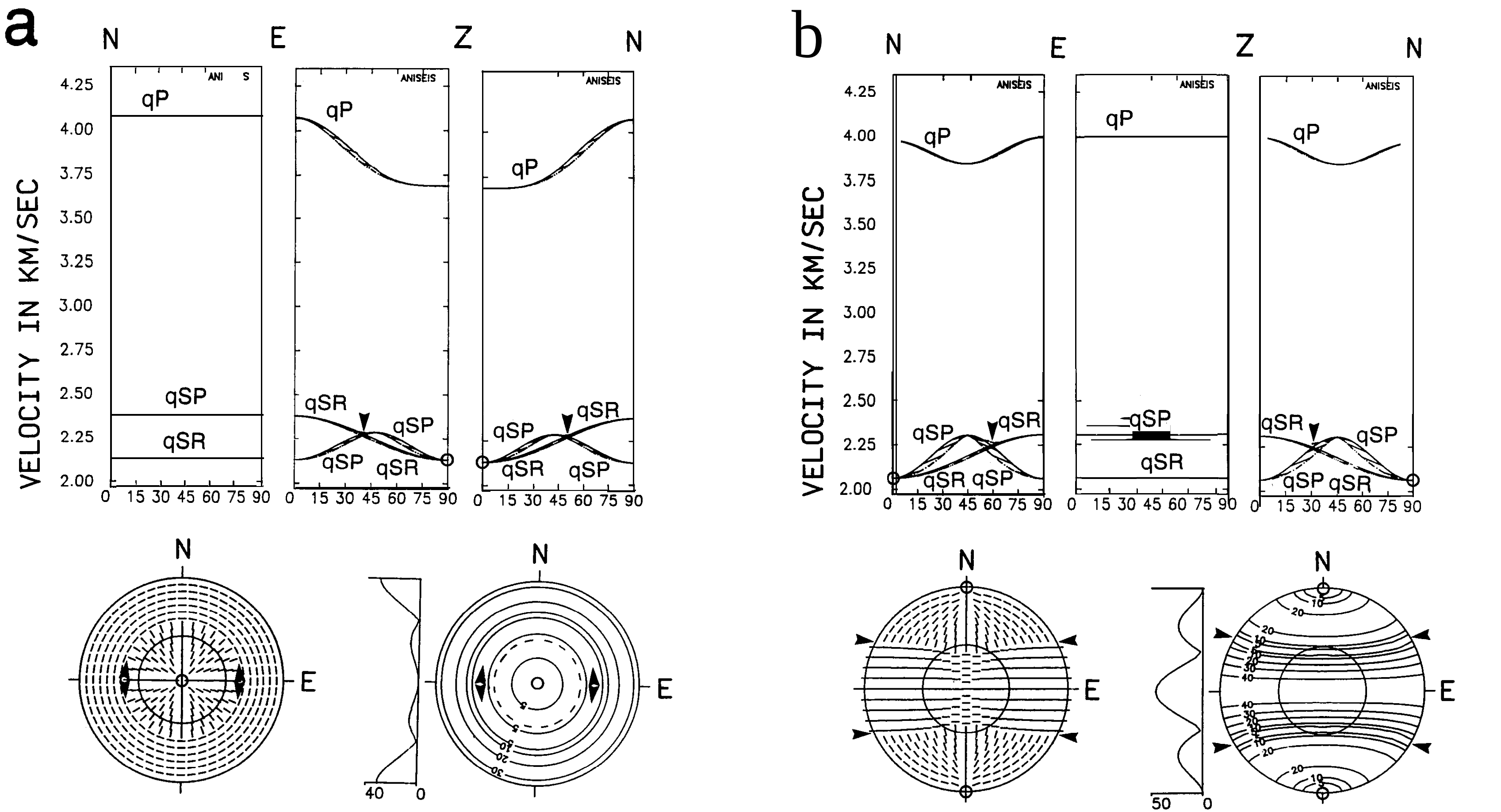


Figure 3. Properties of (a) P07, with PTL anisotropy, and (b) **C10,** with **EDA** anisotropy where the vertical cracks strike east-west. Upper diagrams are velocity variations in the three mutually orthogonal symmetry planes: **(N)orth–(E)ast; (E)ast–(Z)Vertical;** and **(Z)Vertical–(N)orth;** where the upper solid lines are phase velocity, and the lower dot-dash lines are group velocity joined to the appropriate phase velocity every 10”. The three body waves are a quasi-P-wave, ***qP,*** and two quasi-shear waves, ***qSP,*** polarized **(p)arallel,** and ***qSR,*** polarized at **(r)ight** angles, to the symmetry plane through the symmetry axis (direction 0”). Lower diagrams are, to the left, equal-area projections (polar maps) of a hemisphere of directions of the phase-velocity polarizations of the faster split shear wave, ***qS*1,** and to the right, the delays between the split shear waves contoured in ms km-’ . (Note that in symmetry planes, as in the velocity variations the polarizations of ***qS*1** are sometimes ***qSP*** and sometimes ***qSR.*)** The inner circle (dashed in the contour diagrams) is the theoretical shear-wave window for the free surface at an angle of incidence **35.3°,** and is inserted to give some internal scale. There is a north-south section of the delays to the left of the contoured projections. Kiss singularities are marked by open circles, and the directions of line singularities are indicated by arrowheads.

