Interpreting data matrix asymmetry in near-offset, shear-wave VSP data

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ABSTRACT

Poor experimental control in shear-wave VSPs may contribute to unreliable estimates of shear-wave splitting and possible misinterpretation of the medium anisotropy. To avoid this, the acquisition and processing of multicomponent shear-wave data needs special care and attention. Measurement of asymmetry in the recorded data matrix using singular-value decomposition (SVD) provides a useful way of examining possible acquisition inaccuracies and may help guide data conditioning and interpretation to ensure more reliable estimates of shear-wave polarization azimuth.

Three examples demonstrate how variations in shear-wave polarization and acquisition inaccuracies affect the SVD results in different ways. In the first example, analysis of synthetic seismograms with known depth changes in the polarization azimuth show how these may be detected. In the second example, a known source re-orientation and polarity reversal is detected by applying SVD to near-offset, shear-wave VSP data, recorded in the Romashkino field, Tatar Republic. Additional information on a polarization change in the overburden is also obtained by comparing the SVD results with those for full-wave synthetic seismograms. The polarization azimuth changes from N160°E in the overburden to N117°E within the VSP depth range. Most of the shear-wave splitting is built up over the VSP depth range. The final example is a near-offset, shear-wave VSP data set from Lost Hills, California. Here, most of the shear-wave splitting is in the shallow layers before the VSP depth range. SVD revealed a known correction for horizontal reorientation of the sources, but also exhibited results with a distinct oscillatory behavior. Stripping the overburden effects reduces but does not eliminate these oscillations. There appears to be a polarization change from N45°E in the overburden to N125°E in the VSP section.

The details in these examples would be difficult to detect by visual inspection of the seismograms or polarization diagrams. Results from these preliminary analyses are encouraging and suggest that it may be possible to routinely use this, or a similar technique, to resolve changes in the subsurface anisotropy from multicomponent experiments where acquisition has not been carefully controlled.

INTRODUCTION

Shear-wave splitting in multicomponent seismic experiments is diagnostic of propagation in an effectively anisotropic structure, and is frequently analyzed in terms of the internal crack- and stress-geometry of the subsurface rocks (Crampin and Lovell, 1991). These data, if interpreted correctly, can provide valuable information that can be combined with geological and other geophysical data to enhance reservoir description. In recent years, many techniques have evolved to estimate and interpret this splitting so that its potential can be evaluated. However, to achieve the necessary accuracy for such estimates, more care must be taken during acquisition than with standard single-component (P-wave) data to minimize errors arising from incorrect scaling and other anomalies between the different components. This requirement becomes more important when attempting to detect changes in the shear-wave splitting symptomatic of changes in local crack or stress conditions or
singularities (Crampin, 1991), as the inaccuracies may also lead to misinterpretations. This paper presents a technique applicable to near-offset vertical seismic profiling (VSP) experiments, which may provide a way of examining and accounting for the effect of acquisition error, even in the presence of changes in the anisotropic behavior, to obtain reliable estimates of polarization azimuth.

The mathematics for the analysis procedure is developed by representing the recorded multicomponent data as a matrix of traces (Tatham and McCormack, 1991). This is particularly convenient for our present work, as it facilitates the application of matrix algebra and the vector convolutional model for anisotropic shear-wave propagation (Queen and Rizer, 1990). The most common configuration is the four-component matrix, which is an arrangement of recordings from two horizontal geophone components generated by two horizontal, orthogonal, shear-wave source orientations (Alford, 1986) (or nine-component seismograms if vertical source and vertical geophone are included). These four horizontal traces are then arranged into a 2 x 2 data matrix, $\mathbf{D}(t)$:

$$\mathbf{D}(t) = \begin{bmatrix} d_{XX}(t) & d_{XY}(t) \\ d_{XY}(t) & d_{YY}(t) \end{bmatrix},$$

with the top row $d_{XX}(t)$ and $d_{XY}(t)$ corresponding to the in-line (X) geophone traces from the in-line (X) and crossline (Y) sources. The bottom row, $d_{XY}(t)$ and $d_{YY}(t)$, corresponds to the crossline (Y) traces from the in-line (X) and crossline (Y) sources. For a noise-free, four-component VSP experiment with ideal downhole geophones and ideal sources, in a plane-layered isotropic medium with source motions and geophone components aligned, there should be no energy in the off-diagonal components $d_{ij}(t)$ and $dy(t)$ of this data matrix. When anisotropy is present, there is off-diagonal energy in the data matrix whenever the source and geophone components are not parallel to the shear-wave polarization directions in the anisotropic medium (Thomsen, 1988). For uniform anisotropy the data matrix is symmetric, and the off-diagonal components are equal. The presence of off-diagonal energy in the data matrix is not a sole diagnostic for anisotropy as, for example, misalignments of source motions and geophone components can also give significant off-diagonals in an isotropic medium. In this case, however, $d_{ij}(t)$ and $dy(t)$ are different.

An asymmetry in the recorded data matrix may arise because of factors of experimental control such as: geophone coupling to the borehole, coupling of the source pad to the ground, differential gain and phase response between different horizontal geophones, differential amplitudes between different source activations (Lewis, 1989; Choi and Gangi, 1991), directional response of receivers (Ghose and Takahashi, 1991), polarity reversals, and misorientations of the geophone tool (Queen, 1987), misorientations of the source (Queen and Rizer, 1990), borehole deviation, etc. Some of these factors may also be avoided or reduced with careful field practice, although others must be tolerated and corrected in the recorded data by conditioning steps after acquisition, such as amplitude adjustments from source balancing computations (Winterstein and Meadows, 1991a). However, even with proper field practice, certain unavoidable inaccuracies may still be present in the multicomponent data set, which limit the accuracy of the anisotropic measurement, especially in the presence of depth changes in the anisotropic properties.

