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The mechanical properties of materials with interconnected cracks and pores

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SUMMARY

This paper studies the effect on the overall properties of a cracked solid of the existence of connections between otherwise isolated cracks and of small-scale porosity within the 'solid' material. The intention is to provide effective medium models for the calculation of elastic wave propagation with wavelengths greater than the dimensions of the cracks. The method follows that of earlier papers in which the overall elastic properties are directly related to parameters governing the microstructure, such as crack number density and the mean radius and spacing distance of the cracks. Expressions derived by the method of smoothing are evaluated to second order in the number density of cracks, thereby incorporating crack-crack interactions through both the strain field in the solid and the flow field of fluids in the pores.

Flow of interstitial liquids tends to weaken the material; the limit of zero flow is equivalent to isolating the cracks and the limit of free flow is equivalent to dry (gas-filled) cracks. It also introduces additional attenuation. The inclusion of small-scale porosity gives a model of 'equant porosity' which is more closely constrained by the details of crack dynamics than earlier models.

Key words: cracks, porosity, seismic waves.

1 INTRODUCTION

The overall or mean properties of materials with isolated cracks are now well-established in a series of papers (beginning with Hudson 1980), based on the method of smoothing (Keller 1964) which takes into account interactions between cracks and is generally accurate for values of the crack number density ν less than about $(0.1)/a^3$, where a is the mean radius of the cracks (Crampin 1984). Aligned cracks at such number densities give rise to differential shear-wave velocity anisotropy up to about 10 per cent. This value represents such pervasive cracking that any natural solid such as rock *in situ* would be so weakened as to be close to desegregation (Crampin 1994). This means that the theoretical results appear to apply to the full range of cracked rocks in the Earth's crust up to the limit of complete failure.

Various interior conditions may be applied to the cracks, such as dry (gas-filled), liquid-filled, and a weak solid fill (Hudson 1981). (It is conventional to use the description 'dry' or 'empty' for a crack which is filled with an inviscid gas whose compressibility is such that pressure variations within the crack can be ignored. We shall also refer to such cracks as 'gas-filled'.) A further model with a partial gas/liquid-fill (Hudson 1988) allows liquid to flow from one part of a cavity into another when the crack is distorted by the stress field of an incoming wave-the 'squirt' mechanism of Mavko & Nur (1979). Up to now, however, the cracks have been assumed to be isolated from each other and no transfer of fluid or fluid pressure has been envisaged.

At the other extreme, Biot theory (Biot 1956) allows for complete connections throughout the solid by means of a network of pores. This method has the disadvantage that it is difficult to relate the parameters of the macroscopic behaviour to the details of the microstructure, in spite of more recent efforts to relate the two (see, for example, Burridge & Keller 1981). The difficulty lies in the complexity of the appropriate calculations. A combination of the squirt and the Biot models (the BISQ model) has been developed by Dvorkin & Nur (1993).

The method used in this paper is similar to that of O'Connell & Budiansky (1977), who modified the formulae for isolated cracks to allow for interconnections. Their interest was in static deformation and they dealt with three models. The first is the limit where connections are so inefficient that flow between cracks can be ignored (liquid-filled isolated cracks). The second is the opposite limit, in which the cracks are completely drained. This is equivalent to the case of isolated empty (gas-filled) cracks, and both these models are dealt with by a theory based on isolated cracks. The last model is where the cracks are undrained but the internal pressures equilibrate to a single value. We shall be dealing with the corresponding dynamic case where the pores allow for pressure relaxation up to a limit of distance imposed by the frequency of the wave. As such, this study is equivalent to those of Mavko & Jizba (1991) and Mukerji & Mavko (1994), who dealt with the frequency-dependent effects of local relaxation of fluid pressure in dynamic situations but did not use a specific model for the pore shapes.

The concept of interconnections between cracks is based on a model in which the material contains a random distribution of isolated cracks which are connected by fluid pathways that are mechanically invisible; that is, they have no effect on the overall properties of the material if the fluid is a compressible inviscid gas. In general, rocks are fractured on a whole range of scales. Large fractures will scatter and diffract the incoming waves. In the present model we assume that cracks of this size are absent and that only micro-fracturing exists, on scale-lengths small compared with a seismic wavelength. The effect of isolated microcracks depends essentially on the parameter va^3, where v is the number density and a is the crack radius. In our model we are assuming that interconnections between the cracks and pores exist with values of (va^3) so small that they have a negligible effect on a gas-filled (dry) system, but which modify the internal pressure when the structure is saturated with liquid. We do not specify the shape or size of these interconnections, but it is natural to assume that they are even smaller in scale than the so-called microstructure; that is, the distribution of cracks that they link together. They may form isolated pathways or they may be present as a network of pores. If the latter, they will affect the properties of the material as 'equant porosity' (Thomsen 1986) even if they do not form effective connections between cracks.

2 OVERALL PROPERTIES OF CRACKED MATERIALS

Expressions for the overall parameters accurate to second order in the number density of cracks have been given in a general form by Hudson (1986); they are

$$\boldsymbol{c} = \boldsymbol{c}^0 + \boldsymbol{c}^1 + \boldsymbol{c}^2, \tag{1}$$

where $\boldsymbol{c}^0$ are the elastic parameters of the matrix (uncracked) material, $\boldsymbol{c}^1$ are the first-order corrections for the effect of the embedded cracks and $\boldsymbol{c}^2$ are the second-order terms. If the cracks are aligned with normals $\boldsymbol{q}$,

$$c^1_{ijkl} = -\frac{(va^3)}{\mu}\ell_{sm}q_r c^0_{srij}\ell_{tu}q_v c^0_{tvkl}\bar{U}_{mu}, \tag{2}$$

where v is number density of cracks, a the mean radius, μ the shear modulus of the matrix, and $\{\bar{U}_{mu}\} = \mathrm{diag}\{\bar{U}_{11}, \bar{U}_{11}, \bar{U}_{33}\}$, where $\bar{U}_{11}$ and $\bar{U}_{33}$ are generated by the response of a circular crack width normal in the 3-direction to shear stress and normal stress respectively; $\{\ell_{ij}\}$ is the rotation matrix which takes the 3-axis into the direction $\boldsymbol{q}$ ($\ell_{i3} = q_i$, $i = 1, 2, 3$). The second-order term is given by

$$c^2_{ijkl} = \frac{1}{\mu}c^1_{ijrs}\chi_{rstu}c^1_{tukl}, \tag{3}$$

where

$$\chi_{ijkl} = \{\delta_{ik}\delta_{jl}(4+\beta^2/\alpha^2) - (\delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk})(1-\beta^2/\alpha^2)\}/15,$$

a and β being the wave speed in the uncracked matrix.

