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Seismic properties of a general fracture

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ABSTRACT: In modelling the wave behaviour through fractured and jointed rocks, different models have been proposed to describe the fractures. A fracture can be modelled (1) as a parallel-walled thin layer, or (2) as a planar array of distributions of small cracks or voids (rough surface), or (3) empirically as a linear slip interface. For the first two cases, approximate boundary conditions can be derived and we find that they are similar to the third empirical linear slip interface model. The results according to exact elastodynamic theory have been compared with solutions based on the approximate boundary conditions. Good agreement is obtained. Consequently, these three models can be cast into a universal form. The relatively simple boundary conditions that we derive can be used to model the much more complicated microstructures of natural fractures, joints, and faults in the Earth.

1 INTRODUCTION

Fractures and faults are very common in the subsurface of the Earth's crust, and they control much of the mechanical strength and transport properties of its solid structure. Fractures and fracture systems are also crucial for hydrocarbon production, control and manipulation of water supplies, dispersal of pollutants, storage of nuclear waste, and others. Much of our knowledge about the Earth's crust is obtained from seismic waves. One of the most successful methods for the detection and characterization of fractures and prediction of fluid flow directions is the use of seismic shear-waves (Crampin 1994; Liu et al. 1991; Queen & Rizer 1990).

In modelling the seismic response of natural and induced fractures, it is essential to understand the microstructure of fractures. The general understanding is that a fracture is a cluster of small cracks, and a fault is a cluster of fractures. Several fracture models have been proposed in geophysics, acoustics, and non-destructive test in mechanics to study the mechanical, transport, and seismic properties of fractures. In this paper, we show that various fracture models are seismically equivalent, and therefore propose a general boundary condition which is based on analysis of details of the microstructure of fracture surfaces in an attempt to consolidate the inter-relationship between various fracture models.

2 PHYSICAL DESCRIPTION OF A FRACTURE

Intensive studies have addressed this problem because of its conceptual and practical importance. Newmark et al. (195 1), and Jones & Whittier (1967) were among the earlier attempts to study an 'incomplete' interface (fracture), from which the boundary conditions relating stresses and displacements were derived (such interfaces are also called slip, imperfect, loosely bonded, or unbonded interfaces). Murty (1975, 1976) later considered a vanishing thin layer of viscous fluid and introduced similar boundary condition to that of Newmark et al. (1957). Murty also gave numerical results for the reflected and transmitted energy for an incident plane wave. The transmission and reflection properties of seismic waves across a thin layer of viscous fluid were studied by Fehler (1982) specifically to interpret acoustic events induced by hydraulic fracturing. The effect of a thin weak elastic layer was investigated by Jones & Whittier (1967), who considered the propagation of Stoneley waves along the interface, and recently by Rokhlin & Wang (1991). Schoenberg (1980) and Pyrak-Nolte et al. (1990) extended Murty's (1976) model, and assumed that the discontinuity in each component of displacements is proportional to the corresponding component of traction. Further generality was introduced by Tleukenov (199 1) who considered thin layers filled with anisotropic elastic or linearly viscoelastic material, and also derived a non-linear relationship between displacement discontinuity and traction for thin layers of non-linearly viscoelastic and visco-plastic material. Angel & Achenbach (1985) studied the problem of elastic-wave diffraction by interface imperfections due to a periodical array of cracks. We have obtained similar boundary conditions to the previous authors for a fracture surface with distribution of microcracks or with distribution of welded areas (Hudson et al. 1994).

In summary, published fracture models can be broadly classified into three groups. The simplest model is a parallel-walled, thin-layer of weak visco-elastic material (the Fehler model). Usually, the fracture thickness can be very small in the order of mm or cm (much smaller than the seismic wave-length). The second model is a plane boundary with a distribution of crack-like voids, or its inverse, a thin fluid layer with a distribution of contact points and/or areas (the Hudson model), which is similar to the rough surface model of Gangi (1981). Finally, a fracture can be represented as an interface across which stresses are continuous but displacements are discontinuous (known as the linear slip model or displacement discontinuity model, the Murty, Schoenberg, and Pyrak-Nolte model). A fracture has both a normal and a shear fracture stiffness.

