Linear-transform techniques for processing shear-wave anisotropy in four-component seismic data

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ABSTRACT
Most published techniques for analyzing shear-wave splitting tend to be computing intensive, and make assumptions, such as the orthogonality of the two split shear waves, which are not necessarily correct. We present a fast linear-transform technique for analyzing shear-wave splitting in four-component (two sources/two receivers) seismic data, which is flexible and widely applicable.

We transform the four-component data by simple linear transforms so that the complicated shear-wave motion is linearized in a wide variety of circumstances. This allows various attributes to be measured, including the polarizations of faster split shear waves and the time delays between faster and slower split shear waves, as well as allowing the time series of the faster and slower split shear waves to be separated deterministically. In addition, with minimal assumptions, the geophone orientations can be estimated for zero-offset vertical seismic profiles (VSPs), and the polarizations of the slower split shear waves can be measured for offset VSPs. The time series of the split shear waves can be separated before stack for reflection surveys. The technique has been successfully applied to a number of field VSPs and reflection data sets. Applications to a zero-offset VSP, an offset VSP, and a reflection data set will be presented to illustrate the technique.

INTRODUCTION
Identification and quantification of shear-wave splitting in seismic data can provide information about the internal structure of a reservoir including fracture intensity and fracture orientation (Crampin, 1987; Mueller, 1991). In some cases, production can be correlated with the degree of shear-wave splitting (Clet et al., 1990; Davis and Lewis, 1990). However, the success of such applications depends on the success of the processing techniques for determining the faster and slower split shear waves $q_{S1}$ and $q_{S2}$.


We present a new fast technique for analyzing shear-wave splitting in four-component seismic data that we call the linear-transform technique. We assume an acquisition geometry for four-component data of two-horizontal sources and two horizontal receivers, where the orientations of receivers may be different from the source orientations, and the two split shear waves may not be polarized orthogonally. In homogeneous crack-induced anisotropy, caused, say, by a single set of parallel vertical cracks in an isotropic matrix rock, the recorded four-components can be ex-
pressed in terms of the properties of the uncracked matrix and the properties of the faster and slower split shear waves. Thus, in principle, the properties of the matrix and the faster and slower split shear waves can be determined from the recorded components. Alford (1986) and Thomsen (1988) did this by rotation scanning where, if the two split shear waves are orthogonally polarized, the sources and receivers can be synchronously rotated until they are parallel to the polarizations of the split shear waves, thus minimizing the off-diagonal energy in the data matrix.

We transform the data by simple linear transforms so that the complicated shear-wave motion is linearized in a wide variety of circumstances. This allows a number of attributes to be calculated, including the polarizations of fast split shear waves, the time delays between faster and slower split shear waves, and it also allows deterministic separation of the principal time series into the faster and slower split shear waves. In addition, with minimal assumptions, geophone orientations can be estimated in zero-offset vertical seismic profiles (VSPs) if the two split shear waves are orthogonally polarized, and the polarization of the slower split shear waves can be determined in offset VSPs. The time series of split shear waves can be separated before stacking in reflection surveys. The technique has been applied successfully to a number of field VSPs and reflection surveys, and will be illustrated with a zero-offset VSP, an offset VSP, and a reflection survey.

**Figure 1.** Diagrams showing the acquisition geometry and coordinate system in the horizontal plane. (a) Source geometry, where X and Y are two orthogonal sources with signature \( F(t) \), \( e_1 \) and \( e_2 \) are the directions of faster and slower split shear waves received at the geophone position, and \( F_1 \) and \( F_2 \) are two decompositions of the source vector \( F \). (b) Geophone geometry, where \( x \) and \( y \) are two orthogonal geophones possibly in a different orientation from the sources, and \( e_1 \) and \( e_2 \) are the same as (a). (c) Overlay of (a) and (b) for a homogeneous medium where the effective shear-wave polarization does not change in the medium between the source and geophone.
We further assume that the angle between the faster and slower split shear waves is preserved throughout the homogeneous material. Consequently, in overlaying Figures 1a and 1b, we have:

$$\beta' - \alpha' = \beta - \alpha.$$  \hspace{1cm} (1)

**Basic assumptions**

**Anisotropy of the earth.**-We assume anisotropy in the earth is caused by stress-aligned fluid-filled inclusions (EDA-cracks), and the inclusions are uniformly distributed between the source and geophone. If this is satisfied the medium between the source and geophone is referred to as homogeneous. If the polarizations of the shear waves change with depth or angle of incidence, as reported in Winterstein and Meadows (1990) (and interpreted by them as implying changing crack orientations), we call the medium inhomogeneous. In such inhomogeneous anisotropy, it is possible to extrapolate the source downwards, as demonstrated in Winterstein and Meadows (1991).

**Source signature.**-We use $F(t)$ to represent the signature of the source, which we assume is the same for both X- and Y-source orientations. This is an assumption in field acquisition that many of the reported multicomponent data sets appear to satisfy, at least approximately (Alford, 1986; Squires et al., 1989; Winterstein and Meadows, 1991).

**Geophone orientations.**-The geophone orientation may change between different locations, particularly at different levels in VSPs, but for a given location, the orientation is fixed for both X- and Y-sources. Again, this requirement is usually satisfied in field acquisition.