For a nonuniform anisotropy, caused by depth changes in shear-wave polarization directions, the data matrix is no longer symmetric. Such changes in shear-wave polarizations have been observed and estimated in field data by Martin and Davis (1987), Squires et al. (1989), Winterstein and Meadows (1991a, b), and Lefeuvre et al. (1991). Two approaches may be used to deal specifically with the problem of identifying the parameters of shear-wave splitting in the presence of changes in the polarization with depth so that this asymmetry is avoided. The first approach is a propagator matrix technique that analyzes the shear-waves by constructing a local tensor function connecting the recorded data matrices at two geophone levels, thus avoiding overburden complications. Procedures to implement this have been developed by Nicoletis et al. (1988), Lefeuvre et al. (1989 and 1991), and Lefeuvre and Mandal (1991). The second approach offers an alternative using a layer stripping scheme for direct shear-waves (Winterstein and Meadows, 1991a), where one compensates for the anisotropy in the overlying layers to simplify the interpretation of the data from the lower layers. The layer stripping procedure works remarkably well (Winterstein and Meadows, 1991a, b) and agrees closely with the results of the propagator matrix method when applied to near-offset, shear-wave VSP data acquired at the Railroad Gap and Lost Hills fields, California (Lefeuvre et al., 1991). In spite of this agreement, these techniques are sensitive to acquisition inaccuracy and are dependent on producing high quality data.

The motivation for this present work is grounded in the desire to obtain reliable estimates of changes in polarization azimuth in spite of acquisition inaccuracy. From the above, it seems natural that a technique for separating the effects of acquisition errors and polarization changes should be based upon measuring the lack of symmetry between the off-diagonal elements of the recorded data. This is accomplished by performing singular-value decomposition (SVD) on the recorded data matrix. The SVD is implemented in the dual-independent technique (DIT) originally introduced by MacBeth and Crampin (1989 and 1991) from the concept of independent source-geophone rotation (Igel and Crampin, 1990). An alternative form exists as the linear transform technique (LTT) of Li and Crampin (1993).

### CONVOLUTIONAL MODEL FOR SHEAR-WAVE SPLITTING

The vector convolutional model described by Queen and Rizer (1990) is a convenient practical simplification of the mathematics of shear-wave propagation through an anisotropic medium. Here, we shall use a specialized four-component form of this model for orthogonal shear-wave sources and orthogonal Q1 and Q2 polarization directions. For a VSP, it expresses the data matrix of displacements recorded at the nth level, $\mathbf{D}_n(t)$, generated by horizontal source motions at the surface as:

$$\mathbf{D}_n(t) = \mathbf{G}(t) \ast \mathbf{M}_n(t) \ast \mathbf{S}(t) + \mathbf{N}(t);$$

where $\ast$ denotes convolution.
where $\mathbf{M}_i(t)$ is the cumulative anisotropic medium response that is direction dependent, $\mathbf{G}(t)$ is a diagonal matrix of the horizontal geophone responses, $\mathbf{S}(t)$ is a diagonal source matrix, constructed from the motions induced by two orthogonal sources that are assumed point forces, and $\mathbf{N}(t)$ is a matrix of noise terms. The operation denoted by the asterisk (*) is a multiplication in the frequency domain or convolution in the time domain. Ignoring the upgoing wavefield and multiple reflections and considering only direct waves, equation (2) can be rewritten for a uniform anisotropic half-space as:

$$\mathbf{D}_i(t) = \mathbf{G}(t) * \{ \mathbf{R}^T(\theta) \Delta(i; \tau_1, \tau_2) \mathbf{R}(\theta) \} * \mathbf{S}(t) + \mathbf{N}(t).$$

(3)

The terms in curly brackets represent the directional medium response for a uniform half-space where the matrix $\mathbf{R}(\theta)$ is the standard 2 x 2 rotation matrix, and the superscript $T$ refers to the matrix transpose. $\mathbf{A}$ is the polarization direction of the leading split shear-wave qS1 measured with respect to the X-axis of the acquisition coordinate frame in the horizontal plane (with X-Y-axes horizontal and the Z-axis pointing downwards). If the polarizations of qS1 and the slower split shear-wave qS2 are not orthogonal, $\mathbf{R}(\theta)$ can be readily replaced by a more general transformation matrix. The expression is strictly relevant for a one-dimensional (1-D) model of the earth, and consequently neglects medium inhomogeneities, dipping layers, or anisotropic materials that do not have a horizontal plane of symmetry. Although this implies that the shear-waves qS1 and qS2 may be replaced by SV and SH, we retain the terminology of qS1 and qS2 for faster and slower split shear-waves in this present work as the vector convolutional model is also applicable to oblique incidence where the terms qS1 and qS2 are relevant. Equation (3) is simplified by the alignment of the source directions and geophone axes along a common acquisition coordinate frame. Element $\mathbf{A}$ is a diagonal matrix that contains the time-shift operators for the shear-wave motion:

$$\mathbf{A} = \begin{bmatrix} \lambda_1(t) & 0 \\ 0 & \lambda_2(t) \end{bmatrix},$$