3 BOUNDARY CONDITIONS

3.1 Schoenberg and Pyrak-Nolte model

Using a model based empirically on static laboratory tests, Pyrak-Nolte et al. (1990) studied the implications of the continuity equations proposed by Schoenberg (1980):

$$[u_i] = \frac{1}{K_i}\tau_i, \qquad i = 1,2,3 \tag{1}$$

where $[u_i]$ is ith displacement discontinuity, $\tau_i = \sigma_{i3}$ is the stress component, and K_i is fracture stiffness. In order to relate the parameters K_i to the microstructure, Schoenberg & Douma (1988) use Hudson's (198 1) first-order results. In general K_i were taken to be real. The same model, in which K_i is replaced by $K_i + i\omega\eta_i$ (i = 1, 2, 3), was also studied (Myer et al. 1990).

3.2 Fehler model

Fehler (1982) took as a very simple fracture model of a thin plane layer (however he did not derive the following boundary conditions). If the ratio of the thickness d of the layer to the wavelength of radiation is very small, then we may approximate the physical conditions on the fracture by taking the stress components σ_{i3}, where the 3-component is normal to the fracture, to be constant throughout the layer. We then obtain the following boundary conditions (omitting the mathematical derivations):

$$[u_1] = \frac{d}{\mu' + i\omega\eta}\tau_1, \tag{2}$$

$$[u_2] = \frac{d}{\mu' + i\omega\eta}\tau_2, \tag{3}$$

and

$$[u_3] = \frac{d}{\kappa' + \frac{4}{3}\mu' + \frac{4}{3}i\omega\eta}\tau_3, \tag{4}$$

where κ' and μ' are respectively the bulk and shear moduli of the thin layer, η is the viscosity, and ω is the frequency.

3.3 Murty model.

One of the earliest attempts to characterize a 'loosely bonded interface was by Murty (1975, 1976) who envisaged a thin layer filled with viscous fluid. We obtain Murty's boundary condition by putting μ' = 0 in equations 2 and 3, that is, the fluid has zero rigidity. Murty also assumed that the bulk modulus of the fluid is sufficiently large that [u,] = 0 in place of equation (4). A similar model, in which Murty's viscous fluid is replaced by an inviscid elastic solid, was suggested by Yanovskaya & Dmitriyeva (1991). That is, in equations 2 and 3, η is set to 0 , again assuming $[u_3]$ = 0. It is clear that both these models are simplified forms of the Fehler model.

3.4 Hudson model

On the assumption that a fracture is a cluster of small cracks and a fault is a cluster of fractures (Fig. I), we propose models that assume that a fracture consists of areas of contact interspersed with areas of slip, or its inverse that a fracture can be modelled as a distribution of small (circular) cracks (voids). Hudson et al. (1994) calculated the condition which would hold for these two models. The discontinuity in $[u_i]$ in ith displacements u_i (i = 1, 2, 3) is give by:

$$[u_1] = \frac{\alpha}{\mu\omega}A\tau_1, \tag{5}$$

$$[u_2] = \frac{\alpha}{\mu\omega}A\tau_2, \tag{6}$$

and

$$[u_3] = \frac{\alpha}{\mu\omega}B\tau_3, \tag{7}$$

where α is the P-wave velocity in the coherent material (the same on both sides of the fracture plane), μ is the corresponding modulus of rigidity of the rock matrix. ***A*** and ***B*** are given by:

$$A = \frac{\gamma k_\alpha d\mu}{\left[\mu' + \frac{3\pi d\mu}{16a}(3-2\beta^2/\alpha^2)(1-\gamma^{3/2})\right]}, \qquad (8)$$

and

$$B = \frac{\gamma k_\alpha d\mu}{\left[\kappa' + \frac{4}{3}\mu' + \frac{3\pi d\mu}{4a}(1-\beta^2/\alpha^2)(1-\gamma^{3/2})\right]}, \qquad (9)$$

where $k_\alpha = \omega/\alpha$ is the ***P*** wavenumber, and γ is the slip ratio or percentage of fluid on the fracture surface defined as:

$$\gamma = \frac{\text{Open area}}{\text{Total area}} = \nu^s\,(\pi a^2), \qquad (10)$$

where a is crack radius, $d = (4/3)\,c$, c is half thickness of cracks. ν^s = N/S is the number density of cracks - the number of cracks N in area S, and κ' is bulk modulus and $\mu' = \mu_f + i\omega\eta$ (μ_f is elastic rigidity and η is viscosity) of crack-infill.