Expressions for $\bar{U}_{11}$ and $\bar{U}_{33}$ are given by Hudson (198 1) for gas- and liquid-filled cracks and by Hudson (19.88) for partially saturated cracks. All these formulae apply only to isolated cracks.

The existence of connections between cracks, or of an equant porosity, changes the response of each crack to an imposed stress. That is, the matrix $\{\bar{U}_{ij}\}$ above will be changed. In all other respects, the formulae for the overall properties of the cracked solid remain the same. In what follows, therefore, we develop new formulae for the $\bar{U}_{ij}$ for the cases where pressure in the liquid within a crack is relaxed owing to flow into other cracks or into the surrounding porous matrix.

3 CONSERVATION OF FLUID MASS

We consider a solid material with cracks and pores that are interconnected by a network of fine channels or perhaps by random pathways along the unwelded contact of the rough faces of a fault plane. The pores, cracks and pathways are filled with a uniform fluid (gas or liquid). It is assumed that the structure is homogeneous on a scale l which is large compared with the scale a of the pore or crack geometry. On the scale l, we may define an average fluid density ρ_f and mass flux $\rho_f \boldsymbol{w}$. Let ϕ be the porosity of the material-the ratio of fluid volume to total volume-then $\rho_f\phi$ is the mass of fluid per unit volume.

Conservation of fluid mass in an arbitrary volume V may be stated by

$$\frac{d}{dt}\left(\int_V \rho_f\phi\, dV\right) = -\int_S \rho_f \boldsymbol{w}\cdot\boldsymbol{n}\, dS, \tag{4}$$

where S is the surface of V and $\boldsymbol{n}$ the outward normal. Equivalently, this may be stated by

$$\frac{\partial}{\partial t}(\rho_f\phi) = -\,\mathrm{div}(\rho_f \boldsymbol{w}). \tag{5}$$

If $\rho_f \boldsymbol{w}$ is the average mass flux, it is reasonable to assume that w is the volume flux. This, in turn, may be expected to follow D'Arcy's law:

$$\boldsymbol{w} = -D_r\,\mathrm{grad}\,p_f, \tag{6}$$

where D_r is a diffusivity parameter depending on the geometry of the connecting channels and the viscosity of the fluid, and p_f is the average pressure in the fluid. Since the pathways deform under pressure, we may expect D_r to depend also on p_f. Eq. (5) now becomes

$$\frac{\partial}{\partial t}(\rho_f\phi) = \mathrm{div}(\rho_f D_r\,\mathrm{grad}\,p_f). \tag{7}$$

We assume to begin with that the cracks and pores are similarly shaped and orientated so that the pressure in near neighbours is the same and remains the same even after the imposition of external stress; the local pressure is thus equal to the average pressure p_f.

The mass of fluid in a unit volume of material is $\rho_f\phi$, and so the volume of the fluid in a unit volume of material in its unstressed state is $\rho_f\phi/\rho_0$, where ρ_0 is the density of the unstressed fluid. The relative increase of volume of the fluid above its volume in the unstressed state is therefore $(\rho_0/\rho_f - 1)$. This is related to the fluid pressure by

$$\frac{\rho_0}{\rho_f} - 1 = -p_f/\kappa_f, \tag{8}$$

where κ_f is the bulk modulus of the fluid. It follows, of course, from this equation applied to each crack, that the fluid density is also the same in adjacent cracks and equal to the average density ρ_f. Substituting for ρ_f in eq. (7), we get

$$\frac{\partial}{\partial t}\rho_0\phi(1-p_f/\kappa_f)^{-1} = \mathrm{div}\{\rho_0 D_r(1-p_f/\kappa_f)^{-1}\,\mathrm{grad}\,p_f\}. \tag{9}$$

4 EFFECT OF AN IMPOSED STRESS

If a static stress field $\boldsymbol{\sigma}^0$ is imposed on a volume of scale size 1 or greater, the pressure p_f in the fluid changes, due both to a change in porosity and to fluid flow. Suppose that, in the absence of the fluid and with a stress-free condition on the surface of the pores, the porosity ϕ changes according to

$$\begin{aligned}\phi &= \phi(\boldsymbol{\sigma}^0)\\ &= \phi^0 + \boldsymbol{\phi}^1\boldsymbol{\sigma}^0\end{aligned} \tag{10}$$

approximately (to first order) for small imposed stress, where ϕ^0 and $\boldsymbol{\phi}^1$ are material constants.

If fluid is present, the surface condition on the pores is that the traction should be $-p_f\boldsymbol{n}$, where n is normal to the surface. We subtract a uniform field $\boldsymbol{\sigma} = -p_f\boldsymbol{I}$ from the stress field in the solid material, leaving a structure with imposed stress $\boldsymbol{\sigma}^0 + p_f\boldsymbol{I}$ and zero traction on the surfaces of the pores, with a resultant porosity that can be calculated from eq. (10). The porosity due to the superposition of these two fields is

$$\phi + \phi^0 p_f/\kappa = \phi^0 + \boldsymbol{\phi}^1(\boldsymbol{\sigma}^0 + p_f\boldsymbol{I}) \tag{11}$$

to first order, where κ is the bulk modulus of the solid $(\kappa = \lambda + 2\mu/3)$.