(4)

where, for a lossless medium, $\lambda_1$ and $\lambda_2$ define operators that convolve with the source wavelet such that $\lambda_1 * s(t) = a_1(t) * s(t - \tau_1)$; $\tau_1$ is the traveltime and $d_1(t)$ the attenuation of the qS1 wave, with similar operators for the qS2 wave. It can be assumed, in the ideal case of linear, in-line and crossline source excitations, that $\mathbf{S}(t)$ is a diagonal matrix with the principal diagonals given by the scalar source functions $s_{1}(t)$ and $s_{c}(t)$, for the in-line and crossline source wavelets, respectively:

$$\mathbf{S} = \begin{bmatrix} s_{1}(t) & 0 \\ 0 & s_{c}(t) \end{bmatrix}.$$  

(5)

Nonlinear source motions produce nonzero off-diagonal terms. The principal diagonal terms of $\mathbf{G}(t)$ are given by the in-line and crossline geophone responses:

$$\mathbf{G} = \begin{bmatrix} g_{1}(t) & 0 \\ 0 & g_{c}(t) \end{bmatrix}.$$  

(6)

These responses may not be equal for geophones oriented parallel and perpendicular to the geophone clamping arms. If the geophone responses can be assumed equal and flat with no phase distortion within the signal bandwidth, $\mathbf{G}(t)$ is represented by a unit matrix multiplied by a scalar.

In conditions of good experimental control for which the source components are orthogonal and aligned in-line and crossline and for which $s_{1}(t) = s_{c}(t)$, the shear-wave polarization direction (0) can be obtained by methods that perform a least-squares decomposition of the recorded data matrix $\mathbf{D}_i(t)$ using the expression in equation (3). $\mathbf{D}_i(t)$ is assumed to be symmetric, and an eigenvalue-eigenvector analysis yields the desired solutions. The time-delay ($\Delta \tau = \tau_2 - \tau_1$) between the two split shear-waves is then obtained by finding the maximum cross correlation between the estimates of $s(t - \tau_1)$ and $s(t - \tau_2)$, which are derived from the principal diagonal components of $\mathbf{A}$. This particular decomposition can be achieved through numerical rotation (Alford, 1986; Thomsen, 1988); the faster algebraic techniques of Murtha (1988) or the linear transformations of Li and Crampin (1993).

The four-component orthogonal convolutional model of equation (3) may be valid strictly for vertical or near-vertical, shear-wave propagation where the shear-wave polarization directions are likely to be orthogonal provided the anisotropy possesses a horizontal plane of symmetry. However, the model may be extended to oblique incidence using a transformation into the dynamic plane or nine-component data, as the eigenmodes will be mutually orthogonal for weak anisotropic material and the data matrix will be symmetric. Some complications may arise, however, in sedimentary basins, where a combination of crack- and matrix-layer-induced anisotropy is expected (Bush and Crampin, 1991). Here, the behavior of shear waves propagating with ray directions close to the directions of point singularities will be sensitive to small changes in ray direction (Crampin, 1991; Wild and Crampin, 1991). These singularities may be quite close to normal incidence for many combinations of the two types of anisotropy (Wild and Crampin, 1991). Nonorthogonal polarizations for qS1 and qS2 produce an asymmetric data matrix. Independent tests have shown that equation (3) may still be valid for departures from orthogonality of up to 20 degrees.

CAUSES OF ASYMMETRY IN THE DATA MATRIX

Changes in polarization direction

Equation (3) can be rewritten to accommodate an abrupt change in the polarization direction for normally incident shear-waves by adapting a two-dimensional (2-D), plane-wave subset of the anisotropic wave propagation expressions of Fryer and Frazer (1984) for a single interface:

$$\mathbf{D}_i(t) = \mathbf{G}(t) * \{ \mathbf{R}^T(\theta_2) \mathbf{A}_2(t) \} * \mathbf{Z}(\theta_2, \theta_1),$$

$$\rho_1, \nu_1, \rho_2, \nu_2, \eta_1, \eta_2) * \{ \mathbf{A}_1(t) \mathbf{R}_1(\theta_1) \} * \mathbf{S}(t);$$

(7)

where $\mathbf{Z}$ is the transmission matrix between layer 1 and layer 2. The upper anisotropic layer has a qS1 polarization direction of $\theta_1$ and the lower anisotropic layer has a qS1
polarization direction of $\theta_2$. The background media have shear-wave velocities $V_1$ and $V_2$, and densities $\rho_1$ and $\rho_2$, and birefringences of $\eta_1$ and $\eta_2$. The transmission matrix relates the $qS1$ and $qS2$ waves to each other at the interface. Here, normal incidence is assumed and the interface is horizontal. The main diagonal elements of $Z$ (a $2 \times 2$ matrix in this case) give the transmission coefficients for $qS1$ to $qS1'$ and $qS2$ to $qS2'$, while the off-diagonal elements are the transmission coefficients for $qS1$ to $qS2'$ and $qS2$ to $qS1'$. Li and Crampin (1992) derive expressions for these coefficients and describe their behavior for different medium parameters. As with the convolutional model, these are strictly valid for anisotropies with a horizontal plane of symmetry. The transmission matrix may be approximated by:

$$Z \approx (2\rho_1 V_1/(\rho_1 V_1 + \rho_2 V_2))R(\theta_2)R^T(\theta_1); \quad (8)$$

if the weaker dependence on $n_1$ and $n_2$ is neglected [see Li and Crampin (1992) for an illustration of the dependencies on the different parameters of this expression]. The degree of interconversion between $qS1$ and $qS2$ waves at the interface is determined by the off-diagonals of the matrix $R^T(\theta_2 - \theta_1)$, and hence the sine of the polarization change, $\sin (\theta_2 - \theta_1)$. The modified equation (7) now becomes:

$$D_i(t) = AG(t) \ast \{R_2^T(\theta_2)A_2(t; \tau_2)R(\theta_2)\}$$

$$\ast \{R^T(\theta_1)A_1(t; \tau_1)R(\theta_1)\} \ast S(t), \quad (9)$$

where $A$ is a scalar constant, and $\tau_1$ and $\tau_2$ are the travel-times of the waves through each layer. The time-series product of these matrices is, in general, asymmetric, as the data matrix is now the product of the two symmetric matrices (each matrix formed by $R^TAR$ in curly brackets), and it need not be symmetric unless these matrices are commutative. Consequently, the effect of a polarization change at an interface may destroy the symmetry of the data matrix.