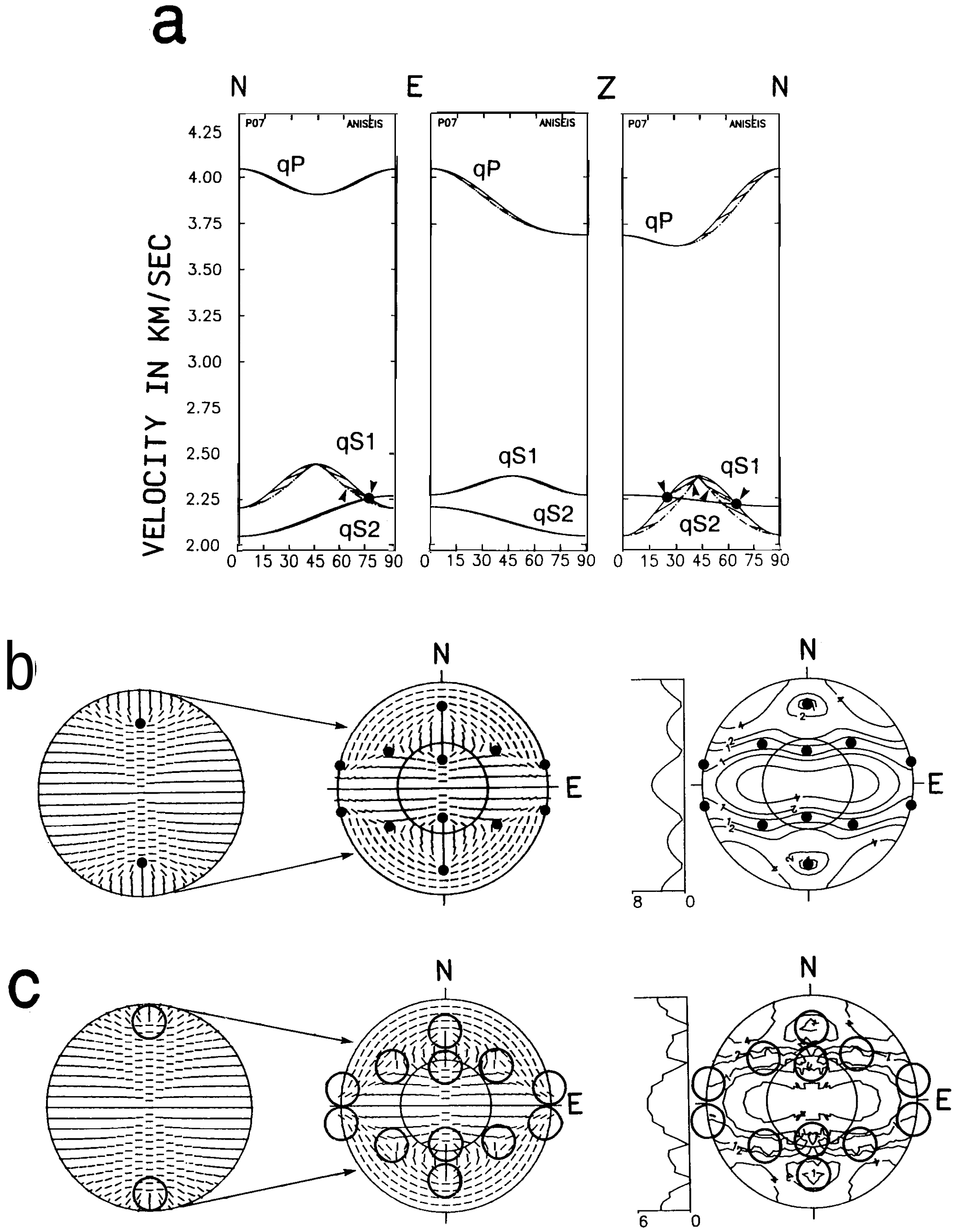


Figure 4. Properties of P07C10 with orthorhombic symmetry: (a) velocity variations in three mutually orthogonal symmetry planes, and equal-area projections of (b) phase-velocity, and (c) group-velocity variations. The irregular contours in (c) are discussed in the text. Notation as in Fig. 3. Solid circles mark the positions of point singularities on the phase-velocity variations in (a) and (b). Open circles mark schematically the centres of the anomalies on the group-velocity variations in (c). There are insets of the shear-wave window to the left of the equal-area projections of the polarizations.

represent concentrations of energy (larger amplitude signals, assuming uniform energy radiation from a point source), and the stretchings represent regions of lower than average energy (smaller amplitude signals).

[Note that hidden-line plotting algorithms might have displayed the details of Fig. 5 more clearly, but the range of hidden-line algorithms available to us could not handle the complexities of the group-velocity wave surfaces for equal phase-velocity grids as calculated here. A satisfactory display of these multilayered group-velocity surfaces of polarized waves has not been devised. Alternative techniques would be to plot the smooth group-velocity surface, or to plot the polarizations, as Dellinger (1991) has done. These would give a more easily visualized picture, but would hide the complexities in Fig. 5. In all cases, such detailed group-velocity surfaces are expensive to compute (Dellinger 1991).]

The directions of point singularities on the phase-velocity domain transform to the generators of cones of solid angles in the group-velocity domain (with approximately elliptical