On the assumption that wavelengths are long compared with the scale-length 1, we may apply the static result given in eq. (11) to dynamic situations, where $\boldsymbol{\sigma}^0$ varies with time, corresponding to the passage of a seismic wave. So we now substitute for ϕ from eq. (11) in eq. (7) to obtain a differential equation satisfied by p_f:

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\phi^0 + \boldsymbol{\phi}^1\boldsymbol{\sigma}^0 + p_f\left(\phi^1_{kk} - \frac{\phi^0}{\kappa}\right)\right\}(1-p_f/\kappa_f)^{-1}\\ &\quad = \mathrm{div}\,\boldsymbol{D}_r\{(1-p_f/\kappa_f)^{-1}\,\mathrm{grad}\,p_f\}.\end{aligned} \tag{12}$$

To first order in p_f/κ and $\boldsymbol{\phi}^1\boldsymbol{\sigma}^0/\phi_0$ we have

$$\boldsymbol{\phi}^1\frac{\partial\boldsymbol{\sigma}^0}{\partial t} + \frac{\partial p_f}{\partial t}\left(\phi^1_{kk} + \frac{\phi^0}{\kappa_f} - \frac{\phi^0}{\kappa}\right) = D_r\nabla^2 p_f, \tag{13}$$

and, if the frequency of the seismic wave is ω and the wavenumber k, we obtain a relation between p_f and $\boldsymbol{\sigma}^0$:

$$p_f\left\{\phi^1_{jj} + \frac{\phi^0}{\kappa_f} - \frac{\phi^0}{\kappa} - \frac{ik^2}{\omega}D_r\right\} = -\boldsymbol{\phi}^1\boldsymbol{\sigma}^0. \tag{14}$$

So pressure variations will be the same as for the isolated pores—that is, flow between pores may be ignored—if

$$\frac{\kappa_f D_r k^2}{\omega\phi^0} \ll 1. \tag{15}$$

Alternatively, pressure variations may be ignored if

$$\frac{\kappa_f D_r k^2}{\omega\phi^0} \gg 1. \tag{16}$$

In both cases, the appropriate expressions of Hudson (1980, 1981) for isolated cracks apply.

5 CALCULATION OF OVERALL POROSITY FOR MIXED POPULATIONS OF CRACKS

If, as assumed above, all individual crack and pore spaces have the same shape and orientation, the quantities ϕ^0 and $\boldsymbol{\phi}^1$, which appear in eq. (7) and which govern changes in porosity ϕ, may be calculated for a single crack and will be the same for all. Similarly, the fluid pressure will be the same in neighbouring pores or cracks and will be equal to the average fluid pressure p_f, even in a dynamic experiment. If, however, pore shapes are different and the pores are orientated differently, the changes in porosity as a result of stress will be different, and will need to be averaged to obtain overall porosity, and fluid pressure would vary between cracks if the cracks were isolated. Interconnections between nearby cracks will tend to equalize pressure to the average value p_f as in the static situation. If the time-scale for pressure equalization is short compared with the seismic wave period, the static result may be used. If, on the other hand, the interconnections are constricted, this time-scale may be of the same order as, or longer than, the period of seismic waves.

In such a case (and now we abandon the assumption of similar pores), we designate the fluid pressure in the nth crack, or nth family of similarly shaped or orientated cracks, by p^n_f. The corresponding porosity is by virtue of eq. (11) applied to the nth family of cracks only:

$$\phi_n = \phi^0_n + \boldsymbol{\phi}^1_n(\boldsymbol{\sigma}^0 + p^n_f\boldsymbol{I}) - \phi^0_n p^n_f/\kappa, \qquad n = 1, 2, 3, \ldots, \tag{17}$$

where ϕ^0_n and $\boldsymbol{\phi}^1_n$ may, if the cracks are sufficiently well-spaced, be calculated for isolated single cracks.

Multiplying by ρ^n_f, the fluid density in the nth crack set, and summing over n, we obtain the total mass concentration of fluid:

$$\begin{aligned}m_f = \sum_n \phi_n\rho^n_f &= m^0 + \boldsymbol{m}^1\boldsymbol{\sigma}^0 + \sum_n \rho^n_f p^n_f(\phi^1_n)_{jj}\\ &\quad - \frac{1}{\kappa}\sum_n \rho^n_f p^n_f \phi^0_n,\end{aligned} \tag{18}$$

where

$$m^0 = \sum_n \phi^0_n\rho^n_f, \qquad \boldsymbol{m}^1 = \sum_n \boldsymbol{\phi}^1_n\rho^n_f.$$

We need to evaluate the sums over n on the right-hand side of eq. (18) in terms of the average pressure p_f and parameters relating to the geometry of the pore space.

The mass flow out of the nth family of cracks is driven by the pressure difference between p^n_f and the fluid pressure in neighbouring cracks, and we assume that this may be approximated by

$$\frac{\partial}{\partial t}(\rho^n_f\phi_n) = -\frac{\phi^0_n\rho_0}{\kappa_f\tau}(p^n_f - p_f), \tag{19}$$

where τ is a time-scale (relaxation) parameter. The fluid density is related to the pressure by the equivalent of eq. (8):

$$\frac{\rho_0}{\rho_f^n} - 1 = -p_f^n/\kappa_f. \tag{20}$$

Substituting from eqs (17) and (20) in eq. (19), we obtain

$$\frac{\partial}{\partial t} p_f^n \left\{\frac{(\phi_n^1)_{jj}}{\phi_n^0} - \frac{1}{\kappa} + \frac{1}{\kappa_f}\right\} + \frac{\partial}{\partial t}\frac{\phi_n^1\sigma^0}{\phi_n^0} = -\frac{1}{\kappa_f\tau}(p_f^n - p_f) \tag{21}$$

to first order in p_f^n/κ_f and $(\phi_n/\phi_n^0 - 1)$. Writing

$$\gamma_n = 1 - \frac{\kappa_f}{\kappa} + \frac{\kappa_f(\phi_n^1)_{jj}}{\phi_n^0}, \tag{22}$$

we have

$$\gamma_n\tau\frac{\partial p_f^n}{\partial t} + p_f^n = p_f - \kappa_f\tau\frac{\partial}{\partial t}\frac{\phi_n^1\sigma^0}{\phi_n^0}. \tag{23}$$