**Source and geophone effects**

The degree of experimental control also influences the asymmetry of the matrix, hence the interpretation of the results from the DIT algorithm. For example, source or geophone misalignment or misorientation may manifest as angular errors $\varepsilon_S$ and $\varepsilon_G$ in $\theta_S$ and $\theta_G$, if data for which the assumption of a uniform homogeneous structure holds is analysed by assuming exactly in-line and crossline source and geophone orientations. This can be interpreted using the model of equation (3) by premultiplying $S(t)$ by $R(\varepsilon_S)$ so that the equation now becomes:

$$D_i(t) = G(t) \ast \{R^T(\theta)A(t; \tau)R(\theta)\} \ast R(\varepsilon_S)S(t)$$

$$= \{R^T(\theta)A(t; \tau + \varepsilon_S)\} \ast S(t) \quad \text{(10a)}$$

for source misorientation; or premultiplying $G(t)$ by $R(\varepsilon_G)$ for geophone misorientation,

$$D_i(t) = R(\varepsilon_G)G(t) \ast \{R^T(\theta)A(t; \tau)R(\theta)\} \ast S(t)$$

$$= G(t) \ast \{R^T(\theta - \varepsilon_G)A(t; \tau)R(\theta)\} \ast S(t), \quad \text{(10b)}$$

or by applying a combination of both. This also leads to asymmetric off-diagonal traces in the four-component matrix. Such misalignment effects have been noted and corrected for in-field data. For example, Queen (1987) and Queen and Rizer (1990) found that the error for source orientation could be as large as 30 degrees if aligned by operator sight control, or as small as ±5 degrees if the source truck is driven along a preoriented line. For borehole geophones oriented by far-offset P-wave arrivals, EG could be 15 degrees, with more control if a time-consuming and expensive gyroscopic orientation tool is used. However, using a gyroscope often introduces resonance that makes the relative amplitudes of the horizontal components unusable. Consequently, if the data matrix is analysed using techniques based upon the model in equation (3) that forces a symmetry for $D_i(t)$, errors will be introduced into the estimated polarization direction $\theta$ and a small error into the time-delay.

Source imbalance caused by unequal coupling (possibly frequency dependent) for the different positions or orientations of the shear sources produces a static scaling factor as well as unequal source wavelets ($s_C(t) = \kappa s_S(t) = \kappa s(t)$). A constant coupling factor $\kappa$ gives the source matrix:

$$S = \begin{bmatrix} s(t) & 0 \\ 0 & \kappa s(t) \end{bmatrix}, \quad (11)$$

with corresponding data matrix:

$$D_i = \begin{bmatrix} d_{XX} & \kappa d_{XY} \\ d_{YY} & \kappa d_{YY} \end{bmatrix}, \quad (12)$$

where matrix $\kappa S/s(t)$ multiplies the symmetric data matrix $D_i(t)$, creating an asymmetry in the off-diagonal trace amplitudes (although the traces have the same phase relation). This imbalance affects the polarization $\theta$ and to a lesser extent the time-delay $\Delta\tau$ estimated from the decomposition of the data matrix using equation (3). If the cause of the asymmetry can be identified as source coupling only, the errors may be reduced by employing a balancing procedure based upon energies. Choi and Gangi (1991) conclude that consideration of source balancing can reduce the variability of the estimated polarization directions from different source components from ±15 degrees to less than ±5 degrees. The extent to which the results are affected depends on the magnitude of the polarization direction of the shear-waves and the relative time delay.

Similar effects arise for an unequal source function caused by near-surface conditions:

$$S = \begin{bmatrix} s_f(t) & 0 \\ 0 & s_C(t) \end{bmatrix}, \quad (13)$$

where $s_f(t) \neq s_C(t)$ for all $t$. Such consistency problems may be significant when the source pad position is moved to slightly different positions to preserve coupling. This is emphasized by Winterstein and Meadows (1991a), who find that even small movements in the source position can make shear-wave data unusable for layer stripping. They attribute this finding to the variability of the near-surface layering. Different source functions for different source motions may also be caused by differential attenuation in the near-surface layering.
Differential gain in geophone components produces a similar result to the source imbalance, changing both polarization values and time delays. The data matrix now becomes:

\[
\mathbf{D}_t = \begin{bmatrix} d_{XX}^i & d_{XY}^i \\ \zeta d_{XY}^i & \zeta d_{YY}^i \end{bmatrix},
\]

where, again, \( \zeta \) may be frequency dependent. Polarity reversals in the geophone and source components affect the results to a greater extent. The polarity change effectively sets \( \zeta \) in the above equal to -1. An example of this is described below in the case study of the Romashkino VSP data, which shows that this type of error may give rise to significant anomalies in the shear-wave estimates that cannot be represented by a simple rotational change as obtained from equation (3).