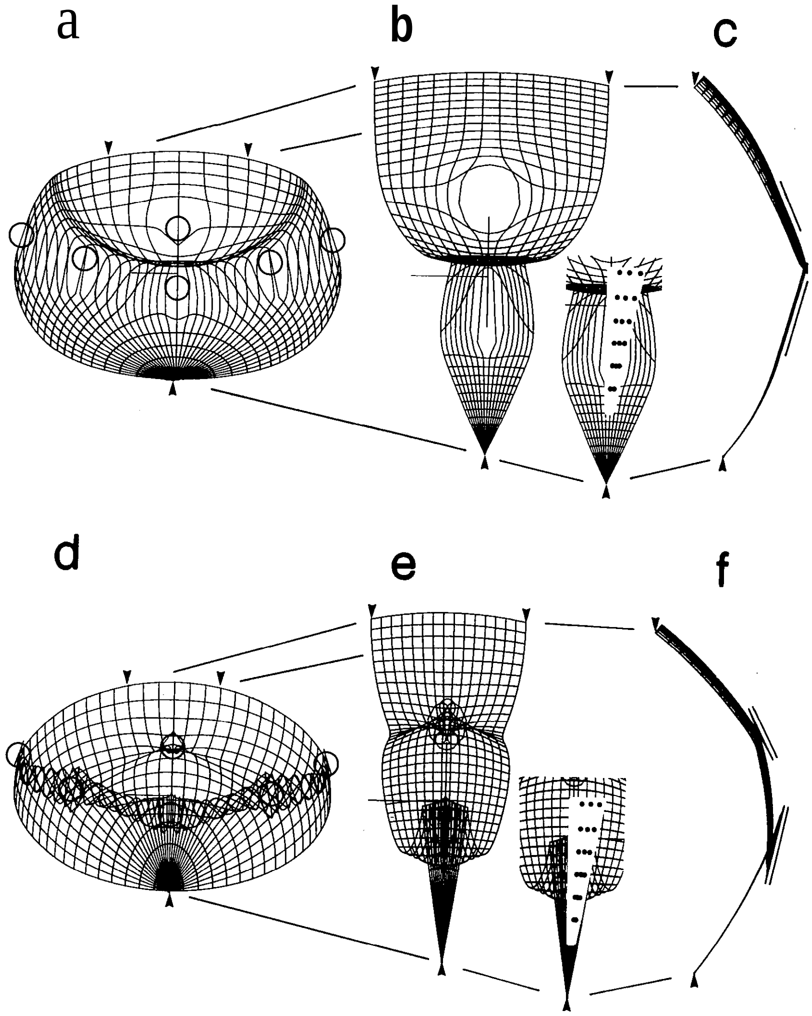


Figure 5. Various projections of the two shear-wave group-velocity sheets (wave surfaces) in **P07C10**. (a), (b), and (c) are of the outer (faster) sheet, $qS1$; (d), (e), and (f) of the inner (slower) sheet, $qS2$. (a) and (d) are half hemispheres with lines every 5" of phase-velocity direction; (b), and (e) are 30" half lunes with lines every 2" of phase-velocity direction; and (c), and (f) are orthogonal views of (b), and (e), respectively. The view point is in the centre with 'T', and the arrowheads mark corresponding points on the different sections. The approximate centres of features associated with point singularities are indicated schematically by open circles in (a), and (d). Lines are drawn parallel to the planar cross-sections of the holes in (c) and the lids in (f). The dots on the inset to (b) and (e) are the directions of the ray paths for the synthetic seismograms in Figs 8 and 9.

cross-sections). The locus of group velocities (ray velocities) in the neighbourhood of a point singularity on the inner group-velocity sheet in Figs 5(d), (e), and (f) is a flat approximately elliptically shaped lid (funnel), with a cuspoidal cross-section, that fits exactly into an opening in the outer sheet in Fig 5(a), (b) and (c). The direction of a point singularity on the phase-velocity sheet transforms to the edge of the lid on the inner group-velocity sheet and the rim of the hole on the outer sheet. The rim of the hole, and the edge of the lid, is approximately planar, as is indicated by the lines parallel to the cross-sections in Figs 5(c), (f), and 6. There are no direct body-wave arrivals corresponding to the planar top surface of the lid, but Burridge (1967) has calculated the contribution of the non-geometrical arrival from the top of the plane lid to shear waves in cubic symmetry (where the lids are circular).

Note that the directions of point singularities on the phase-velocity sheets do not necessarily coincide with the centres of the corresponding features on the group-velocity sheets. This is most clearly seen in the velocity variations in Fig. 4(a), where the point singularities (solid circles) in the phase-velocity sheets are to one side of the hole or lid (delimited by arrowheads) in the group-velocity sheets. Note that the direction of the line singularity in the phase-velocity sheet of hexagonal symmetry also does not coincide exactly with the direction of the apparent intersection of the group-velocity sheets. The concentrations of lines at the base of the lids on the inner sheets are not isolated directions but are tight constrictions ('fasces') through which many directions pass.

The narrow cuspoidal fin on the outer group-velocity sheet in Figs 5(a), (b), and (c) is a conventional cusp caused by the relatively high curvature of the phase-velocity sheet. This cusp is also seen as the sharp peak in the group-velocity variations in the faster quasi- shear wave, $qS1$, in the Vertical/North section of the velocity variations in Fig. 4(a).