If the propagating wave has frequency ω, eq. (23) shows that

$$p_f^n = \left(p_f - i\omega\tau\kappa_f\frac{\phi_n^1\sigma^0}{\phi_n^0}\right)\Bigg/(1 + i\omega\gamma_n\tau). \tag{24}$$

The time-scale for pressure equalization is $\gamma_n\tau$, and if this is short compared with the period of the wave, i.e.

$$\omega\gamma_n\tau \ll 1, \tag{25}$$

the local pressure p_f^n is equal to the average pressure p_f, as noted above. Otherwise, we need to use eq. (24) for p_f^n, and eq. (20) for ρ_f^n:

$$\rho_f^n = \rho_0(1 - p_f^n/\kappa_f)^{-1}. \tag{26}$$

Overall conservation of mass now takes the form (instead of eq. 5)

$$\frac{\partial m_f}{\partial t} = -\operatorname{div}(\rho_f \boldsymbol{w}), \tag{27}$$

and this in turn, by virtue of D'Arcy's law (eq. 6), becomes

$$\frac{\partial m_f}{\partial t} = \operatorname{div}(\rho_f D_r \operatorname{grad} p_f). \tag{28}$$

We obtain the relationship between p_f and σ^0, corresponding to eq. (14) and to the same accuracy, by substitution for m_f from eq. (18):

$$m_f = \sum_n \phi_n^0\rho_0 + \sum_n \phi_n^1\sigma^0\rho_0 + \frac{\rho_0}{\kappa_f}\sum_n \phi_n^0 p_f^n + \rho_0\sum_n(\phi_n^1)_{jj}p_f^n - \frac{\rho_0}{\kappa}\sum_n p_f^n\phi_n^0. \tag{29}$$

Finally, substituting for p_f^n in terms of p_f and σ^0 from eq. (24), we get

$$\frac{m_f}{\rho_0} = \phi^0 + \frac{p_f}{\kappa_f}\left\{\left(1 - \frac{\kappa_f}{\kappa}\right)\sum_n\frac{\phi_n^0}{1 + i\omega\gamma_n\tau} + \kappa_f\sum_n\frac{(\phi_n^1)_{jj}}{1 + i\omega\gamma_n\tau}\right\} + \left(\sum_n\frac{\phi_n^1}{1 + i\omega\gamma_n\tau}\right)\sigma^0, \tag{30}$$

and eq. (28) gives

$$\frac{p_f}{k_f}\left\{\left(1 - \frac{\kappa_f}{\kappa}\right)\sum_n\frac{\phi_n^0}{1 + i\omega\gamma_n\tau} + \kappa_f\sum_n\frac{(\phi_n^1)_{jj}}{1 + i\omega\gamma_n\tau} - \frac{ik^2}{\omega}D_r\kappa_f\right\} = -\left(\sum_n\frac{\phi_n^1}{1 + i\omega\gamma_n\tau}\right)\sigma^0 \tag{31}$$

for motion with frequency ω and wavenumber k. Once again we see that, if $\omega\gamma_n\tau \ll 1$ for all n, we return to the earlier model, where pressure does not vary significantly between the pores, represented by eq. (14).

6 EFFECTIVE ELASTIC PARAMETERS OF MATERIAL WITH INTERCONNECTED PARALLEL CRACKS

If we assume that the imperfections in the material consist of flat elliptical cracks, and that these are well-spaced, we may approximate the parameter ϕ^1 by using the result for a single crack. The relative increase in volume of any empty crack under the imposed stress σ^0 is given by

$$\frac{\delta V}{V} = \frac{\phi - \phi^0}{\phi^0} = \frac{\phi^1\sigma^0}{\phi^0} = \frac{1}{V_c}\int_{V_c}\operatorname{div}\boldsymbol{u}\,dV, \tag{32}$$

where V_c is the volume of the crack, and $\boldsymbol{u}$ the displacements.

Integrating over the thickness of the crack, we obtain

$$\frac{\delta V}{V} = \frac{1}{V_c}\int_{S_c}[u_3]\,dS \tag{33}$$

if the aspect ratio of the crack is small, where $[u_3]$ indicates the discontinuity across the crack face S_c, and the crack normal is assumed to lie in the x_3-direction.

Thus,

$$\frac{\phi^1\sigma^0}{\phi^0} = \frac{1}{V_c}\int_{S_c}[u_3]\,dS = \frac{3a}{4\pi\mu c}\bar{U}_{33}^d\sigma_{33}^0, \tag{34}$$

where a is the crack radius, c/a its aspect ratio, and $\bar{U}_{33}^d$ is given for a dry crack by Hudson (1981):

$$\bar{U}_{33}^d = \frac{4}{3}\left(\frac{\lambda + 2\mu}{\lambda + \mu}\right), \tag{35}$$

λ and μ being the Lamé parameters of the solid material. So ϕ^1 has only one non-zero component, and that is

$$\phi_{33}^1 = \frac{\phi^0 a}{\pi c}\frac{\lambda + 2\mu}{\mu(\lambda + \mu)}. \tag{36}$$

When the crack is filled with fluid at pressure p_f, then the relative volume change is, on the assumption that all cracks are similarly orientated,

$$\frac{\delta V}{V} = \frac{\phi - \phi^0}{\phi^0} = \frac{\phi^1\sigma^0}{\phi^0} + p_f\left\{\frac{(\phi^1)_{kk}}{\phi^0} - \frac{1}{\kappa}\right\} = \frac{3a}{4\pi\mu c}\bar{U}_{33}\sigma_{33}^0 \tag{37}$$

(eq. 11), where $\bar{U}_{33}$ is the crack-opening parameter needed for the calculation of the overall parameters of the material (Hudson 1981).