Asymmetry estimation using DIT algorithm

The dual independent source-geophone rotation technique (DIT) technique of MacBeth and Crampin (1989 and 1991) or the linear transform technique (LTT) of Li and Crampin (1993) effectively perform a least-squares singular value decomposition (Lawson and Hanson, 1974) on the asymmetric data matrix of traces \( \mathbf{D}_t(t) \) with the result being:

\[
\mathbf{D}_t(t) = \mathbf{G}(t) \ast \left\{ \mathbf{R}^T(\theta_G) \mathbf{A}_{AV}(t) \mathbf{R}(\theta_S) \right\} \ast \mathbf{S}(t),
\]

where \( \mathbf{A}_{AV} \) is as close to a diagonal matrix as possible, with a minimum in the combined off-diagonal energies. Application to data with the acquisition uncertainties outlined above, should, in theory, give principal diagonal components that are time-shift operators for the raypaths. The subscripts S and G in this equation refer to the source and geophone and relate to the asymptotic behavior of this relation for the propagation of a delta function through a polarization change (MacBeth and Yardley, 1992). Physically, this technique may be thought of as an independent rotation of both source and geophone until the data matrix becomes diagonal or closest to diagonal form (Igel and Crampin, 1990; MacBeth and Crampin, 1989 and 1991). This particular interpretation may be understood by reference to the previous section on source and geophone effects and equations (10a) and (10b).

The behavior of \( \theta_S \) and \( \theta_G \) derived by this technique is not fully understood in all cases, but the relative values of \( \theta_S \) and \( \theta_G \) can be related to the degree of experimental control and changes in the shear-wave polarizations with depth. If there is no change in the shear-wave polarizations and the acquisition system is correctly aligned with correct instrumentation, \( \theta_S \) and \( \theta_G \) have identical values. If the shear-wave polarization directions change, the divergence of \( \theta_S \) and \( \theta_G \) estimates provides a sensitive quantitative indicator of the depth at which this occurs. This is illustrated in Figure 1, adapted from MacBeth and Yardley (1992). Although this behavior depends upon the relative birefringence and thickness of the two layers, together with the peak frequency and type of source wavelet, one salient feature of this analysis is clear: \( \theta_S \) and \( \theta_G \) diverge at the depth where the change in polarization occurs. A special exception to this is when the polarization change is 90 degrees, when \( \theta_S \) equals \( \theta_G \), although this can be detected by examining the slope of the time delays that change sign at this depth. When the polarization change is greater than 45 degrees, \( \theta_S \) and \( \theta_G \) may also contain 90 degrees jumps, and will not assume the asymptotic behavior. Once the point at which the polarization direction change occurs is detected, results from shallower geophones can be used in a layer-stripping approach. Layer stripping can then be implemented by applying a deconvolution operator derived from equation (9):

\[
\mathbf{L}_{STRIP}(t) = \mathbf{R}^T(\theta_1) \mathbf{A}_{AV}^{-1}(1) \mathbf{R}(\theta_1)
\]

to the recorded data matrix \( \mathbf{D}_t(t) \). A necessary assumption for the application of this operator is that \( \mathbf{S}(t) \) can be written as a unit matrix multiplied by the scalar function \( s(t) \). In favorable cases, the stripping operator could also be evaluated analytically by the requirement that the final transformed matrix should be symmetric.

If the medium can be assumed to have uniform anisotropic symmetry, which we may expect for recordings made below the depth at which vertical stress is equal to the minimum horizontal stress (Crampin, 1990), the implementation of equation (3) is justified. In this situation, it is possible to identify acquisition errors by examining the amount of asymmetry in the data matrix through the values of \( \theta_S \) and \( \theta_G \). It may then be possible to compensate for any misorientations in the geophone and source, and for polarity reversals, so that reliable shear-wave estimates may be obtained from the multicomponent data. The effect of con-

![Figure 1](image-url)
Interpreting Data Matrix Asymmetry

The geophone orientations can be determined using this technique. If the source polarizations are fixed and no other effects are significant (Li, personal communication), the effects of misorientation, coupling, and medium changes affect $\theta_S$ and $\theta_G$ differently. It is therefore, in principle, possible to separate or attribute measured asymmetries caused by acquisition errors and medium changes on the basis of the relative behavior of $\theta_S$ and $\theta_G$. This technique is not, however, without ambiguity in interpretation. For example, the results in Figure 1 for the deeper geophones embedded in the lower medium also appear as a simple displacement between $\theta_S$ and $\theta_G$. It is therefore possible for this to occur at deep geophones below variable anisotropy in thin near-surface layers. The examples below have been selected to illustrate different DIT results, and how these may be interpreted as acquisition errors and medium changes.

Example of Polarization Changes with Depth in Synthetic Data

To demonstrate how the DIT algorithm may be used to detect polarization changes attributed to the medium, it is applied to synthetic shear-wave data computed by ANISEIS (Taylor, 1991, ANISEIS II Manual: Applied Geophysical Software Inc.) for the anisotropic model proposed by Winterstein and Meadows (1991a) for Lost Hills, California. Our model is shown in Table 1. The background velocity structure used in the computations is derived from zero-offset VSPs obtained from a borehole in the vicinity (Yardley, 1992). Two shear-wave sources (simulated by horizontal point forces) are positioned at a 15 m offset from the borehole, with in-line and crossline directions of motion. Geophones are distributed in a vertical borehole from 651 to 31 m in steps of 31 m. The data acquisition geometry and dimensions are similar to those used in the field acquisition of this data set (Winterstein and Meadows, 1991a). The geophone displacements for downwardly propagating plane waves are computed for each shear-wave source, and analyzed with the DIT algorithm in combination with layer stripping. The results from each stage of this analysis are shown in Figures 2a through 2d. For each stage of this procedure, the data are also analyzed according to equation (3) with the assumption of symmetry in the matrix of traces (hence, $\theta_S = \theta_G$). This process yields a best estimate of $\theta$ and the diagonal phase matrix $A(t)$ by fitting the best symmetric matrix by least-squares to the data matrix. If the symmetry assumption is justified, $A(t)$ will be a diagonal matrix of separated $qS1$ and $qS2$ waves. The estimates of $A(t)$ corresponding to each set of DIT results in Figure 2 are shown in Figures 3a through 3d.