**Polarizations of the split shear waves.**-The polarizations of the split shear waves are fixed for a given raypath direction. This implies that the angles $\alpha'$, $\beta'$, $\alpha$, and $\beta$ are invariant over a time window that covers a specific shear-wave arrival, but may change for different arrivals as different raypaths are involved. These conditions are believed to be generally satisfied in most anisotropic materials.

**Principle of superposition.**-We assume that a source vector $F$ with signature $F(t)$ can be decomposed into two subsources $F_1(t)$ and $F_2(t)$ along $e_1$ and $e_2$ with signatures $F_1(t)$ and $F_2(t)$, respectively, (Figure 1a) and the wavefield excited by source vector $F$ in the medium is equivalent to the wavefield excited simultaneously by subsources $F_1(t)$ and $F_2(t)$. We call this assumption the principle of superposition.

With the above assumptions, where $\alpha'$ and $\beta'$ in Figure 1 are the angles of the faster and slower split shear waves from the X-direction, we have (see Appendix B):

$$F_1(t) = -F(t) \cos \beta'/\sin (\beta' - \alpha');$$

and

$$F_2(t) = F(t) \cos \alpha'/\sin (\beta' - \alpha').$$  \hspace{1cm} (2)

We define the principal time series $qS1(t)$ of the faster split shear wave as the time series received at a receiver when the receiver and a source vector $F$ with signature $F(t)$ are both polarized along $e_1$. Similarly, we define time series $qS2(t)$ of the slower split shear waves as the time series received when the receiver and the source vector $F$ are both polarized along $e_2$. The concept of the principal time series was introduced in Alford (1986) and Thomsen (1988). Here we give an alternative geometric definition.

We introduce two transformed time series $V_1(t)$ and $V_2(t)$ as the sum and difference, respectively, of the principal time series $qS1(t)$ and $qS2(t)$:

$$V_1(t) = qS1(t) + qS2(t),$$

$$V_2(t) = qS1(t) - qS2(t).$$  \hspace{1cm} (3)

**THE LINEAR-TRANSFORM TECHNIQUE**

**Equations of four-component data**

With the acquisition geometry in homogeneous anisotropic medium shown in Figure 1, and the principle of superposition, we can decompose the X-source into subsources as shown Figure 1a. The amplitudes of the faster and slower split shear waves excited by the X-source can be written as (see Appendix B):

$$qS1(t) \sin \beta'/\sin (\beta' - \alpha'),$$

and

$$-qS2(t) \sin \alpha'/\sin (\beta' - \alpha'),$$

respectively, where $qS1(t)$, and $qS2(t)$ are the principal time series of faster and slower split shear waves, respectively; and $\alpha'$ and $\beta'$ are the angles in Figure 1a and 1c.

We define the four-component time series $s(t)$ recorded from X- and Y-sources ($j = 1, 2$) at X- and Y-geophones ($i = 1, 2$). The two geophone components $s_{11}(t)$ and $s_{21}(t)$ from the X-source can be written as:

$$s_{11}(t) = [qS1(t) \sin \beta' \cos \alpha]$$

$$- qS2(t) \sin \alpha' \cos \beta'/\sin (\beta' - \alpha'),$$

and

$$s_{21}(t) = [qS1(t) \sin \beta' \sin \alpha]$$

$$- qS2(t) \sin \alpha' \sin \beta'/\sin (\beta' - \alpha'),$$

respectively, where $qS1(t)$, and $qS2(t)$ are the principal time series of faster and slower split shear waves, respectively; and $\alpha'$ and $\beta'$ are the angles in Figure 1a and 1c.

Similarly, the amplitudes of the faster and slower split shear waves excited by the Y-source are (see Appendix B):

$$-qS1(t) \cos \beta'/\sin (\beta' - \alpha'),$$

and

$$qS2(t) \cos \alpha'/\sin (\beta' - \alpha'),$$

the and two components of $s_{12}(t)$ and $s_{22}(t)$ can be written as:

$$s_{12}(t) = [-qS1(t) \cos \beta' \cos \alpha]$$

$$+ qS2(t) \cos \alpha' \cos \beta'/\sin (\beta' - \alpha'),$$

and...
These are basic relations between the recorded components and the principal time series of split shear waves. The equations could be solved by computer-intensive rotation scanning techniques, as suggested in Alford (1986) and Thomsen (1988). Here, we apply linear transforms to the four-component data sets:

\[
\begin{align*}
\xi(t) &= s_{11}(t) - s_{22}(t), \\
\eta(t) &= s_{21}(t) + s_{12}(t), \\
\zeta(t) &= s_{11}(t) + s_{22}(t), \\
\chi(t) &= s_{12}(t) - s_{21}(t).
\end{align*}
\]

Combining these various equations, we have:

\[
\begin{align*}
\xi(t) &= [qS1(t) \sin (\alpha + \beta')] \\
&\quad - qS2(t) \sin (CY' + \beta])/\sin (\beta' - \alpha'), \\
\eta(t) &= -[qS1(t) \cos (\alpha + \beta')] \\
&\quad - qS2(t) \cos (\alpha' + \beta])/\sin (\beta' - \alpha'), \\
\zeta(t) &= [qS1(t) \sin (\beta' - \alpha) \\
&\quad + qS2(t) \sin (\beta - \alpha')]/\sin (\beta' - \alpha), \\
\chi(t) &= -[qS1(t) \cos (\beta' - \alpha) \\
&\quad - qS2(t) \cos (\beta - \alpha')]/\sin (\beta' - \alpha).
\end{align*}
\]