Thus,

$$\frac{3a}{4\pi\mu c}\bar{U}_{33}\sigma_{33}^0 = \frac{a}{\pi c}\frac{(\lambda + 2\mu)}{\mu(\lambda + \mu)}\sigma_{33}^0 + p_f\left\{\frac{a}{\pi c}\frac{(\lambda + 2\mu)}{\mu(\lambda + \mu)} - \frac{1}{\kappa}\right\} \tag{38}$$

and, in order to obtain the appropriate value of $\bar{U}_{33}$, we substitute for p_f in terms of σ^0 from eq. (14):

$$p_f = -\frac{a}{\pi c}\frac{(\lambda+2\mu)}{\mu(\lambda+\mu)}\sigma^0_{33}\left\{\frac{a}{\pi c}\frac{(\lambda+2\mu)}{\mu(\lambda+\mu)}+\frac{1}{\kappa_f}-\frac{1}{\kappa}-\frac{ik^2}{\omega}\frac{D_r}{\phi^0}\right\}^{-1}. \tag{39}$$

Using

$$\phi^0 = v\tfrac{4}{3}\pi a^2 c, \tag{40}$$

where v is the number density of cracks, we then have

$$\bar{U}_{33} = \frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\left(1-\frac{3i\kappa_f k^2 D_r}{4\pi v a^2 c\omega}\right)$$

$$\times\left\{1-\frac{\kappa_f}{\kappa}+\frac{a}{\pi c}\frac{\kappa_f}{\mu}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)-\frac{3i\kappa_f k^2 D_r}{4\pi v a c\omega}\right\}^{-1}. \tag{41}$$

We see once again that the parameter governing fluid flow between cracks is

$$\frac{3\kappa_f k^2 D_r}{4\pi v a^2 c\omega} \tag{42}$$

(see eqs 15, 16). If this is small, then the cracks may be regarded as being isolated for the purpose of measuring overall parameters. In which case, eq. (41) becomes that given by Hudson (1981) for cracks filled with weak material since κ_f/κ is clearly small and may be neglected compared with the following term because c/a is small. If the expression (42) is not small, eq. (41) may be written as

$$\bar{U}_{33} = \frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\Big/(1+K) \tag{41a}$$

(following Hudson 1981), where

$$K = \left\{\frac{a}{\pi c}\frac{\kappa_f}{\mu}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\right\}\left\{1-\frac{3i\kappa_f k^2 D_r}{4\pi v a^2 c\omega}\right\}^{-1}. \tag{41b}$$

The diffusion of pressure between cracks is equivalent to the presence of damping in compression of the material within the crack. The effects are similar to those due to the presence of isolated, but partially saturated, cracks (Hudson 1988), a not unexpected result.

The second crack parameter, $\bar{U}_{11}$, which appears in the formulae for overall elastic parameters (Hudson 1981) is unaffected by interconnections between pores, since shear stress applied to a crack face does not give rise to a volume change.

7 EFFECTIVE ELASTIC PARAMETERS OF MATERIALS WITH NON-PARALLEL CRACKS

If we divide the population of cracks into families of parallel cracks numbered by $n = 1, 2, \ldots$ as before, we need to calculate ϕ^1_n/ϕ^0_n for each family. Let the normal to the nth family be $\boldsymbol{q}^n$; then eq. (36) gives

$$\frac{(\phi^1_n)_{ij}}{\phi^0_n} = \frac{a}{\pi c}\frac{(\lambda+2\mu)}{\mu(\lambda+\mu)}q^n_i q^n_j. \tag{43}$$

The parameter γ_n, given by eq. (22), is

$$\gamma_n = 1-\frac{\kappa_f}{\kappa}+\frac{a}{\pi c}\frac{\kappa_f}{\mu}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right) \tag{44}$$

and is therefore the same for all n and we drop the suffix.

The relation between the average pressure p_f and the imposed stress σ^0 is, from eq. (31),

$$p_f\left\{\gamma\phi^0-\frac{ik^2 D_r}{\omega}\kappa_f(1+i\omega\gamma\tau)\right\}$$

$$= -\frac{a}{\pi c}\frac{\kappa_f}{\mu}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\sum_n \phi^0_n q^n_i q^n_j \sigma^0_{ij}, \tag{45}$$

where, once again

$$\phi^0 = \sum_n \phi^0_n;$$

ϕ^0 is the total porosity.

The pressure in the nth set of cracks is given by eq. (21):

$$p^n_f = -\frac{a}{\pi c}\frac{\kappa_f}{\mu}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\frac{\sigma^0_{ij}}{(1+i\omega\gamma\tau)}$$

$$\times\left\{i\omega\tau q^n_i q^n_j+\sum\phi^0_n q^n_i q^n_j\left(\gamma\phi^0-\frac{ik^2 D_r\kappa_f}{\omega}(1+i\omega\gamma\tau)\right)^{-1}\right\}. \tag{46}$$

The corresponding relative change in porosity is (eq. 17)

$$\frac{\phi_n-\phi^0_n}{\phi^0_n} = \frac{\phi^1_n\sigma^0}{\phi^0_n}+p^n_f\left\{\frac{a}{\pi c\mu}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)-\frac{1}{\kappa}\right\}$$

$$= \frac{a}{\pi c\mu}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\left\{q^n_i q^n_j(1+i\omega\tau)\right.$$

$$\left.-\frac{(\gamma-1)\sum\phi^0_n q^n_i q^n_j}{\{\gamma\phi^0-(ik^2 D_r\kappa_f/\omega)(1+i\omega\gamma\tau)\}}\right\}\frac{\sigma^0_{ij}}{1+i\omega\gamma\tau}. \tag{47}$$