During this processing, the time-delay profile changes from a slow increase with depth to a series of linear increases, punctuated by jumps to lower values. As pointed out by Winterstein and Meadows (1991a) and MacBeth and Crampin (1991), layer stripping reduces the time delays to zero for Geophones 1, 9, and 12, which lie on the interfaces between the two different media. In these diagrams, we have chosen to represent the time delays at each geophone level by the cumulative time delay for the overburden at that depth after stripping and processing.

The profile of polarization directions obtained from the layer stripping guided by DIT is in good agreement with the actual model values, with a slight decrease in resolution for the thin 70-degree layer with 3-percent birefringence. The accuracy of the fit between the actual model profiles and processed shear-wave results, together with the diagonal form of the final $A(t)$ estimate, confirms the usefulness of the layer stripping procedure.

Field Data Examples

In the sections below we analyze two different sets of VSP field data using DIT. Both data sets reveal polarization changes that combine in different ways with the experimental inaccuracies.

Romashkino Reservoir, Tatar Republic, Russia

The first data set was acquired as part of a seismic survey in the Romashkino oil field, in the Tatar Republic, Russia, east of the Russian platform (Polskov et al., 1980; Cliet et al., 1991). This is a fractured carbonate reservoir and is currently in production. Although the data consisted of VSPs recorded at three wells and a surface line, we consider only VSP data acquired at the vertical well 15037. The data in this particular example were generated by two horizontal, orthogonal, shear-wave sources (“VEIP” S-wave electrodynamic pulse sources, Cliet et al., 1991) positioned at near offset. The shear-wave sources used in this example are aligned in the directions N47°E and N137°E. The data were recorded at 5 m intervals between 975 m and 605 m. Geophone reorientation was performed as a preprocessing step, using offset

<table>
<thead>
<tr>
<th>Thickness (m)</th>
<th>Model polarization</th>
<th>Model anisotropy</th>
<th>Stripped polarization</th>
<th>Stripped time delay (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>7%</td>
<td>7°</td>
<td>7°</td>
<td>5.2</td>
</tr>
<tr>
<td>240</td>
<td>3%</td>
<td>43°</td>
<td>43°</td>
<td>14.0</td>
</tr>
<tr>
<td>100</td>
<td>3%</td>
<td>70°</td>
<td>68°</td>
<td>6.0</td>
</tr>
<tr>
<td>HS</td>
<td>3%</td>
<td>43°</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Numbers on the left give polarization directions of leading shear-wave, together with birefringence values for each layer. Numbers to the right give polarization directions and time-delays that are determined in the automatic layer stripping procedure. The model is taken from the study of Winterstein and Meadows (1991a).
FIG. 2. DIT results from the analysis of synthetic seismograms computed using the model in Table 1. Each diagram shows the polarization azimuths $\theta_G$ and $\theta_S$ as a function of depth, together with the differential travelt ime, computed for the data during various stages of layer stripping. The horizontal arrows mark the depth points at which $\theta_G$ and $\theta_S$ are judged to diverge and at which layer stripping is applied. Solid line corresponds to actual model values. Results are for: (a) original data, (b) data with the thin top layer stripped, (c) further stripping for next deepest layer, and (d) further stripping for next deepest layer.
P-wave data, so that the horizontal geophones were aligned along the geographical coordinates (north and east). Down-going and upgoing waves were separated by $f-k$ filtering applied as a further stage in preprocessing. Tests of synthetic data (not shown) showed that for the frequencies and geophone spacing in this experiment, this procedure did not affect the results adversely.

The results of the DIT algorithm applied to these data are shown in Figures 4a through 4d. These results are analyzed together with the best estimates of $A(t)$ obtained, as in the synthetic examples by using equation (3) (Figures 5a through 5d). The $\theta_A$ and $\theta_G$ estimates in Figure 4a corresponding to the original recorded data are different but approximately parallel below 800 m. Above this level, the estimates are more scattered but exhibit a linear divergence. The two sets of estimates appear to be symmetric about a polarization azimuth of 60 degrees. The original rotated data matrix in Figure 5a has only a small amount of residual energy on the off-diagonal traces, suggesting that there are no severe changes in polarization with depth. Taking into account the known difference in alignment between the source and geophone axes by rotating the source directions into the geographical axes, changes the DIT results to those in Figure 4b. The estimates of $\theta_A$ increase by 47 degrees, with the effect of pulling $\theta_A$ further apart from $\theta_G$. The estimate of $A(t)$ is now considerably worse (Figure 5b). The time delays estimated for these two cases are identical, increasing linearly from values of 16 ms to about 32 ms. These DIT results highlight an observation that is clear from initial inspection of the principal traces, the in-line and crossline geophones have different polarities. A left-handed set used in acquisition would explain these polarities. The principal diagonal traces in Figures 5a and 5b have an opposite polarity. A polarity reversal for the in-line traces is confirmed by horizontal recordings of the P-waves generated by an offset source, and was also later confirmed by Compagnie Generale de Geophysique. Figure 4c shows the effect of this polarity reversal on the initially unrotated data where the $\theta_A$
and $\theta_G$ estimates are now approximately parallel separated on average by an average of 47 degrees, which is the value of the correction term for the source orientation. This processing step equalizes the polarity of the principal diagonal traces at the expense of focusing more energy on one of the off-diagonal trace sets (Figure 5c). The time-delay estimates now increase linearly from -6 to 10 ms as the reversal causes the algorithm to pick the cross-correlation on an earlier half cycle (about 20 ms difference). Figure 5d shows the final results that combine the polarity reversal with the source reorientation. The $\theta_S$ and $\theta_G$ estimates are now close at most depths. The alignment of source and geophone components has the desired effect of bringing $\theta_S$ and $\theta_G$ together, indicating data matrix symmetry (Figure 5d). The scatter of values in the shallow section above 750 m reflect the poorer estimate in $\Delta(t)$. The average polarization direction is 134 degrees for depths below 800 m, and the time-delay estimates for the complete depth range agree with the analysis of Cliet et al. (1991).