Equation (13) shows that the time series \(V2(t) = qS1 - qS2\) represents linear motion in coordinate system \((\xi, \eta)\) with angle \(\alpha' + \alpha\) to the axis \(\xi\), and equation (14) shows that the time series \(V1(t) = qS1 + qS2\) represents linear motion in coordinate system \((\xi, \chi)\) with angle \(\alpha' - \alpha\) to the \(\xi\) axis. From the properties of linear motion in Appendix A, we can uniquely determine \(V1\) and \(V2\), and \(\alpha' + \alpha\) and \(\alpha' - \alpha\). Thus, \(qS1 = (V1 + V2)/2\) and \(qS2 = (V1 - V2)/2\), and \(\alpha\) and \(\alpha'\) can be determined, and the orientation of the geophone can be estimated from \(\alpha - \alpha'\).

**Nonorthogonal split shear waves**

In off-vertical incidence, the polarizations of the faster and slower split shear waves in the horizontal plane may not be orthogonal. Such cases include offset-VSPs, and prestack data in reflection surveys, but nonorthogonally polarized shear waves may also be present in zero-offset VSPs if the structure is not horizontal. [Note that there are two principal reasons for the nonorthogonality of shear waves. The polarizations of split shear waves along raypaths in directions without axial symmetry are not usually orthogonal (Crampin, 1981), except in media with transverse isotropy, and in any case, the orientations of nonvertically propagating shear waves will be further distorted in the horizontal plane.]

In almost all cases of nonorthogonality, it is reasonable to assume that the orientation of the geophone is the same as the source. This is usually true in reflection surveys. In offset VSPs, even without data, the orientations of geophones can usually be determined from the polarization of P-waves, so that the horizontal recordings can be digitally rotated so that the axes of source and geophone do effectively coincide. If the orientation of a geophone is the same as a source, that is, \(\alpha = \alpha'\), and \(\beta = \beta'\), equations (12) can be rewritten as:

\[
\begin{align*}
\xi(t) &= [qS1(t) - qS2(t)] \sin (\alpha + \beta)/\sin (\beta - \alpha), \\
\eta(t) &= -[qS1(t) - qS2(t)] \cos (\alpha + \beta)/\sin (\beta - \alpha), \\
\zeta(t) &= qS1(t) + qS2(t), \\
x(t) &= -[qS1(t) - qS2(t)] \cos (\beta - \alpha)/\sin (\beta - \alpha).
\end{align*}
\]

Thus, time series \(V1 = qS1 + qS2 = \xi(t)\) is directly given by equation (16), and \(V2 = qS1 - qS2\) can be determined as follows. We introduce a time series \(U(t)\):

\[
U(t) = [qS1(t) - qS2(t)]/\sin (\beta - \alpha);
\]

so that equations (15), (16), and (17) can be written:
\[ \xi(t) = U(t) \sin(\alpha + \beta), \]
\[ \eta(t) = -U(t) \cos(\alpha + \beta), \]

and
\[ \chi = -U(t) \cos(\beta - \alpha). \]

Similarly, equations (17) and (18) show that U(t) is also linear in coordinate system (\( \xi, -\eta \)). Consequently, \( V_1 = qS1 + qS2 \), \( V_2 = qS1 - qS2 \), and angles \( \alpha - \beta \) and \( \alpha + \beta \) can also be estimated from linear transforms. Thus, the faster split shear wave \( qS1 \) and polarization angle \( \alpha \), and the slower split shear wave, \( qS2 \) and angle \( \beta \), can be directly determined. Angle \( \beta - \alpha \) is the angle between the faster and slower split shear waves and can be used to test the orthogonality of the split shear waves. Thus, we refer to it as the degree of orthogonality.

In summary, instead of solving for \( qS1 \) and \( qS2 \) in zero-offset or wider-offset VSPs with rotation, with linear transforms the time series of the split shear-waves, \( qS1 = (V1 + V2)/2 \) and \( qS2 = (V1 - V2)/2 \), can be separated directly, and their various polarizations estimated, even if the split shear waves are not orthogonal.

**VERIFYING THE LINEAR-TRANSFORM TECHNIQUE**

To verify this technique, we demonstrate the linearity of the transformed time series in field VSPs, and apply the technique to a synthetic VSP, where the polarizations and amplitudes of the split shear waves are known.

**Linearity of \( V_1 \) and \( V_2 \)**

Equations (13) and (14) show that \( V_1 \) and \( V_2 \) are expected to have linear-motion in the transformed coordinate system for zero-offset VSPs. We test this with field data in Figure 2, which shows polarization diagrams (PDs, or hodograms) in the horizontal plane of a zero-offset VSP from British Petroleum’s (BP’s) test site at Devine, Texas (Raikes, 1991). Figure 2a shows PDs of the in-line (X-) source (the particle motion of \( s_{11} \) and \( s_{21} \)), and Figure 2b shows PDs of the cross-line (Y-) source (particle motion of \( s_{12} \) and \( s_{22} \)), at the same geophones. Shear-wave splitting is difficult to identify directly, and their various polarizations estimated, even if the split shear waves are not orthogonal.