This, in turn, is equal to

$$\frac{3}{4\pi}\frac{a}{\mu c}\bar{N}^n_{ij}\sigma^0_{ij}, \tag{48}$$

where $\{\bar{N}^n_{ij}\}$ are the crack-opening parameters for the nth set of cracks; that is, if $[u_n]$ is the discontinuity in normal displacement across the crack, then

$$\bar{N}^n_{ij}\sigma^0_{ij} = \frac{\mu}{a^3}\int_{S_c}[u_n]\,dS. \tag{49}$$

The form of the expression on the left, in which $\bar{N}^n_{ij}$ is independent of σ^0, is due to the first-order nature of the solution. We now have the following expression for $\bar{N}^n_{ij}$:

$$\bar{N}^n_{ij} = \frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)(1+i\omega\gamma\tau)^{-1}$$

$$\times\left\{q^n_i q^n_j(1+i\omega\tau)-\frac{(\gamma-1)\sum\phi^0_n q^n_i q^n_j}{\{\gamma\phi^0-(ik^2 D_r\kappa_f/\omega)(1+i\omega\gamma\tau)\}}\right\}. \tag{50}$$

The introduction of the $\bar{N}^n_{ij}$ implies an increase in complexity not covered by standard formulae for overall elastic parameters of materials with cracks. A flat crack with normal in the x,-direction is affected by the imposed tractions in the l-2 plane only; that is, by σ^0_{3j}, $j = 1, 2, 3$. If the cracks are non-parallel and connected, however, any stress component may create a crack opening on one set of cracks which, by the communication of pressure, affects all neighbouring cracks whatever their orientation.

In order to find expressions for the overall parameters of the material, we calculate $\boldsymbol{c}^1$, the variation of the elastic parameters from those of the matrix material, to first order in number density, for each set of cracks separately and then add them in order to obtain the response to all sets simultaneously. The second order term, c^2, may be found directly from c^1. We have, for isolated cracks (Hudson 1986),

$$c^1_{ipjq} = -\sum \frac{v^n(a^n)^3}{\mu} \ell^n_{km} q^n_r c^0_{krip} \ell^n_{ul} q^n_s c^0_{usjq} \bar{U}^n_{ml}, \tag{51}$$

where the $\bar{U}^n_{ml}$ correspond to discontinuities across a crack with normal in the x,-direction; the only non-zero components are $\bar{U}^n_{11} = \bar{U}^n_{22}$ and $\bar{U}^n_{33}$. The sum is over differently orientated sets of cracks with normals $\boldsymbol{q}^n$, while ℓ^n is the rotation matrix from the background axes to axes fixed in the crack and with the 3-axis along the normal:

For connected sets of cracks, we must replace eq. (51) by

$$c^1_{ipjq} = -\sum_n \frac{v^n(a^n)^3}{\mu} \ell^n_{km} q^n_r c^0_{krip} \ell^n_{ul} \ell^n_{st} c^0_{usjq} \bar{U}^n_{mlt}, \tag{52}$$

where $\bar{U}^n_{mlt}$ relates the discontinuity in the x,-direction across a crack to the lt component of the applied stress; the normal to this crack is in the x_3-direction and so discontinuities in the x_1- and x,-directions are unaffected by crack interconnections:

$$\bar{U}^n_{1lt} = \bar{U}^n_{11}\delta_{l1}\delta_{t3}$$
$$\bar{U}^n_{2lt} = \bar{U}^n_{11}\delta_{l2}\delta_{t3}, \tag{53}$$

where $\bar{U}^n_{11}$ is unchanged from the expression given by Hudson (1981). On the other hand,

$$\bar{U}^n_{3lt} = \bar{N}^n_{lt}. \tag{54}$$

We write

$$\bar{U}^n_{mlt} = \delta_{t3}\bar{U}^n_{ml} + \delta_{m3}(\bar{N}^n_{lt} - \delta_{t3}\bar{U}^n_{l3}), \tag{55}$$

where $\bar{N}^n_{lt}$ is calculated for a crack with normal $\boldsymbol{q}$ in the x,-direction:

$$\bar{N}^n_{lt} = \frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)(1+i\omega\gamma\tau)^{-1} \times \left\{\delta_{l3}\delta_{t3}(1+i\omega\tau) - \frac{(\gamma-1)\sum^n \phi^0_n \ell^n_{il}\ell^n_{jt} q^n_i q^n_j}{\{\gamma\phi^0 - (ik^2 D_r \kappa_f/\omega)(1+i\omega\gamma\tau)\}}\right\}. \tag{56}$$

We choose $\bar{U}_{33}$ to be

$$\bar{U}_{33} = \frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\left(\frac{1+i\omega\tau}{1+i\omega\gamma\tau}\right) \tag{57}$$

and

$$\{\bar{U}_{ij}\} = \mathrm{diag}\{\bar{U}_{11}, \bar{U}_{11}, \bar{U}_{33}\}, \tag{58}$$

where $\bar{U}_{11}$ is calculated for a dry crack (Hudson 1981) and $\bar{U}_{33}$ comes from eq. (57). Then

$$\bar{N}'_{lt} = \bar{N}_{lt} - \delta_{t3}\bar{U}_{l3} = -\frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\left(\frac{\gamma-1}{1+i\omega\gamma\tau}\right)\frac{\sum_n \phi^0_n \ell^n_{il}\ell^n_{jt} q^n_i q^n_j}{\{\gamma\phi^0 - (ik^2 D_r \kappa_f/\omega)(1+i\omega\gamma\tau)\}}. \tag{59}$$

Substituting from eq. (55) for $\bar{U}_{mlt}$ in (52), we get

$$c^1_{ipjq} = -\sum_n \frac{v^n(a^n)^3}{\mu} \ell^n_{km} q^n_r c^0_{krip} \ell^n_{ul} q^n_s c^0_{usjq} \bar{U}_{ml} - \sum_n \frac{v^n(a^n)^3}{\mu} q^n_k q^n_r c^0_{krip} \ell^n_{ul} \ell^n_{st} c^0_{usjq} \bar{N}'_{lt}, \tag{60}$$

since $\ell^n_{j3} = q^n_j$.

