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Comment on "Crack models for a transversely isotropic medium" by C. H. Cheng and comment by C. M. Sayers

Stuart Crampin[1]

Department of Geology and Geophysics, University of Edinburgh, Scotland

Introduction

Sayers [1993] commented that the expressions of *Cheng* [1993a] for extending the equations of *Hudson* [1980, 1981] for the elastic constants of distributions of parallel cracks to large crack densities were nonphysical. He offered the formulations of *Sayers and Kachanov* [1991] as preferable alternatives, and *Cheng [* 1993b] accepted these comments. In this further comment, I argue that the large crack densities suggested by both Sayers and Cheng when applied to real rock represent such pronounced fracturing that the rock would be fragmentary, the long-wavelength limit would be exceeded, and effective media concepts would no longer apply.

Crampin [1978] and *Hudson* [1980, 1981] developed expressions for the effective elastic constants, in the long-wavelength limit, modeling seismic wave propagation through solids containing weak distributions of parallel noninteracting cracks. The elastic constants specify purely elastic anisotropic solids whose variations of velocity match those of two-phase isotropic solids containing regular distributions of parallel cracks. Such effective medium models are important because they allow the behavior of complicated two-phase solids, which are difficult to calculate, to be simulated by calculations using uniform elastic models. These models are valid when the seismic wavelengths are much greater than the dimensions of the cracks (5 or 10 times greater are usually considered sufficient).

The expressions of Hudson are correct to the second order in perturbations from isotropy and it is found that the series approximations begin to diverge for crack densities greater than about ε = 0.1 *[Crampin,* 1984]. *Hudson* [1986] gives expressions for crack-to-crack interactions which are clearly essential for large crack densities but, as expected, make little difference to the behavior for crack densities less than about ε = 0.1. One of the obvious signs of the failure of the *Hudson* [1980, 1981] expressions for larger crack densities is that for $\varepsilon \geq 0.15$, some of the elastic constants, and hence the seismic velocities, begin to increase with increasing crack density *[Sayers and Kachanov,* 1991]. This behavior is considered by Sayers and Kachanov to be nonphysical. (This may be a plausible interpretation, but since there is no appeal to the actual physical behavior of cracks, the criticism is essentially not proven.)

[1]Also at Edinburgh Anisotropy Project, British Geological Survey, Scotland.

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Both *Sayers and Kachanov* [1991] and *Cheng* [1992, 1993a] extend the validity of the *Hudson* [1980, 1981] expressions to larger crack densities by removing the "nonphysical" increase of elastic constants with increasing crack density. *Sayers and Kachanov* [1991] eliminate the nonphysical behavior by introducing a second-rank tensor generalization of the crack density, which is dependent on the particular model being investigated. *Cheng* [1992, 1993a] removes the nonphysical behavior by extending Hudson's expressions by Padé approximations.

Physical Realization of Crack Densities

Crack density is usually written as $\varepsilon = n\,a^3/v$, where n is the number of, in this case isolated (unconnected) circular, parallel cracks of radius a in volume v . Aspect ratio and pore fluid properties are also required to fully specify the elastic constants *[Hudson,* 1980, 1981]. The crack density can be normalized to $\varepsilon = a^3$, where a is the radius of an average crack in a unit cube of the matrix rock. Since ε is dimensionless, the volume of the unit cube is the volume of the cracked solid divided by the number of cracks. (*Crampin* [1993] shows that when the *Vp*/*Vs* ratio is 1.732, the percentage of differential shear wave velocity anisotropy in a distribution of parallel cracks is approximately equal to $\varepsilon \times 100$.)

Following *Crampin and Leary* [1993] and *Crampin* [1994], Figure 1 indicates the physical realization of normalized crack densities as distributions of average cracks with crack radius a for values of crack density from ε = 0.1 to 0.5. These diagrams are sections through regular distributions of parallel equally sized (average) cracks with the specified crack densities and percentages of anisotropy. The diagrams in Figure 1 are not images of real rocks, which would have irregular distributions of crack size, crack shape, and crack orientation; however, they do indicate the amount of alignment implied by the crack densities and, in particular, the amount of uncracked matrix rock around the average crack. Seismic wavelengths are usually so much larger than the cracks giving rise to the azimuthal anisotropy in the Earth's crust that there is extraordinarily good statistical averaging, and synthetic seismograms through models of effective media can match observed seismograms through cracked rock with considerable accuracy *[Crampin,* 1994].