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**Fig. 2.** Polarization diagrams (PDs) of a zero-offset VSP from BP’s test site at Devine, Texas, showing the linearity of the transformed motions: (a) in-line source (X-source), \( s_{11}(t) \) and \( s_{21}(t) \); (b) cross-line source (Y-source), \( s_{12}(t) \) and \( s_{22}(t) \); (c) transformed components, \( \xi(t) = s_{11}(t) - s_{21}(t) \) and \( \eta(t) = s_{12}(t) + s_{21}(t) \); and (c) transformed components \( \xi(t) = s_{11}(t) + s_{22}(t) \) and \( \chi(t) = s_{12}(t) - s_{22}(t) \).
because the small delay between the two split shear waves makes the motion elliptical. Figure 2c shows PDs of the transformed motions, $V_2(t)$ (particle motion of $\xi = s_{11} - s_{22}$ and $\eta = s_{12} + s_{31}$); and Figure 2d shows PDs of the transformed motions, $V_1(t)$ (particle motion of $\zeta = s_{11} + s_{22}$ and $\chi = s_{12} - s_{31}$). Almost all of the patterns of the transformed motions in the PDs of Figures 2c and 2d are essentially linear as expected, despite the ellipticity of the original particle motion.

Note that if the anisotropy is small, similar to the case of an isotropic medium, the orientations of PDs (the polarization direction of the main ellipse) of recorded shear-wave components at near vertical incidence will often preserve the orientations of the sources. Thus, the orientations of PDs in (a) and (b) in Figure 2 are expected to be orthogonal for a given geophone level because PDs in (a) and (b) correspond to two orthogonal sources and time delay between the two split shear-waves, hence, anisotropy is small. However, the orientations of transformed PDs in (c) and (d) of Figure 2 are dependent on angles $\alpha' + \alpha$ and $\alpha' - \alpha$, respectively, as shown in equations (13) and (14). If geophone x-direction happens to be close to $e_l$ in Figure 1b, angle $\alpha$ will be small and angles $\alpha' + \alpha$ and $\alpha' - \alpha$ will be close. Thus the orientations of the transformed PDs in this case will be almost parallel as shown in geophone 32, 39, etc. in (c) and (d) of Figure 2. As $\alpha'$ (source X-direction) is fixed and $e_x$ (geophone x-direction) varies with depth, the orientation of PDs of the transformed components in Figure 2c and 2d will vary with depth.

Figure 3 shows similar PDs in the horizontal plane of an offset-VSP, also from Devine, Texas. Figures 3a and 3b show the polarizations of in-line and cross-line (X- and Y-) sources received at horizontal geophones, as in Figure 2. Again the recorded shear motion is elliptical. Figure 3c shows PDs of the transformed motion in the particle motion of $V_2(t)$ in coordinate system $(\xi, \eta)$ from equations (15). Again the motion in Figure 3c is essentially linear. Note that the essentially linear motion of $V_1(t)$ is already determined by equation (16). Also note that in Figure 3, the geophones are rotated so that all the geophones are effectively oriented in the same direction as the sources; thus the orientations of the PDs in Figure 3 do not change with depth.

**FIG. 3.** PDs of an offset VSP [offset 600 ft (190 m)], also from Devine, Texas, showing the linearity of the transformed motions: (a) in-line source $s_{11}(t)$ and $s_{21}(t)$; (b) cross-line source $s_{12}(t)$ and $s_{22}(t)$; and (c) transformed components $\xi(t) = s_{11}(t) - s_{22}(t)$ and $\eta(t) = s_{12}(t) + s_{21}(t).$
In summary, although the particle motion of shear waves as recorded are frequently elliptical, the linear transforms of the sum and difference of the split shear waves V1 and V2, respectively, are linear. Consequently, the time series of the split shear waves can be directly separated by recombining V1 and V2.

**Testing with synthetic data**

We test the technique for measuring the polarization, time-delay, geophone orientation, and degree of orthogonality with synthetic data. The data are modeled on VSP Model 1 of the Anisotropy Modeling Collaboration (Thomsen et al., 1989) by the Edinburgh Anisotropy Project. The VSP has a 500 m offset, to a maximum depth of 2500 m. For the purposes of this analysis, it is treated as a zero-offset VSP. That is, the polarizations of the two split shear waves in the horizontal plane are assumed to be orthogonal.

Figure 4a shows the comparison of measured and expected time delay between the split shear waves; Figure 4b shows the comparison of measured and expected polarizations (where the top layer of the model is isotropic); Figure 4c shows the measured and expected degree of orthogonality; and Figure 4d shows the comparison of the measured geophone orientation with the known orientation, which has been deliberately modified for the purposes of the test. The match of measured to expected results for time delays (Figure 4a) is good, except for the top two geophones that are too close to the source and are noisy. The match for polarizations (Figure 4b) is good for all geophones. The match for orthogonalities is good, except for geophones 5 to 7 which are located at, or close to the isotropic/anisotropic interface where the time delay starts to build up as shown in Figure 4a, and where some interferences of split shear waves exist. The match for geophone orientations is good for all geophones (Figure 4d), but for the top 15 geophones there are 10 to 15 degree differences between the measured and expected results, which is possible a result of the small time delays between the two split shear waves for the top 15 geophones as shown in Figure 4a, and the likely existence of some interferences of split shear waves. Despite this, the overall match is good.