The continuity equations 5, 6, and 7 appear to be general and are accurate in describing non-welded interfaces provided that the wavelength is much larger than the dimensions of the asperities of the interface. Using simple algebra, the several models described above can be also written in the same form as equations 5, 6 and 7 with appropriate expressions for ***A*** and ***B***. In particular, the form of equations 8 and 9 is identical to that for a uniform thin layer, of thickness ***d***, containing material with shear modulus:

$$\mu' + \frac{3\pi d\mu}{16a}(3-2\beta^2/\alpha^2)(1-\gamma^{3/2}), \qquad (11)$$

d

and bulk modulus

$$\kappa' - \frac{\pi d\mu}{4a}\frac{\beta^2}{\alpha^2}(1-\gamma^{3/2}) \qquad (12)$$

except that the magnitudes of the displacement discontinuities are reduced by a factor γ, the relative area of slip. Thus the elastic rigidity of the crack infill is increased by the presence of the crack edges whereas, its bulk modulus is decreased. However, the resistance of the layer to both tension and shear is increased. If $\gamma \to 0$, both ***A*** and ***B*** $\to 0$, so that $[u_i] \to 0$ (i = 1, 2, 3), and the boundary degenerates to the

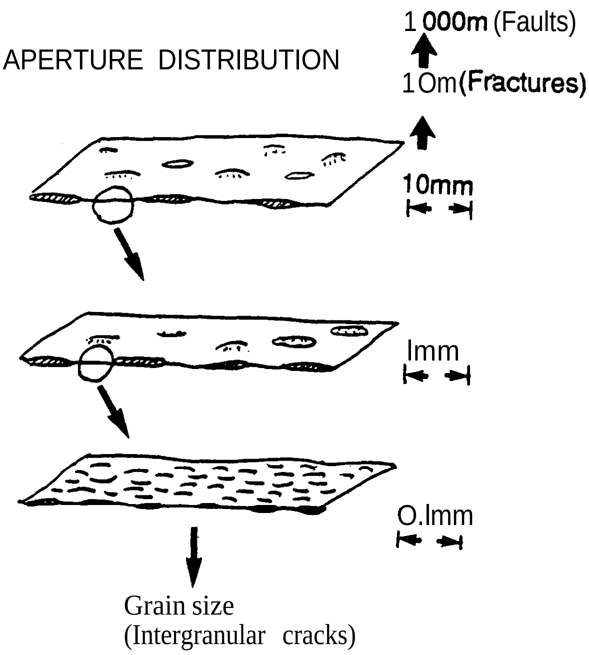

Figure 1. Schematic illustration of aperture distribution of a fracture surface at different scales.

conventional interface; that is, the displacements are continuous across the boundary (lower limit). If, on the other hand, $\gamma \to 1$, ***A*** and ***B*** become:

$$A = \frac{K_\alpha d\mu}{\mu'}, \qquad B = \frac{K_\alpha d\mu}{\kappa' + \frac{4}{3}\mu'}, \qquad (13)$$

and in this case, ***A*** and ***B*** are exactly reduced to those of the Fehler model; in other words, the Hudson model can be represented by a thin layer filled with fluid, and the equivalent thickness of the layer is ***d*** (upper limit), and bulk and shear moduli are given in equations 11 and 12. In general, the value of γ is between 0 and 1.