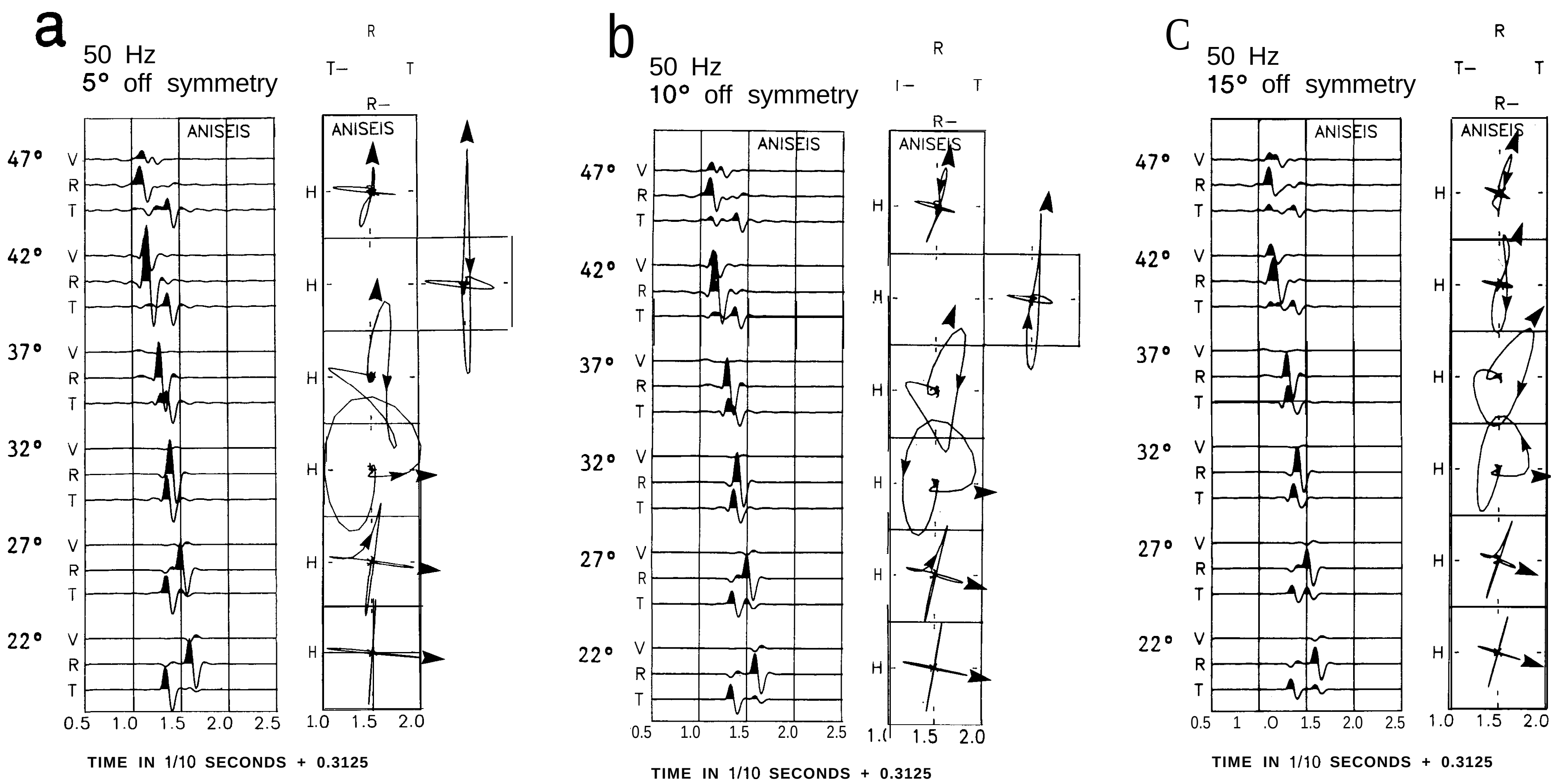