**APPLICATIONS TO FIELD DATA**

We now apply the technique to field data sets: (1) zero-offset and wide offset VSPs from BP’s test site at Devine, Texas; and (2) a shot gather from the Kim-Tech Lost Hills reflection line.

**Zero-offset VSP**

The technique has been applied with satisfactory results to a number of zero-offset VSPs without gyro data where the geophone orientations are unknown. We choose a VSP from Devine as an example.

Figure 5a shows the four-component seismic data matrix from a Devine VSP, and Figure 5b shows the faster and slower split shear waves separated by the linear transform technique. Note that in displaying Figure 5a, the gain applied to components s11 and s12 is four times larger than the gain applied to components s21 and s12 to show the variation of waveforms in component s21 and s12 clearly. A comparison of Figures 5a and 5b shows that the overall signal-to-noise ratio is slightly improved by the separation, with the direct shear arrivals being more consistent between geophone levels. Detailed comparison of the separated split shear-waves shows almost identical waveforms with a very small time delay.

Figure 5c shows the estimated time delays. The trend of variation of the time delay agrees with the analysis based on PDs by Yardley (1992). Figure 5d shows the polarization of the leading split shear wave, which is consistent between geophones with a gradually linear change of polarization angle with depth. This linear change of polarization angle with depth may represent some kind of crack orientation changing with depth, but may also represent the differential attenuations of faster and slower split shear waves. The slower split shear wave tends to be attenuated more than the fast split shear wave (Thomsen, 1988; Mueller, 1991). As depth increases, such attenuation of slower split shear waves will increase, resulting in a gradual change of polarization angle with depth. There are also other factors, such as irregularities in field acquisition, that may cause polarization change. It is possible to separate these factors by full-wave forward modeling (Yardley, 1992).

Figure 5e shows geophone orientations (x-direction measured from the X-source direction) determined by the linear transform technique (solid line) compared with those estimated from PDs (dotted line). There are 10 to 20 degree differences between the two kinds of measurements, for which there are two possible explanations. On one hand, when the time delay is small, measurements of geophone orientation by the linear transform technique may yield 10 to 15 degree differences, as shown in the synthetic results of Figure 4d. On the other hand, a visual examination of polarization diagrams may easily introduce an error of 5 to 10 degrees. However, the trend of the variation of the two kinds of measurements matches very well, and the results are generally acceptable.

**Offset VSP**

The technique has been applied to several offset VSPs from Devine BP and elsewhere, where the geophone orientation is determined from the polarizations of P-waves. We present one example from Devine. Figure 6a shows the seismic data matrix, and Figure 6b shows the separated split shear waves. The same gains as in Figure 5a and 5b were used to display Figure 6a and 6b. Again the signal-to-noise ratio is improved, and the two split shear waves are consistent in the separated data. Figure 6c compares the delay between the split shear waves by assuming orthogonal measurements in equations (13) and (14), and the delay from allowing nonorthogonal shear wave polarizations in equations (15) and (16). The time delays from assuming orthogonal and nonorthogonal polarizations are almost identical, presumably because the polarizations are nearly orthogonal. As in the zero-offset data in Figure 5, the delays increase gradually with depth. Figure 6d shows the polarizations of the shear waves and the estimated degree of orthogonality. The estimated polarizations are similar to those of the zero offset VSP (Figure 5d). The two split shear waves are about 90 degrees, as expected from the comparison of the delays in Figure 6c.
Reflection survey

Again the technique has been applied to several reflection data sets, where the geophone orientations are the same as the source orientations. We choose a shot gather from the Kim-Tech Lost Hills reflection line as an illustration.

Figure 7a shows the seismic data matrix, and Figure 7b shows the separated split shear waves. The continuity of events is improved in the separated data, and a comparison of the faster and slower section shows a clear separation at the major reflection arrivals. A constant gain was applied to display Figure 7a and 7b.

Figure 7c shows a color display of polarizations, using the techniques of Li and Crampin (1990, 1991a, 1991b). First, the variation of color represents the variation of polarization. Two major color events, blue and red-orange, can be identified in Figure 7c, which represent the polarizations of the