Fig. 2a shows the variation of parameter ***A*** with slip ratio γ for different values of the dimensionless wavenumber $k_\alpha a$. Fig. 2b shows the equivalent variation for parameter ***B***. Both figures are calculated for water-filled cracks. As expected, both ***A*** and ***B*** increase as γ and frequency increase, and the range of γ between (0, 1) covers the fracture surface from the conventional interface ($A = B = 0$) to a completely open fracture ($A \to \infty$). ***The*** larger the values of ***A*** and ***B***, ***the*** larger are the jumps in the displacements across the fractures. The values of ***B are*** usually much smaller than ***A*** for a water-filled fracture (of the order 1/10), and numerical calculations of reflection and transmission coefficients have indicated that in most

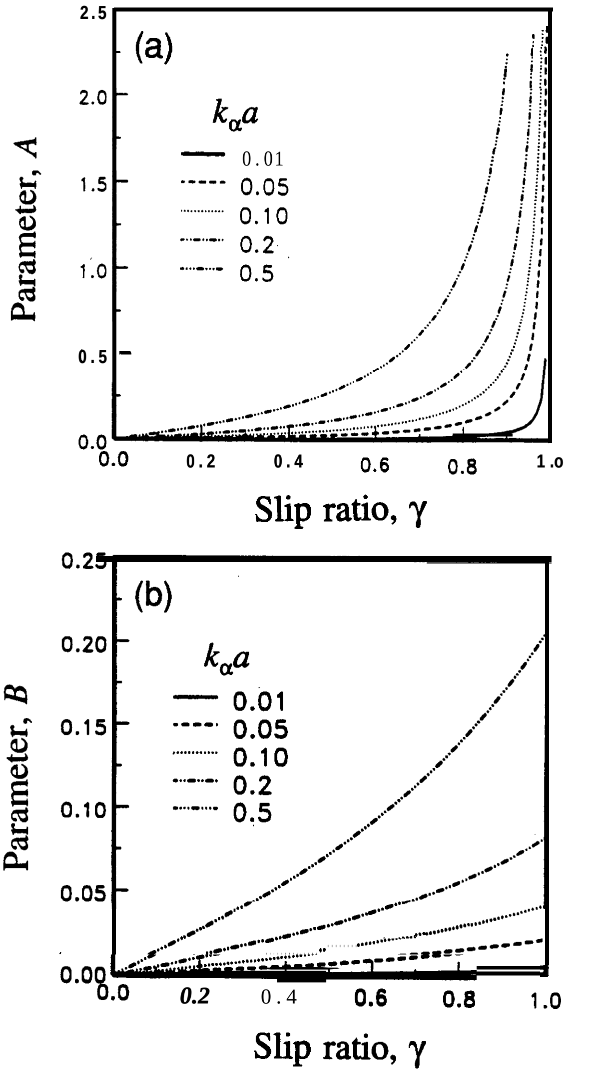

Figure 2. (a) Variation of dimensionless parameter, A, for different slip ratio γ for a water-filled fracture; (b) Similar to (a), but for parameter, ***B. The*** curves correspond to different values of wavenumber $k_\alpha a$.

cases B can be effectively regarded as zero [implying $[u_3] = 0$, as assumed by Murty (1975, 1976)].

4 VERIFICATION

In the above section, we have suggested that these various boundary condition can be reconciled to a single form, the Hudson model. The validity of the general fracture model can be verified by comparing the reflection and transmission (*R/T*) coefficients of seismic waves across single fractures. The ***R/T*** coefficients for the slip interface fracture were studied by Murty (1975), Schoenberg (1980), and Pyrak-Nolte et al. (1990), and for the parallel walled fracture by Fehler (1982). We have corrected some errors (some are typographic) in Fehler's paper and have re-

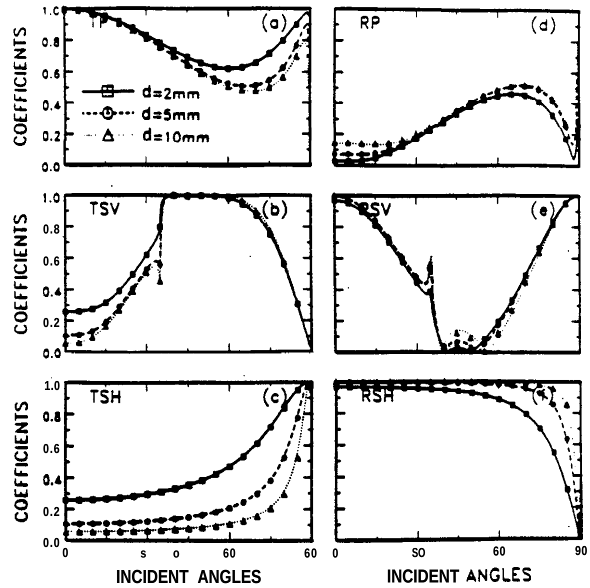

Figure 3. Variation of ***R/T*** coefficients of plane waves across a single fracture for different thickness of fracture calculated using the **fullwave** theory (lines) and the approximate boundary conditions (symbols). Fracture is filled with water ($\mu' = 0$).