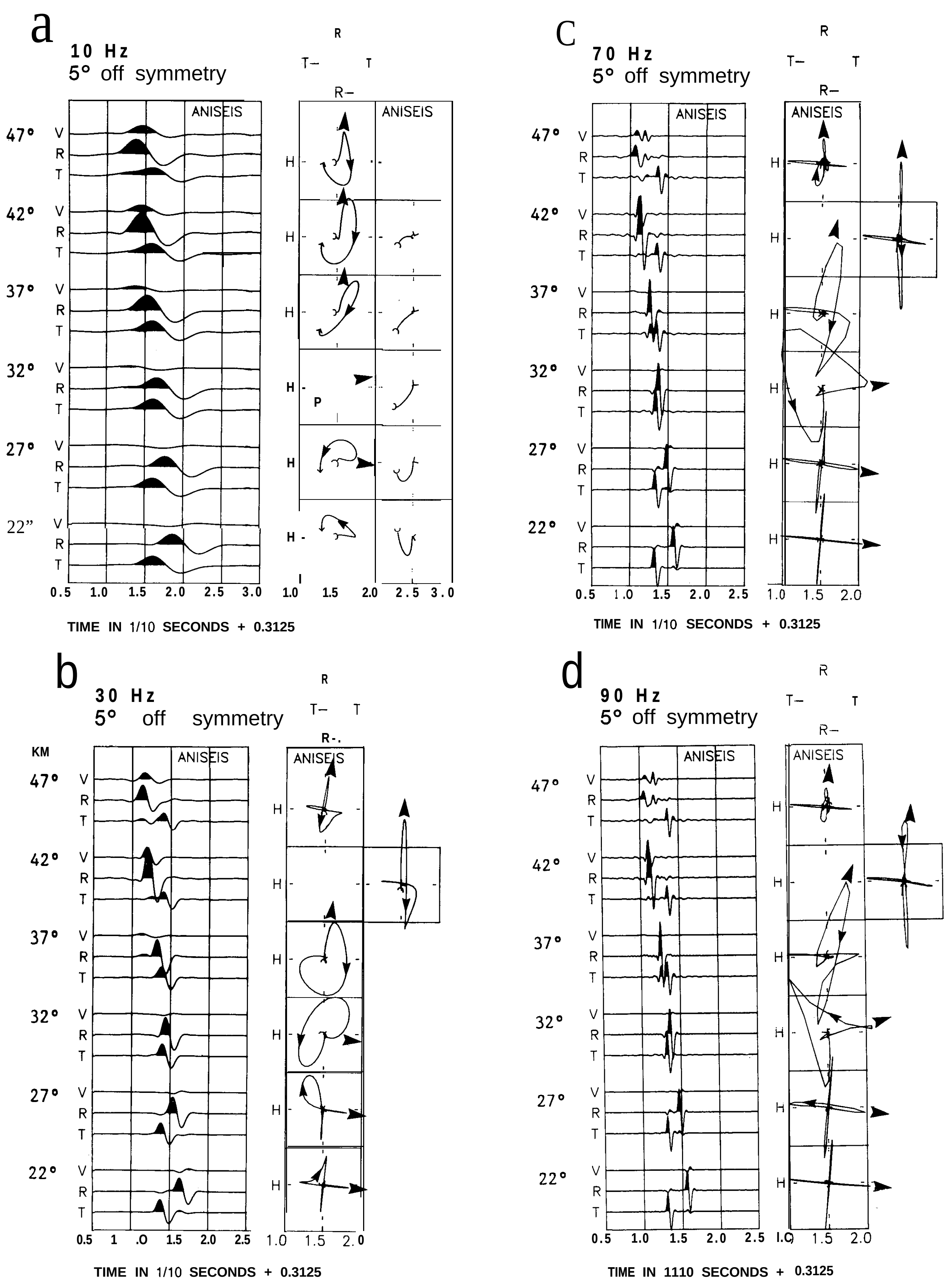