The first term above is exactly the same as in the formula for isolated cracks, but with the changed form of $\bar{U}_{33}$.

8 EQUANT POROSITY

Another model of pressure relaxation in cracks is one where the material in which the crack lies possesses small-scale porosity, where the size of the pores is an order of magnitude smaller than the cracks. Pressure is relieved within a crack by diffusion into the matrix material rather than by flow through connectionsto other cracks. This model has been called equant porosity (Thomsen 1986).

The porosity ϕ_m within the matrix must satisfy the diffusion equation (*cf.* eq. 7):

$$\frac{\partial}{\partial t}(\rho_f\phi_m) = \mathrm{div}(\rho_f D_m \,\mathrm{grad}\, p_f), \tag{61}$$

where ρ_f is the local fluid density and p_f its pressure, while D_m is the coefficient of diffusion relating the volume flux to the pressure gradient within the porous matrix. For simplicity, we neglect the effect of stress and fluid pressure on the porosity ϕ_m, which therefore remains constant. Using eq. (8) and working to first order in p_f/κ_f, *we* get

$$\frac{\phi_m}{\kappa_f}\frac{\partial p_f}{\partial t} = D_m \nabla^2 p_f \tag{62}$$

(*cf.* eq. 13).

We now assume that diffusion is linear, away from a crack face, which we may choose to be in the plane $x_3 = 0$. In this case, and when time variations are harmonic, the fluid pressure p_f within the matrix satisfies

$$p_f = p_c \exp\{-(1+i)qx_3 + i\omega t\}, \tag{63}$$

where

$$q^2 = \frac{\omega\phi_m}{2\kappa_f D_m}, \qquad q > 0,$$

and p_c is the fluid pressure in the crack. The mass flow into the crack, taken to be circular with radius a, is (eq. 6)

$$\dot{m}_c = 2\pi a^2 D_m \rho_f \left.\frac{\partial p_f}{\partial x_3}\right|_{x_3=0} = -2(1+i)\pi a^2 q D_m \rho_c p_c, \tag{64}$$

where ρ_c is the fluid density in the crack. We now calculate the relationships between crack-opening displacements and fluid density, mass and pressure. First of all, we note that only the axial stress σ_{33} gives rise to changes of volume of the crack and, writing the corresponding crack-opening displacement as

$$[u_3] = \frac{a}{\mu}\sigma^\infty_{33} U_{33}(r) \tag{65}$$

(Hudson 1981), where σ^∞_{33} is the value of the axial stress at infinity and r the distance from the centre of the crack, we see that the corresponding change in volume of the crack is, to the first order,

$$\delta V_c = \int_{S_c} [u_3]\, dS = \frac{a}{\mu}\sigma_{33}^{\infty} \int_{S_c} U_{33}(r)\, dS, \tag{66}$$

where S_c is the plane face of the crack.

The corresponding change in density, $\delta\rho_c$, is given by

$$\frac{\delta\rho_c}{\rho_0} = \frac{\delta m_c}{m_0} - \frac{\delta V_c}{V_0} \tag{67}$$

to first order, where ρ_0, m_0, V_0 are the values of ρ_c, m_c, V_c, respectively, in unstressed equilibrium. In addition, we have

$$\frac{\delta\rho_c}{\rho_0} = \frac{p_c}{\kappa_f}. \tag{68}$$

Eqs (67) and (68) give

$$\frac{p_c}{\kappa_f} = \frac{\delta m_c}{m_0} - \frac{\delta V_c}{V_0}, \tag{69}$$

and substitution of $\delta m_c (= \dot{m}_c/i\omega)$ from (64) and δV_c from (66) results in

$$\frac{-p_c}{\kappa_f} = \frac{4\pi c\mu}{\ }\sigma_{33}^{\infty}\bar{U}_{33} + \frac{3(1-i)qD_m p_c}{2c}, \tag{70}$$

where

$$\bar{U}_{33} = \frac{1}{a^2}\int_{S_c} U_{33}(r)\, dS.$$

Following Hudson (1981), we note that the crack-opening displacement is exactly the same as for an empty (dry) crack under an axial stress ($\sigma_{33}^{\infty} + p_c$). Results for a dry crack are given by Eshelby (1957), and we have

$$U_{33}(r) = \frac{2}{\pi}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)(1 - r^2/a^2)^{1/2}(1 + p_c/\sigma_{33}^{\infty}) \tag{71}$$

and so, integrating over S_c, we get

$$\bar{U}_{33} = \frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\left\{1 - \frac{(3a\kappa_f/4\pi c\mu)\bar{U}_{33}}{(1 + 3(1-i)\kappa_f q D_m/2c)}\right\}. \tag{72}$$

Finally, rearranging eq. (72), we obtain the value of $\bar{U}_{33}$ for substitution into eq. (2) for the overall properties of the cracked material:

$$\bar{U}_{33} = \frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\bigg/(1+K), \tag{73}$$

where, in a generalization of the result given by Hudson (1981),

$$K = \frac{1}{\pi}\frac{a}{c}\frac{\kappa_f}{\mu}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\bigg/[1 + 3(1-i)J/2c] \tag{74}$$

and

$$J^2 = \omega\phi_m\kappa_f D_m/2, \qquad J > 0.$$

Clearly, $\bar{U}_{11}$ and $\bar{U}_{22}$ are not affected.

This result is very similar to that for connected parallel cracks (eqs 41a,b) and, as noted there, to the result for partially saturated isolated cracks, although the dependence on frequency is different.