derived his equations. The solution is written in a condensed 4x4 matrix form for SH-waves, and 8x8 matrix form for ***P-SV*** waves. We compare the ***R/T*** coefficients for the parallel-walled fractures using approximate boundary conditions (equations 2, 3 and 4) and the full-wave solution. The results are given in Fig. 3 and 4, which show respectively the variation of ***R/T*** coeffkients for different layer thickness (Fig. 3) and different shear moduli in the thin viscous layer (Fig. 4). There is a good agreement between the linear slip boundary conditions (symbols) and the full-wave theory (lines).

In fact, the validity of the continuity boundary conditions between the jump in displacements and traction can be more clearly demonstrated by calculating the ratio $\tau_i/[u_i]$ (i = 1, 2, 3) from the full-wave theory. For example, for SH-waves, we have:

$$\frac{\tau_2}{[u_2]} = \frac{\mu d T_{SH}}{R_{SH} - T_{SH} + e^{-2dh}} = K_2, \qquad (14)$$

where $h = d/2$, d is layer thickness, and T_{SH} and R_{SH} are the transmission and reflection coefficients of ***SH***-waves, respectively [which can be calculated using the method of Fehler (1982) or Rokhlin & Wang (199 1)]. Similar expressions for $\tau_1/[u_1]$ and $\tau_3/[u_3]$ can also be derived for ***P***- and SV-waves, but they are rather complicated, and will not be written here. Fig. **5a**

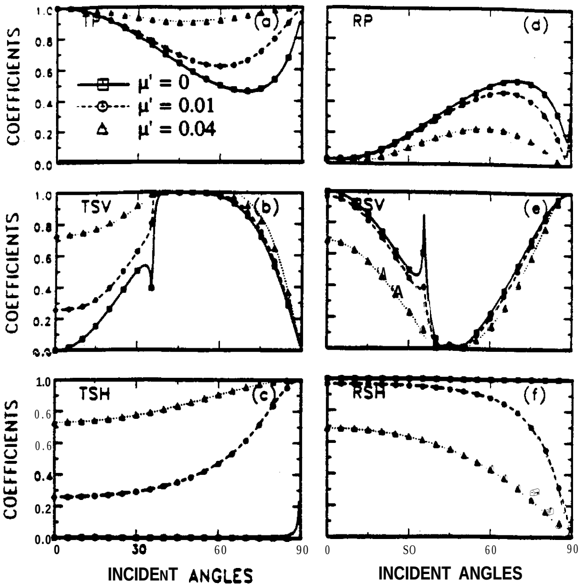

Figure 4. Similar to Figure 3, but showing the variation of *R/T* coefficients for different shear modulus μ' (x10^9 Pascal) of the fracture infill. Fracture thickness is d = 2mm.