Figure 8. Synthetic seismograms of a 50 Hz shear wave from a horizontal source oriented intermediate between *SH* and **SV** (*SH*45*SV*) recorded by an arc of geophones at a radius of 1 km with the geometry of Figs 5, 6, and 7. Record sections along vertical arcs: (a) 5”; (b) 10”; and (c) 15” from sagittal symmetry, marked with dots in Fig. 5. On the right are three-component seismograms with axes vertical and horizontal radial and transverse relative to the rotated frame in Fig. 7. On the left are polarizations diagrams of the horizontal motion for the specified time interval (where the amplitudes are large, the diagrams have been displaced to the right, to avoid overlap). Heavy arrows mark estimates of the polarization of the initial shear-wave motion, and small arrows mark the direction of particle displacement.



Fire 9. Synthetic seismograms for an *SH45SV* source along the ray paths in the vertical arc 5” from sagittal symmetry [the left line of dots in the insets to Figs 5(b) and (e)], for a source with frequencies: (a) 10 Hz; (b) 30 Hz; (c) 70 Hz; and (d) **90** Hz. Notation as in Fig. 8.

irregularity into the patterns of shear-wave polarizations and delays, as is shown in Fig. 8. The change in initial polarization along paths at 37” and 32” and the considerable delays for ray paths only 5” apart, indicates the rapid variations of patterns in polarization diagrams expected for small changes of the direction of the ray path in the neighbourhood of point singularities. This rapid change of polarization along neighbouring ray paths while still preserving significant delays could well be important for identifying the directions of point singularities in sedimentary basins.

5.5 Effects of frequency in Fig. 9

The behaviour of split shear waves is dependent on the frequency of the signal. Fig. 9 shows record sections for a range of frequencies 10, 30, 70 and 90 Hz, at 5” away from the plane of sagittal symmetry. This is for the same ray paths as Fig. 8(a), and the 50 Hz section in Fig. 8(a) is in the middle of the frequency range in Fig. 9. The delay between the split shear waves along any given ray path is a fixed time interval, and its effect on the seismogram depends on the relative size of the time delay relative to the period of the wave. The higher the frequency, the more complete the separation of the split shear waves and the more cruciform the pattern in the polarization diagram. Similarly, the lower the frequency, the less separation and the more elliptical the pattern of polarization. Such subtleties are very difficult to recognize in seismograms displayed in time series, and are more clearly visible in polarization diagrams.

The polarization diagrams in Fig. 9 show considerable variation with frequency. In general, the delay between the split shear waves becomes a larger fraction of the cycle as the frequency increases, and the corresponding polarization patterns become more cruciform. The directions for the 37” and 32” ray paths bracket the constriction (fasces) at the base of the lid and consequently show most change. The initial polarizations along these ray paths show considerable variation with frequency. This again is the result of interference from signals along neighbouring ray paths which have different polarizations and amplitudes from the specific ray path.