A reduction in the off-diagonal energy in $\Delta(t)$, particularly for the shallower section with better agreement between $\theta_S$ and $\theta_G$ values, may be obtained by an additional source rotation of 10 degrees combined with source balancing. Although there are no observer logs or field details available that might suggest misalignment, it is within the margin for error for alignment of the sources (Queen, 1987). However, there may be other factors such as differential attenuation, particularly in the shallow structure, that could also improve $A(t)$, but without additional information this may be merely

**Fig. 4.** DIT results from the analysis of Romashkino near-offset VSP. Each diagram shows the polarization azimuths $\theta_G$ and $\theta_S$ as a function of depth, together with the differential traveltime. Results are for: (a) original data, (b) data corrected for the misaligned source axes, (c) data with polarity reversal for in-line geophone, and (d) data corrected for the source misalignment and polarity reversal.
Interpreting Data Matrix Asymmetry

Fig. 5. Estimates of the phase-shift matrix $\Lambda(t)$ corresponding to each data processing step in Figure 4. Computations use the symmetric decomposition of the data matrix given in equation (3). If the data matrix is symmetric, $\Lambda(t)$ will be a diagonal matrix of traces. The XX and XY traces are in-line ($X$) and crossline ($Y$) recordings of the in-line ($X$) source motion, with YX and YY being the corresponding recordings for the crossline ($Y$) source motion.
Fig. 5. (continued)
a cosmetic effect, which could disguise information about the near surface.

The final results for \( \theta_s \) and \( \theta_G \) in Figure 4d show a linear change in the polarization direction from 174 degrees at 600 m to 134 degrees at 800 m depth, but with the results being quite scattered. It is possible that the \( \theta_s \) and \( \theta_G \) values pull apart a little with depth. Analysis of full-wave modeling results (not shown) demonstrated that a gradual divergence of the DIT results and the linear change in the shallow section can be explained by an overburden layer with a polarization azimuth of N160°E and 2 ms time delay and a polarization throughout the instrumented section of N117°E (17 degrees from the original result) and 5 to 6 percent birefringence. The DIT results for the best model are shown as solid lines on Figure 4d for comparison with the observations. It appears that in this data set, the particular combination of overburden anisotropy and polarization change gives rise to a gradual divergence of the DIT results. This indicates an inherent resolution difficulty with the DIT procedure, which is best resolved by combining the analysis with full-wave modeling. Layer stripping of this data set does straighten out the polarization estimates from the shallower section of the VSP, although the scatter is not reduced.

Lost Hills Field, California

In this second case study, DIT is applied to a four-component subset of a near-offset, nine-component data set acquired at Lost Hills Field in Kern County, California, in 1986 by Kim Tech (now Production Geophysical Services). The reservoir is situated at the southern end of the San Joaquin Valley on the southeast flank of the Lost Hills anticline. Preliminary results from the shear-wave modeling of this data set have been given by Yardley and Crampin (1990), and a full discussion of the data, geology, and full-wave modeling is given by Yardley (1992). Here we describe only the processing and interpretation stage using the DIT algorithm. The source used to generate the data was an Omnipulse air gun, and data for the orthogonal source orientations were acquired on separate runs of the geophone tool. The geophones were three-component, 10-Hz models supplied by Gearhart Inc. Source directions aligned N17°W and N73°E from positions located 155 m from the well head at an azimuth of N253°E were recorded at 42 geophone levels from 1097 to 2347 m at approximately 30-m intervals. There were several small changes (2 to 3 m) in the position of the source pad during acquisition to preserve coupling. The well is slightly deviated giving rise to an azimuthal variation of 12 degrees in the radial direction as drawn horizontally from the source to the surface position of each geophone. This effect being confined mainly to the deeper (below 1800 m) geophones. Tool orientation was determined using the well survey and gyrodata, from which the data were transformed to align along geographical coordinates. The data were preprocessed to separate upgoing from downgoing waves using an f-k filter (Yardley, 1992).

Figures 6a through 6c show the DIT results for various data processing steps with the corresponding \( \Delta(t) \) estimates shown in Figures 7a through 7c. Figure 6a shows the results of applying DIT to the preprocessed direct arrivals. The \( \theta_s \) and \( \theta_G \) estimates lie together only at certain depth points (1.85 km, and 2.1 to 2.2 km), and both exhibit an oscillatory behavior and are mirror images of each other. The time delays are nearly constant at around 42 ms for all the geophones, but they exhibit a slight oscillatory behavior that is in-phase with \( \theta_G \). Figure 7a shows the corresponding estimate of \( \Delta(t) \) obtained using equation (3). The low energy on the off-diagonals suggests that the original data matrix is already symmetric. If the data matrix is corrected for the known misalignment between the source and geophone components so that all of the axes are oriented geographically, the \( \theta_s \) results move closer to the \( \theta_G \) estimates (Figure 6b). This effectively removes the mirror imaging and closes up the estimates for the top of the section (down to 1.8 km), but the oscillations still remain (with a mean of 45 degrees). The estimated time delays are unaffected by this procedure. There is a marginal improvement in the off-diagonals of the corresponding \( \Delta(t) \) in Figure 7b. The analysis of Yardley (1992) using compressional and shear waves suggests that it is highly unlikely that there should be a predominant polarity reversal in this data set. If there was a 180-degree change in the gyroscopic estimate, this would be detected in the DIT results. The rotated matrices of traces in Figures 7a and 7b confirm this analysis.