Fig. 4. Test with synthetic AMC VSP data showing comparison of measured attributes (solid line) with expected values (dotted line): (a) time delay between the faster and slower split shear-waves; (b) polarization of the faster split shear-wave measured from the source X-direction; (c) degree of orthogonality, angle between the faster and slower split shear waves; and (d) geophone orientation (x-direction) measured from X-source direction where the expected values have been disturbed.
Fig. 5. Results from the zero-offset VSP from Devine in Figure 2. (a) Four-component seismic data matrix showing four record sections $s_{11}(t), s_{31}(t), s_{12}(t),$ and $s_{22}(t);$ note that the gain applied to $s_{31}$ and $s_{12}$ is four times larger than the gain applied to $s_{11}$ and $s_{22};$ (b) record section of faster and slower split shear waves $qS_1(t)$ and $qS_2(t)$ computed from data matrix (a); a same constant gain as used for components $s_{11}, s_{31},$ and $s_{22}$ was applied to components $qS_1$ and $qS_2;$ (c) measured time delay against depth; (d) measured polarization direction of faster split shear waves; and (e) comparison of geophone orientations (x-direction) estimated by the linear transform technique (solid line) and by visual examination of polarization diagrams, PDs (dotted line), where both attributes are measured from the source X-direction.
FIG. 5. (continued)
Fig. 6. Results from the offset VSP from Devine in Figure 3. The offset is 600 ft (190 m). (a) and (b) have same notations as in Figures 5a, and 5b; the gain is also the same as in Figure 5a and 5b; (c) shows a comparison of the time delay estimated using the orthogonal algorithm [equations (13) and (14)] (solid line); and using the nonorthogonal algorithm [equations (15) and (16)] (dotted line); and (d) shows the estimated degree of orthogonality, or angle between the two split shear-waves (dotted line), and polarization angles of the faster split shear waves (solid line), where both attributes are measured from the source X-direction.
Fig. 7. Results from Lost Hills reflection data. (a) Four-component data matrix of a shot record from the Lost Hills data set showing four record sections, $s_{11}(t)$, $s_{21}(t)$, $s_{12}(t)$, and $s_{22}(t)$; a constant gain was applied to all four components; (b) computed amplitude section of faster and slower shear waves, $qS_1(t)$ and $qS_2(t)$; a same constant gain as (a) was used here; and (c) color plot of polarization variations of split shear waves.
DISCUSSION AND CONCLUSIONS

The above examples show that the polarizations and amplitudes of the faster and slower split shear waves in VSPs and reflection surveys can be estimated by linear-transform techniques. We have only discussed the situation of homogeneous anisotropic media, that is, where the polarizations of the split shear waves do not vary with depth. The technique can be extended to media where the effective polarizations change with depth. In this case we can either extrapolate the sources downward into the lower layers, so that the medium between the extrapolated sources and the geophones is homogeneous, as was demonstrated by Winterstein and Meadows (1991), or we can spectrally transform the lower by the upper geophone displacement to remove the effects of the upper layer, as demonstrated by Zeng and MacBeth (1992). Linear transforms are then performed in the frequency domain.

This technique recognizes the linearity of the transformed shear-wave motion in the transformed coordinate system. These (four) linear transforms are deterministic and can be efficiently implemented. We suggest that two features of the linear-transform technique should be noted: (1) the speed and directness of the calculations; and (2) the remarkable consistency of the measurements of the attributes in the field data sets, as shown in Figures 5c, 5d, 5e, 6c, and 6d. Conventional methods employ a rotation scanning procedure (Alford, 1986) and tend to be computing intensive.

We conclude from our studies that the linear-transform technique separates the faster and slower split shear waves efficiently, and measures the parameters of shear-wave splitting with satisfactory accuracy in four-component seismic data. It is flexible and can be used in a variety of ways to treat both orthogonally and nonorthogonally polarized split shear waves. The technique estimates the orientation of downhole geophones in zero-offset VSPs where the split shear waves can be assumed, and measures the degree of orthogonality of the two split shear waves in offset VSPs where the orientation of the geophones can be estimated by other techniques. In reflection surveys, the faster and slower split shear wave can be separated deterministically before stack, which simplifies the processing procedure of multicomponent reflection data in the presence of anisotropy (Li, 1992).

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REFERENCES


APPENDIX A

ANALYSIS OF LINEAR MOTION

A shear wave is said to be linearly polarized if the trajectory of the particle motion is a straight line. Such linear motion has simple mathematical properties which make it easy to measure polarization attributes.

Figure A-1 shows a linear motion in the horizontal plane. The arrow marks the initial direction of the particle motion; $A(t)$ is the displacement at time $t$, $x(t)$ and $y(t)$ are the projections of $A(t)$ on the two horizontal axes, and $\phi$ is the angle that the initial motion makes with the x-axis. We define $A(t)$ as positive when particle displacement is in the $\alpha$ direction, and negative when particle displacement is reversed; $\alpha$ is defined over $(-\pi, \pi)$, positive when it is measured clockwise from the x-direction, and negative when it is measured counter-clockwise. We have:

$$x(t) = A(t) \cos \alpha,$$

and

$$y(t) = A(t) \sin \alpha,$$

(A1)

or

$$\tan \alpha = \frac{y(t)}{x(t)},$$

(A2)

where we assume $\alpha$ is time invariant. Note that $A(t)$ can be positive, or negative. This is different from the conventional usage where $A(t)$ is usually positive and $\alpha$ is a function of time. Also note that if $\alpha$ is a function of time and $A(t)$ is constant, equation (A-1) will represent a circular motion.

Mathematical properties

Some useful properties can be immediately derived from equation (A-1) and Figure A-1. [Note that if a shear-wave motion has a displacement $A(t)$, we will refer to this as "motion $A(t)$".]

Zero crossings.-If motion $A(t)$ is a linear motion, $x(t)$, $y(t)$, and $A(t)$ have zero crossings at the same time.