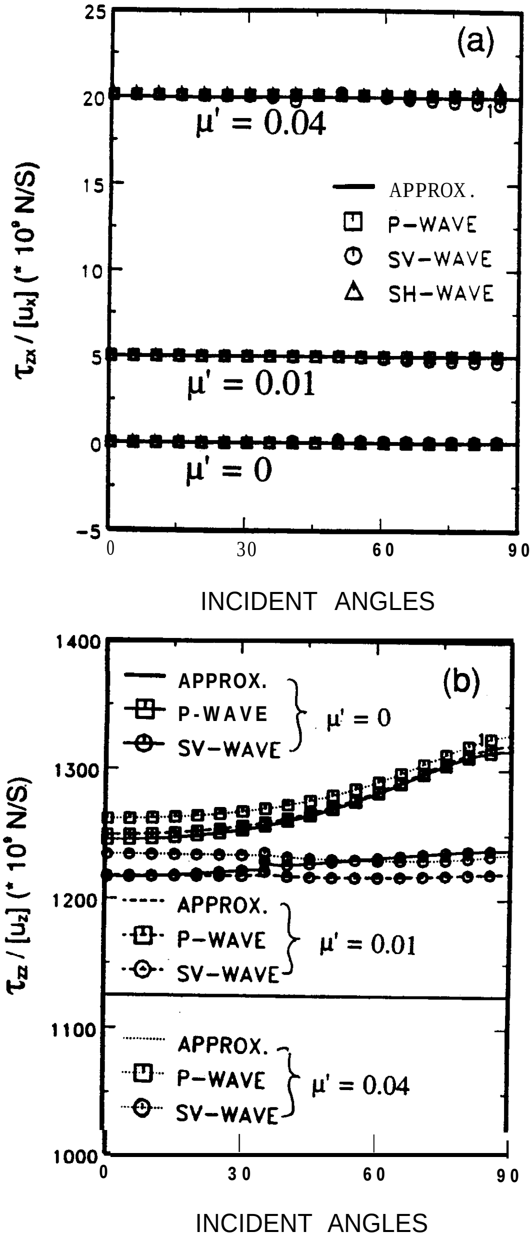

Figure 5. (a) Variation of $|\tau_1/[u_1]| = |\tau_2/[u_2]|$ against angles of incidence calculated from the exact solutions for different shear modulus μ' (x10^9 Pascal) of fracture infill; (b) Similar to (a), but for $|\tau_3/[u_3]|$. Solid lines are calculated using approximate boundary conditions. Symbols are numerical solutions.

shows the variation of moduli of $\tau_1/[u_1]$ and $\tau_2/[u_2]$ with angle of incidence for different values of shear modulus of the weak viscous layer (μ'). Similar plot for $\tau_3/[u_3]$ is given in Fig. 5b. They show that $\tau_1/[u_1]$ and $\tau_2/[u_2]$ are equal and constant, and almost exactly equal to those calculated using the approximate boundary conditions 2 and 3, implying that the linear relationship between the jump in displacement and traction holds. However, there is a slight difference in $\tau_3/[u_3]$ calculated using the two theories (Fig. 5b). We also find that $\tau_3/[u_3]$ is about 10 times larger than $\tau_1/[u_1]$ or $\tau_2/[u_2]$, which can be effectively considered to be infinite, this implies $[u_3] = 0$ (assumed in the Murty model). By comparison with the full-wave theory, we have proved there is indeed a linear relationship between displacement discontinuity and traction.

5 DISCUSSION AND CONCLUSIONS

We have introduced boundary conditions for parallel-wailed and rough surface fractures where the scale of roughness is small compared with a wavelength. For a parallel-walled fracture, the two fracture stiffnesses are related to the fracture thickness as well as to the viscosity, bulk and shear moduli of the fracture infill. Similarly, for a rough surface fracture, the fracture stiffnesses are functions of the distribution of voids (or contact areas) and the dimension of these voids. We have shown that the boundary conditions for the parallel-walled model and rough-surface model can be cast into a general form; i.e. a rough surface can be replaced by an equivalent parallel-walled fracture in which the thickness and elastic moduli of fracture infill are related to the distribution, dimension and infill parameters of voids.

We now have a consistent model of single fracture surfaces. The theory has important implications. Its physical concept is simple, and mathematically it is both simple and concise. More importantly, the model is based on a far more detailed analysis of fracture microstructures than most available models, and for the first time the boundary conditions on the fractures are directly related to the microstructure of the fracture plane. Our model is also useful for establishing the interrelationships among hydraulic, mechanical, and seismic properties of single fractures. These properties are linked by an essential parameter called slip ratio which is defined as the percentage area of fluids of the total fracture surface.

Finally, reflection and transmission coefficients are functions of both frequency and fracture properties, and elastic interface waves may propagate along a fracture. Jones & Whittier (1967); Murty (1975); and Pyrak-Nolte & Cook (1987) investigated the possibility of such interface waves. Pyrak-Nolte et al. (1992) observed these interface waves in the laboratory. Nihei et al. (1994) studied the propagation of *SH* guided-waves and suggested some possible applications of these interface waves for fracture characterization.

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