The approximately constant time delay combined with the unequal \( \theta_s \) and \( \theta_G \) values suggest that layer stripping may be an appropriate processing step; although the polarization angles do not diverge in this case. The example in Figure 2b shows that a polarization change may cause \( \theta_s \) and \( \theta_G \) to crossover more than once. A layer with a \( \sigma_\| \) polarization direction of N45°E and 42 ms is stripped from the data matrix using the deconvolution operator \( L_{STRI} \) described in equation (16). The parameters of this operator are obtained from the results for the shallowest two geophones. As the bulk of the anisotropy appears to be above the geophones, this procedure seems to be an obvious choice. Layer stripping reduces the time-delay values to an almost constant 6-8 ms and brings the \( \theta_s \) and \( \theta_G \) results together (Figure 6c). The absence of an increase in time delay makes the calculated polarization angle questionable. The rotated data matrix \( \Delta(t) \) after source rotation combined with this layer stripping shows another marginal improvement (Figure 7c) for the direct arrival within the dotted marker lines. The DIT results show a constant polarization azimuth of 129 degrees down to 1.45 km followed by a sharp decrease and then slow oscillation with extrema between 108 degrees and 140 degrees. This suggests that such minor variations might very well be the result of crack orientations modified by minor variations in the stress field. These variations may not be reliable as the dominant overburden anisotropy means that the stripped data are sensitive to noise.

An alternate explanation is furnished by the analysis of Choi and Gangi (1991) on the same data set. They obtain two differently calculated angular values related to in-line and crossline sources, which display similar symmetric undulations. They attribute this behavior to source imbalance and correct the data using a scalar factor, assuming identical source waveforms, to reduce but not eliminate this variability. This appears to conflict with the expected DIT analysis of source imbalance, which would give two \( \theta_s \) and \( \theta_G \) curves separated by a constant difference. Synthetic analy-
ses (not shown) using different relative scaling factors and source wavelets could not reproduce these results. Source balancing failed to produce any noticeable improvement. There is no correlation between changes of pad position and the oscillations in Figures 6a and 6b (see position changes marked on diagrams). However, the explanation of inconsistent sources is supported by Yardley (1992) who observes different wavelets on the in-line and crossline source components. These could be caused by the directional-dependent attenuation in the near-surface layering. Yardley (1992) applies techniques that process single-source data, and he finds similar results for the in-line and crossline data sets with no oscillatory behavior. The amplitude of the oscillations in Figures 6a and 6b does appear to be correlated with the known borehole deviation (up to 12 degrees for the deeper geophones), being larger for the deeper geophones below 1.8 km, where the well has a larger deviation. It is possible that a combination of source and near-surface anomalies coupled with borehole deviation may create the oscillatory behavior.

DISCUSSION AND CONCLUSIONS

Acquisition and processing of vector wavefield data has different objectives from those of conventional data acquisition and processing. A conventional survey focuses on
FIG. 7. Estimates of the phase-shift matrix $A(t)$ corresponding to each data conditioning step in Figure 6. Computations use the symmetric decomposition of the data matrix given in equation (3). If the data matrix is symmetric, $A(t)$ will be a diagonal matrix of traces. The XX and XY traces are in-line (X) and crossline (Y) recordings of the in-line (X) source motion, with YX and YY being the corresponding recordings for the crossline (Y) source motion.
stratigraphic and structural detail, but the multicomponent survey has the potential of estimating additional information on the internal properties of the reservoir and rock, such as porosity, pore fluid type, and lithology (Crampin, 1985; Tatham and McCormack, 1991), as well as fracture or stress direction and intensity (Crampin and Lovell, 1991). However, interpretation of multicomponent records for anisotropy, from which fracture direction and intensity may be inferred, relies upon accurate generation and recording of vectorial motion. This requires careful preservation and accurate knowledge of the relative scaling effects between different components in the data. It is of consequent importance to be able to explain and eliminate those anomalous effects in the recorded data that do not conform to our framework for interpreting the anisotropy and may be attributed to either acquisition or other effects. If estimates of the orientation of fractures inferred from polarization directions are to be of practical value, then the control over factors such as the orientation of the field equipment must be good (Queen, 1987). In addition, the control parameters are unlikely to be universally applicable to different geographical areas.

Measurement of data matrix asymmetry using a singular-value decomposition has been introduced as a possible technique for checking the degree of experimental control attained and may provide a way of guiding data conditioning procedures for the compensation of inaccurate acquisition. It has been shown that this procedure can indicate polarity changes in data, which may not be readily visible on initial inspection. It also reveals source or geophone orientation errors and is potentially capable of investigating relative coupling between geophone components and the effects of source imbalance. Synthetic seismograms and field data have shown that the asymmetry introduced by depth changes in polarization azimuth may be separated from the asymmetry created by other effects. It is possible that such a technique will help eliminate some of the concern over the reliability of polarization estimates, and help resolve changes in the shear-wave polarization in the subsurface that may be obfuscated by acquisition inaccuracy. In the future, techniques of this nature may also help relieve some of the restrictive conditions for experimental control, so that a multicomponent data set can be acquired with more flexibility and over a shorter time frame.

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