Sign functions and polarities.-If motion $A(t)$ is a linear motion, sign functions $s_x$, $s_y$, $s_A$:

$$s_x = x(t)/|x(t)|, \quad \text{if } x(t) \neq 0.0,$$

$$s_y = y(t)/|y(t)|, \quad \text{if } y(t) \neq 0.0,$$

(A-3)

and

$$s_A = A(t)/|A(t)|, \quad \text{if } A(t) \neq 0.0,$$

have the following relations:
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$$s_A = s_x, \quad \text{if} \quad \pi/2 > \alpha > -\pi/2,$$

$$= -s_x, \quad \text{if} \quad -\pi/2 > \alpha \geq -\pi, \text{or} \quad \pi \geq \alpha > \pi/2,$$  (A-4)

and

$$s_A = s_y, \quad \text{if} \quad \pi > \alpha > 0,$$

$$= -s_y, \quad \text{if} \quad 0 > \alpha \geq -\pi.$$  (A-5)

In other words, time series $A(t)$ either has the same, or reverse polarity as $x(t)$ or $y(t)$. If we allow a polarity difference, $s_A$ can be uniquely determined from $x(t)$ or $y(t)$ without knowing angle $\alpha$. Note that equations (A-4) and (A-5) are valid for all time samples of a linear motion.

Eigenvalues and eigenvectors-

Form covariance matrix $\mathbf{C}$

$$\mathbf{C} = \begin{bmatrix}
\text{Var}[x] & \text{Cov}[x, y] \\
\text{Cov}[x, y] & \text{Var}[y]
\end{bmatrix},$$  (A-6)

where $\text{Var}[x]$ and $\text{Var}[y]$ are, respectively, the variances of $x(t)$ and $y(t)$, and $\text{Cov}[x, y]$ is the covariances of $x(t)$ and $y(t)$. [For definition of variances and covariances, see equation 19.3-2 in Kanasewich (1981)].

If motion $A(t)$ is a linear motion, the smaller eigenvalue of matrix $\mathbf{C}$ is zero (Kanasewich, 1981). Let $\lambda$ be the nonzero, or larger eigenvalue of $\mathbf{C}$, and $\mathbf{N} = (x_0, y_0)^T$ be the corresponding normalized eigenvector, where superscript $T$ represents the transpose. Then, we have:

$$\tan \alpha = y_0/x_0.$$  (A-7)

Estimating angle $\alpha$

For VSP data, the shear-wave splitting process often involves the direct shear arrival. From equation (A-2), a quick estimation of $\alpha$ can be made as the average angle over the time window of direct arrival, or the angle corresponding to the maximum value of $A(t)$ over the specific time window.

But a more accurate and robust estimation of angle $\alpha$ is given by equation (A-7) by estimating eigenvalues and eigenvectors of the covariance matrix $\mathbf{C}$. Because only a second-order matrix is involved, the computing is still very fast.

For reflection data, processing shear-wave splitting usually involves different shear-wave arrivals (events). Because it is often difficult to define time windows of all shear arrivals accurately, a different approach is suggested here. First, angle $\alpha$ is calculated by sample by sample (instantaneously) using equation (A-2) over the entire trace, then the instantaneous values are displayed in color as Figure 7C. From color sections, we not only can estimate angle $\alpha$, but we can also identify shear-wave events as demonstrated by Figure 7C. Thus with an interactive interpretation tool (such as an interactive workstation), a good initial estimation of angle $\alpha$ can be made, and time windows of all shear-wave arrivals can also be defined. Thus more detailed estimations can be achieved using equation (A-7).

Determining time series $A(t)$

It is possible to find a time invariant $\alpha$ by applying equations (A-6) and (A-7) over either the whole trace or a specific time window. Then $A(t)$ is determined from:

$$A(t) = x(t) \cos \alpha + y(t) \sin \alpha.$$  (A-8)

But we can use the mathematical properties of a linear motion to simplify the determination of $A(t)$ by allowing a polarity difference. Clearly the absolute values of $A(t)$ can be estimated as $(x^2 + y^2)^{1/2}$ from equation (A-1), and the only problem is to estimate the polarity of $A(t)$, or the sign function $s_A(t)$. From equations (A-4) and (A-5), there are only two solutions of $s_A(t)$ with a polarity difference. If we ignore the polarity difference, say, choose a positive solution from (A-4), we have

$$A(t) = s_x [x^2(t) + y^2(t)]^{1/2}, \quad \text{if} \quad x \neq 0.0,$$

and

$$A(t) = 0.0, \quad \text{if} \quad x = 0.0.$$  (A-9)

[Note that, if one wishes, the polarity can be determined from angle $\alpha$ by applying equation (A-6) and (A-7) over a whole trace .]

In this way $A(t)$ can be determined sample by sample without knowing $\alpha$. This involves only simple arithmetic, which can be easily implemented for processing trace-ordered seismic data. Note that the results of equation (A-9) may have a polarity difference from the exact solution. But if the polarities of all traces of $x(t)$ and $y(t)$ are consistent, the polarity differences are also consistent, or systematic for all traces. Thus, this polarity difference will not cause severe problems in further processing of the time series $A(t)$. The measurement of polarizations, such as angle $\alpha$, can often be allowed a 180 degree difference.

In most cases we can obtain satisfactory results using equation (A-9), as shown in Figure 5b, 6b, and 7b, which are all determined in this way. Because of its speed and directness, this approach is particularly useful for determining the principal time series of prestacked data for processing multicomponent reflection data in the presence of anisotropy, as demonstrated by Figure 7b.

Fig. A-1. Geometry of a linear motion.