6 DISCUSSION

Synthetic seismograms, showing substantial differences in polarizations and delays in shear-wave splitting, have been calculated for similar ray paths, from similar source orientations, in a uniform anisotropic solid modelling a possible sedimentary basin. The differences are associated with the effects of propagating near the directions of point singularities and are likely to appear even in weak anisotropy. There are also anomalies near (conventional) cusps in the group-velocity sheets, but such cusps will only occur in comparatively strong anisotropy. Different frequencies, different path lengths, different source orientations, different pulse shapes, and different directions of propagation, will all add to the complexity of the behaviour. The effects of the singularities are result of the relationship between the phase and group velocities, where the group velocities are obtained from the phase velocity by a differentiation operator. These relationships may vary widely between different singularities, as can be seen by comparing the different behaviour at the two singularities on the centre line in the group-velocity sheets in Fig. 5. Even the noise free synthetic seismograms in a homogeneous material show disturbed behaviour, and a very wide range of behaviour must be expected near point singularities in sedimentary basins in the field.

Point singularities in the phase-velocity domain are isolated directions, and plane waves propagating at the phase velocity will behave in a regular and predictable manner. Point singularities in the group-velocity sheets become cones of solid angles of disturbed propagation which have approximately elliptical cross-sections, where the first arriving shear wave is expected to be the non-geometrical arrival from the top of the cuspoidal lid on the inner slower group-velocity sheet. Shear waves along a range of ray paths which are within this cone of directions pass from regular splitting for directions outside the cone to disturbed behaviour for directions within the cone. The effective edge of the cone will be diffuse for shear-wave signals with finite wavelengths. It is not possible to positively identify any feature of the seismograms in Figs 8 and 9 which correspond to the top of the lid. The small downward precursor to the main upward motion of the Radial and Transverse components along ray paths at 32” and 37°, which pass through the lid, also appears along the ray path at 42°, which is associated with the conventional cusp, and the precursor is probably due to the interference from neighbouring ray paths.

Within these cones, the delays between the split shear waves radiated from point sources do not vanish, as they do in the direction of point singularities in phase-velocity sheets. One of the most surprising results of this study is the abrupt change in polarization of about 90° for a change in ray path direction of only 5” (between ray paths at 32” and 37” in all record sections in Figs 8 and 9), while still retaining a significant time delay just less than a 0.01 s. Note that polarization diagrams are very sensitive to the differential phase and amplitude of orthogonal traces, so that a time delay of 0.01 s can produce significant changes in polarization diagrams for frequencies as low as 10 Hz (Fig. 9a), at appropriate shear-wave velocities. In such directions, where the polarizations are changing rapidly, the patterns show irregular elliptical or linear motion where the initial take-off directions may be difficult to identify. Similar changes in polarization are also expected to occur for ray paths either side of pinch singularities [that is either side of the cuspoidal fin in slower group-velocity sheet in Fig. 5(d)], but the behaviour in Figs 8 and 9 suggests that the irregularities in the polarization diagrams will be less severe.

Figure 9 shows irregular behaviour within the cone of disturbed propagation for signal frequencies from 10 to 90 Hz, with the higher frequencies showing the more extreme effects. This suggests that shear waves are likely to show anomalous behaviour for a wide range of frequencies for ray paths near singularities in a wide range of possible combinations of PTL and EDA anisotropies.

7 CONCLUSIONS

In most directions of propagation in anisotropic rocks, shear waves split in regular and predictable ways that can be directly interpreted in terms of the anisotropic symmetry. However, all anisotropic symmetries have singular directions, where the two shear-wave phase-velocity sheets touch each other. This paper demonstrates how the amplitudes and polarizations of rays of shear waves propagating at the group velocity may be severely distributed when they propagate close to the commonest type of singularity, the point singularity, even for propagation in homogeneous uniform structures. Nevertheless, it has been shown that the complications caused by the disturbed propagation near singularities can be interpreted in terms of the interactions of rays of shear waves with polarized displacements. It is worth noting that the disturbances to shear waves are independent of the P-wave behaviour. P-waves may propagate uniformly over the range of directions where rays of shear waves are disturbed by singularities.

Perhaps the most significant result of this study, that may help to identify point singularies in the field, is that the polarizations of the leading split shear waves in directions near point singularities may swing through 90" for small changes in ray path direction, while still retaining significant delays between the split shear waves. This was not anticipated, and is a result of the complicated relationship of phase and group velocities near the directions of point singularities.