[Note that it is difficult to accurately represent three-dimensional distributions in two-dimensional plane diagrams. The diagrams in Figure 1 are not average cross sections but represent the approximate separation of cracks in regular three-dimensional distributions of equally sized cracks at the relevant crack densities.]

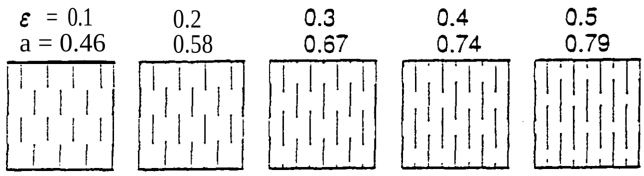

Figure 1. Physical realization of distributions of parallel cracks for normalized crack densities of ε = 0.1 to 0.5. The panels represent two-dimensional schematic cross sections through cracked solids.

The diagrams indicate heavily cracked rock where the average crack for $\varepsilon \geq 0.1$ is within a crack radius of at least eight other cracks. *Crampin* [1994] reviews observations of shear wave anisotropy in crustal rocks. Rock which is otherwise considered to be intact and unfractured below 1 km depth has crack densities less than about ε = 0.035 for sedimentary rocks and less than about ε = 0.045 for igneous and metamorphic rocks. However, heavily fractured rock near the Earth's surface may have crack densities greater than ε = 0.1 (differential shear wave velocity anisotropy greater than about 10%).

Such heavily cracked rock would have little shear strength and would be fragmentary. Cracks would tend to join together and throughgoing fractures easily propagate. Consequently, pore fluids would tend to disperse unless the fluids were constrained in some way, as in overpressurized reservoirs. Except near the surface, where confining pressures are low, such heavily cracked rock would require high pore fluid pressures to keep the crack faces apart. Reduction of pore fluid pressures through fracturing would lead to the closure of cracks and a lowering of the crack density of open cracks. This could occur locally, around a producing hydrocarbon well for example and would inhibit production *[Crampin,* 1994]. In situ natural rocks with the degree of cracking implied by the diagrams in Figure 1 and by the values of crack density in *Sayers* [1993] and *Cheng* [1993a] would be fragmentary, highly irregular and unstable, and could not be modeled by regular distributions of equally sized unconnected circular cracks.

Conclusions

Rocks with crack densities greater than about ε = 0.1 are heavily fractured with the likelihood of throughgoing fractures, severe irregularities of crack shape and aspect ratio, and the likelihood of severe crack-to-crack interactions. These are outside the specifications for the validity of effective media representations of parallel cracks, which are valid for weak crack densities, long-wavelength limits (small cracks), and regular distributions of unconnected circular parallel cracks. In addition, distributions of cracks in such heavily fractured rock are likely to be fragmentary and highly irregular, and the regular distributions implied by effective media approximations are unlikely to be satisfactory. The regular distributions of cracks of the large crack densities implied by *Sayers* [1993] and *Cheng* [1992, 1993a] could only be found in carefully constructed artefacts.

Although large crack densities do exist in natural rock, particularly near the free surface *[Crampin,* 1994], such heavily fractured rock cannot be quantitatively modeled by the purely elastic constants of effective media without taking into account attenuation and severe crack-to-crack interactions. Consequently, any attempt to model large crack densities by expressions, such as those of *Sayers and Kachanov* [1991] and *Cheng* [1992, 1993a], are unlikely to be accurate as the complications introduced by the variations of real rock will far exceed the variations between the various formulations demonstrated by *Sayers and Kachanov* [1991] and *Sayers [* 1993].

Nevertheless, heavily fractured rocks with strong crack-induced velocity anisotropy do exist in the Earth [*Crampin,* 1994] and there is a need to simulate the behavior in some way. Consequently, it may be convenient to obtain appropriate elastic constants for large velocity variations by applying the Hudson expressions to crack densities (or those of *Sayers and Kachanov* [1991] and *Cheng* [1992, 1993a]) outside the limits of validity for effective media modeling, as long as it is realized that there is no longer an exact correspondence between crack density and expressions for elastic constants. On the few occasions that I have used large crack densities ($\varepsilon \approx 0.4$ *Crampin et al.,* 1980]; ε = *0.2 Hudson and Crampin*, 1991], I have always indicated that there is not a causal relationship between crack density and the velocity variations for such large crack densities.

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S. Crampin, Department of Geology and Geophysics, University of Edinburgh, Grant Institute, West Mains Road, Edinburgh EH9 3JW, Scotland UK. (e-mail: scrampin@glg.ed.ac.uk)

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