Derivation of equation (2)

As shown in Figure B-la, a source vector \( \mathbf{F} \) with signature \( F(t) \) is decomposed into two subsources \( F_1 \) and \( F_2 \) with signatures \( F_1(t) \) and \( F_2(t) \), respectively. Given \( F(t) \) and angles \( \gamma_1 \) and \( \gamma_2 \), \( F_1(t) \) and \( F_2(t) \) can be uniquely determined. In the upper triangle of Figure B-la, according to the sine rule of the triangle geometry, and noting \( \gamma = 180^\circ - (\gamma_1 + \gamma_2) \), we have

\[
F(t)/\sin \left[ 180^\circ - (\gamma_1 + \gamma_2) \right] = F_1(t)/\sin \gamma_2,
\]

or

\[
F(t)/\sin (\gamma_1 + \gamma_2) = F_1(t)/\sin \gamma_2.
\]

Similarly, in the lower triangle of Figure B-lb, we have

\[
F(t)/\sin (\gamma_1 + \gamma_2) = F_2(t)/\sin \gamma_1.
\]

Solving equations (B-1) and (B-2) gives

\[
F_1(t) = F(t) \sin \gamma_2/\sin (\gamma_1 + \gamma_2),
\]

and

\[
F_2(t) = F(t) \sin (\gamma_1 + \gamma_2)/\sin (\gamma_1 + \gamma_2).
\]

If source \( \mathbf{F} \) is decomposed in terms of angle \( \alpha' \) and \( \beta' \) as shown in Figure 1a, angles \( \gamma_1 \) and \( \gamma_2 \) in Figure B-la can be expressed as

\[
\gamma_1 = 90^\circ - \alpha',
\]

\[
\gamma_2 = \beta' - 90^\circ,
\]

and

\[
\gamma_1 + \gamma_2 = \beta' - \alpha'.
\]

Substituting equations (B-4) into (B-3) and making some suitable manipulations gives equation (2).

Derivation of equation (4)

Before we can derive equation (4), we first introduce expressions for \( qS_1(t) \) and \( qS_2(t) \), and discuss the decomposition of the in-line \( X \)-source.

Expressions of \( qS_1(t) \) and \( qS_2(t) \).—By definition, \( qS_1(t) \) and \( qS_2(t) \) are the amplitudes of the fast and slower split shear waves, respectively, at a geophone position when source \( \mathbf{F} \) is polarized along \( e_1 \) and \( e_2 \) directions, respectively. Thus following Thomsen (1988), \( qS_1(t) \) and \( qS_2(t) \) can be written as

\[
qS_1(t) = p_1(t)^*F(t),
\]

and

\[
qS_2(t) = p_2(t)^*F(t),
\]

where \( p_1(t) \) and \( p_2(t) \) are the response of the medium to faster and slower split shear waves, respectively, including geometric spreading, attenuation, reflectivity, etc. Equation (B-5) follows the convolution model of a seismic trace (Yilmaz, 1987).

Inline source decomposition.—If source \( \mathbf{F} \) is polarized along the in-line \( X \)-direction, \( \mathbf{F} \) can be decomposed as shown in Figure B-lb. Similar to the derivation of equation (2), the two subsources in Figure B-lb can be written as

\[
F_1(t) = F(t) \sin \beta'/\sin (\beta' - \alpha'),
\]

and

\[
F_2(t) = -F(t) \sin \alpha'/\sin (\beta' - \alpha').
\]

respectively, where the minus sign in \( F_2(t) \) represents the fact that \( F_2 \) in Figure B-lb is polarized along the opposite direction of \( e_2 \).

Derivation of equation (4).—If we decompose the \( X \)-source as shown in Figure B-lb, according to the principle of superposition the amplitudes of the fast and slower split shear waves excited by the \( Y \)-source are equivalent to the amplitudes of fast and slower split shear waves excited by subsources \( F_1 \) and \( F_2 \). Note that in a homogeneous medium as shown Figure 1, a source polarized along the \( e_1 \) direction will only excite fast split shear waves, and a source polarization along the \( e_2 \) direction will only excite slower split shear waves (Crampin, 1981; Thomsen 1988). Thus the amplitudes of the fast and slower split shear waves excited by subsources \( F_1 \) and \( F_2 \) can be written as

\[
p_1(t)^*F_1(t) = [p_1(t)^*F(t)] \sin \beta'/\sin (\beta' - \alpha'),
\]

and

\[
p_2(t)^*F_2(t) = -[p_2(t)^*F(t)] \sin \alpha'/\sin (\beta' - \alpha'),
\]

respectively. Substituting equation (B-5) into (B-7) gives equation (4).

Derivation of equation (6)

Equation (6) can be derived similarly. Figure 1a shows the decomposition of the \( Y \)-source with signature \( F(t) \), and equation (2) shows the two subsources. Thus, the amplitudes of the faster and slower split shear waves excited by the \( Y \)-source can be written as

\[
p_1(t)^*F_1(t) = -[p_1(t)^*F(t)] \cos \beta'/\sin (\beta' + \alpha'),
\]

and

\[
p_2(t)^*F_2(t) = [p_2(t)^*F(t)] \cos \alpha'/\sin (\beta' + \alpha'),
\]

respectively. Substituting equation (B-5) into (B-8) gives equation (6).