Bush & Crampin (1987) and Bush (1990) have shown that combinations of EDA and PTL anisotropy can exist in sedimentary basins. Crampin (1988) and Wild & Crampin (1991) have shown that such combinations necessarily imply the presence of point singularities, which may be along nearly vertical ray paths in sedimentary strata, when the ratio of the percentages of EDA to PTL anisotropy is sufficiently small. This study demonstrates that shear-wave ray paths near point singularities may show significant irregularities. If such anomalous behaviour in a possibly uniform homogenous material is not correctly identified, it might be interpreted as implying non-existent inhomogeneities or discontinuities. If the behaviour is correctly identified, the directions of the singularities place contraints on the ratio of the percentages of EDA and PTL anisotropy, which could be a valuable correlation of estimates derived from separate independent observations.

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REFERENCES

Burridge, R., 1967. The singularity on the plane lids of the wave surface of elastic media with cubic symmetry, Q. ***J. Mech. appl. Math.*, *20*,** 41-56.

Bush, I., 1990. Modelling shear-wave anisotropy in the Paris Basin, ***PhD dissertation,*** University of Edinburgh.

Bush, I. & Crampin, S., 1987. Observations of EDA and PTL anisotropy in shear-wave VSPs, ***57th Ann. Znt. SEG Meeting, New Orleans, Expanded Abstracts,*** pp. 646-649.

Crampin, S., 1981. A review of wave motion in anisotropic and cracked elastic-media, ***Wave Motion, 3,*** 343-391.

Crampin, S., 1984. Effective anisotropic elastic constants for wave propagation through cracked solids, ***Geophys. J. R. astr. Soc.,*** 76, 135-145.

Crampin, S., 1987a. Geological and industrial implications of extensive-dilatancy anisotropy, ***Nature, 328,*** 491-496.

Crampin, S., 1987b. Crack porosity and alignment from shear-wave VSPs, in ***Shear-wave Exploration,*** Geophysical Developments, vol. 1, pp. 227-251, SEG Special Publication, eds Danbom, S. H. & Domenico, S. N.,

Crampin, S., 1988. Non-parallel shear-wave polarizations in sedimentary basins, ***58th Ann. Znt. SEG Meeting, Anaheim, Expanded Abstracts, 2,*** 1130-1132.

Crampin, S., 1989. Suggestions for a consistent terminology for seismic anisotropy, ***Geophys. Prosp., 37, 753-770.***

Crampin, S., 1991. Wave propagation through fluid-filled inclusions of various shapes: interpretation of extensive-dilatancy anisotropy, ***Geophys. J. Znt.,*** 104, 611-623.

Crampin, S. & Kirkwood, S. C., 1981. Velocity variations in systems of anisotropic symmetry, ***J. Geophys., 49, 35-42.***

Crampin, S. & Yedlin, M., 1981. Shear-wave singularities of wave propagation in anisotropic media, ***J. Geophys., 49, 43-46.***

Dellinger, J., 1991. Anisotropic seismic wave propagation, ***PhD dissertation,*** Stanford University.

Hudson, J. A., 1980. Overall properties of a cracked solid, ***Math. Proc. Camb. Phil. Soc., 88,*** 371-384.

Hudson, J. A., 1981. Wave speeds and attenuation of elastic waves in material containing cracks, ***Geophys. J. R. astr. Soc., 64,*** 133-150.

Hudson, J. A., 1986. A higher order approximation to the wave propagation constants for a cracked solid, ***Geophys. J. R. astr. Soc.,*** 87, 265-274.

Taylor, D. B., 1990. ***ANZSEZS Manual: Version*** 4.5, Applied Geophysical Software Inc.

Postma, G. W., 1955. Wave propagation in a stratified medium, ***Geophysics, 20, 780-806.***

Wild, P. & Crampin, S., 1991. Effects of azimuthal isotropy and EDA anisotropy in sedimentary basins, ***Geophys. J. int.,*** 107, this issue.

NOTE ADDED IN PROOF

Note that a discussion of Iain Bush's PhD dissertation (Bush 1990) is published in this issue of ***Geophys. J. Int.***

Bush, I. & Crampin, S., 1991. Paris Basin VSPs: case history establishing combinations of matrix- and crack-anisotropy from modelling shear wavefields near point singularities, ***Geophys. J. Znt., 107,*** this issue.