9 CONCLUSIONS

We have dealt now with three models of the dynamic behaviour of cracked rocks in which a background porosity plays a significant part. The first deals with aligned cracks connected by otherwise invisible pathways and the second consists of a similar system but with the cracks orientated in a general way. The third model is where the matrix rock is porous, but at such a low level that no connections between cracks are made within the time-scale imposed by the period of the wave. The first and third models are the simplest in that very few additional parameters are required to specify the overall dynamic behaviour of the fractured rock. With aligned cracks, only one is required and that is D_r, the overall coefficient of diffusion defining the permeability of the rock-mass (see eq. 41). Equant porosity, the third model, is governed by two parameters; they are ϕ_m and D_m, the porosity and coefficient of diffusion, respectively, of the unfractured rock mass (see eqs 73 and 74). The second model is more complex, requiring for its determination the orientation distribution of the cracks with their associated porosities (eq. 60) as well as the time constant τ which governs the diffusion of pressure between cracks.

The condition that fluid flow between arrays of parallel cracks has no effect on the propagation of waves at frequency ω is (eq. 15)

$$\frac{\kappa_f D_r k^2}{\omega\phi^0} \ll 1. \tag{15}$$

If this condition holds, the cracks can be considered to be isolated. In linear diffusion, the effective diffusion distance from a source of frequency ω is

$$l_d = \left(\frac{\kappa_f D_r}{\omega\phi^0}\right)^{1/2}, \tag{75}$$

and so the condition (15) is equivalent to

$$(kl_d)^2 \ll 1. \tag{76}$$

This gives the condition that diffusion between cracks may be neglected if the linear diffusion distance is much less than a wavelength. Since the pressure gradient between aligned cracks has a scale-length equal to a wavelength, this condition makes good physical sense.

In order to estimate the effect of diffusion between cracks in rocks it will be simpler to convert diffusivity D_r to permeability K_r through the relation (Sheriff 1991)

$$D_r = \frac{K_r}{\eta_f}, \tag{77}$$

where η_f is the viscosity of the diffusing fluid. We shall get relatively high values of the expression in (15) if we take K_r = 10 darcies ($\simeq 10^{-5}$ m^2), η_f = 0.01 poise (= 10^{-3} Pa s), $\phi^0 = 0.1$, $\kappa_f = 2 \times 10^9$ Pa and $k = \omega/c$ with a wave speed $c = 3 \times 10^3$ m s^{-1}. With these values, the inequality becomes

$$\omega \ll 4.5 \times 10^{-2}\ \mathrm{s}^{-1}. \tag{78}$$

Curiously enough, diffusion is ineffective at low frequencies. Although there is more time for the diffusion process at long periods, the distance between high and low pressure-a wavelength-is large. The inequality (78) indicates that diffusion between parallel cracks is important except at frequencies well below 10^{-2} Hz.

If we take a lower permeability (10^{-3} darcies) and higher viscosity (10^{-1} poise) the condition (78) changes to

$$\omega \ll 4.5 \times 10^{3}\ \mathrm{s}^{-1}; \tag{79}$$

that is, diffusion is ineffective for frequencies well below 1 kHz. The range of values for permeability, viscosity and porosity implies that diffusion of fluid between cracks will be important in some cases and not in others.

For populations of cracks that are not aligned, the criterion for treating cracks as isolated becomes (see eq. 25)

$$\omega\gamma\tau \gg 1; \tag{80}$$

the factor γ is given by eq. (44).

The time constant τ may be related to the diffusion distance l-the spacing between cracks-through the matrix rock diffusivity D_m by an equation similar to (75):

$$\tau = \frac{\phi_m l^2}{\kappa_f D_m}, \tag{81}$$

where ϕ_m is the porosity of the matrix rock.

Thus, the inequality becomes

$$\left(\frac{l}{l_m}\right)^2 \gamma \gg 1, \tag{82}$$

where l_m is the diffusion length at frequency ω in the matrix rock:

$$l_m = \left(\frac{\kappa_f D_m}{\omega\phi_m}\right)^{1/2}. \tag{83}$$

Comparing (82) with (76), we see that a similar principle applies in that diffusion between cracks can, in general, be ignored if the diffusion length is much less than the scale-length of the pressure gradient, this time the spacing between cracks.

Re-writing (82), the condition that the cracks may be regarded as isolated is

$$\omega \gg \frac{\kappa_f K_m}{\gamma\phi_m l^2 \eta_f}, \tag{84}$$

where K_m is the permeability of the rock. This time the cracks are isolated at high frequencies since the scale-length of the pressure gradient is fixed.

If we take η_f = 0.01 poise, ϕ_m = 0.1, κ_f = 2 x 10^9 Pa, $K_m = 10^{-1}$ darcies and $1 = 10^{-3}$ m, the inequality becomes

$$\omega \gg \frac{2}{v} \times 10^{12}\ s^{-1}. \tag{85}$$

Even with l = 1, the permeability would need to be lower than 10^{-4} darcies to get the critical frequency down to 1 kHz. (In this we have assumed that γ is roughly equal to unity.) It appears, therefore, that diffusion between cracks cannot be ignored unless they are aligned.

For equant porosity, the criterion that diffusion into the matrix can be ignored is that J/c is small; that is

$$\left(\frac{\phi_m \kappa_f K_m}{\omega c^2 \eta_f}\right)^{1/2} << 1, \tag{86}$$

or

$$\frac{l_m \phi_m}{c} << 1,$$

using the definition (83) of 1,. The second of these conditions states that equant porosity can be ignored if the volume of fluid in the matrix-within- a diffusion length of the crack is small compared with volume within the crack.

If we take, once more, η_f = 0.1 poise, ϕ_m = 0.1, κ_f = 2 x 10^9 Pa, $K_m = 10^{-1}$ darcies and c = 10^{-3} m, the first of (86) becomes

$$\omega \gg 2 \times 10^{10}\ s^{-1}. \tag{87}$$

It would need a very low permeability indeed to bring this value down to 1 kHz, so it appears that equant porosity has a significant effect on seismic waves in most cases.

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