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Effective elastic properties of heavily faulted structures

John A. Hudson* and Enru Liu‡

ABSTRACT

Recent results have shown how to construct the smoothed transmission properties of a plane fault from the parameters of its microstructure in two particular cases. In the first, the fault is modelled as a plane distribution of approximately circular cracks while elsewhere the faces of the fault are held together by the ambient pressure and friction. In the second, the model consists of a plane distribution of approximately circular stuck regions within an area where the faces are separated as for a crack. The averaging method for a sequence of such slip planes enables the construction of overall properties of a material weakened by a series of parallel faults. With the first model, where the distribution of cracks is sparse, this approach leads to exactly the same expressions to first order in the number density as for dilute volume distributions of cracks. The higher-order terms do not agree since they refer to crack–crack interactions and in the Schoenberg–Douma averaging process only the overall interactions between faults are allowed for, not individual interactions between cracks on different faults. Application of this procedure to the second model, in which the fracture density is high, gives for the first time an exact first-order formula for the overall properties of heavily cracked material, the cracks being aligned and confined to the fault planes. These expressions are first order in the (small) parameter, denoting the proportion of each slip plane that is welded. The unwelded part may be free (any cracks) or filled with an incompressible inviscid fluid. An alternative approach in either case is to replace each fault or slip plane by an equivalent thin layer of material whose properties are related, at least in part, to the structure of the fault. The corresponding process of averaging over the layers is, in this case, the original Backus method. Comparison between the properties of the equivalent layers for dilute cracks and for extended cracking leads to an extension of the slip relations on a single heavily cracked fault to cases where the cracks contain secondary material with arbitrary elastic properties. Finally, results for a stack of parallel, heavily cracked faults is identical, to first order in the number density of the contact regions on the faults, to those for a cubical packing of spheres. This further reveals the insensitivity of first-order results to many of the details of the microstructure.

INTRODUCTION

A variety of methods have been applied in recent years to the problem of evaluating the overall or effective properties of cracked material. The effective mechanical properties are those detected by elastodynamic radiation of wavelengths long compared with the scale length of the cracks, or microstructure. At sufficiently long wavelengths, the faulted material appears smoothly varying with properties given by some kind of averaging process applied to the microstructure. At shorter wavelengths, dispersion and scattering occur. Smyshlyaev et al. (1993), using the self-consistent approximation, indicate the variation in phase velocity is $<1\%$ in a material containing dry or liquid-filled cracks for all angular frequencies <0.3 (α/a), where α is the P-wave speed and a is the crack radius.

If the distribution of cracks in the material is sufficiently sparse, the overall properties may be found from expressions for either the stiffnesses or the compliances that are linear in the number density. A number of different routes have been taken to arrive at such expressions (for example, Eshelby, 1957; Walsh, 1965; Garbin and Knopoff, 1973, 1975; Anderson et al. 1974), but all the results appear to agree. The extension to include quadratic terms is provided by Hudson (1980). All of these methods require at some stage the solution for the response of an individual crack, and the first-order theories use a model of an isolated crack in homogeneous matrix material.

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*Department of Applied Mathematics & Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, United Kingdom. E-mail: j.a.hudson@damtp.cam.ac.uk.
‡British Geological Survey, Murchison House, West Mains Road, Edinburgh EH9 3LA, United Kingdom. E-mail: e.liu@bgs.ac.uk.

All crack–crack interactions are ignored, and such methods may be referred to as noninteracting crack theories (Kachanov, 1992). Some methods (e.g., Mori and Tanaka, 1973) provide results that are not linear in the number density but that nevertheless ignore interactions between cracks. The only crack distribution statistics needed for noninteracting crack theories are zero order, namely, the number density. Second-order theories (e.g., Hudson, 1980) require first-order statistics, i.e., the number density of cracks with one crack fixed. First- and second-order theories are exact in this sense; they apply at low number densities with errors of the order of the number density squared and cubed, respectively.

Two of the methods proposed for the approximate evaluation of effective properties at higher concentrations of cracks are the self-consistent method (Budiansky and O'Connell, 1976) and the differential method (Nishizawa, 1982). In general, both methods give plausible (but different) results for all possible values of the number density of cracks, although the self-consistent result, if expanded in terms of the number density, is inaccurate to the second order (Chatterjee et al., 1978). In addition, several of the noninteracting crack theories also give plausible-looking results at high number densities (see, for instance, Sayers and Kachanov, 1991; Berryman and Burge, 1996), even though, strictly speaking, their range of validity is for dilute crack distributions only. Such theories generally agree at low number densities, but at higher values they differ significantly from each other. No one theory of this kind is likely to be accurate for all crack distributions since allowance for higher-order interactions between cracks requires a knowledge of higher-order statistics of the crack distribution (Keller, 1964). Some approximate theories, however, seem to be accurate in certain cases for relatively high number density. For instance, computer studies on 2-D distributions of randomly distributed (but nonintersecting) cracks have shown that the first-order results for the compliances (linear in the number density) are accurate up to $\nu\pi a^2 = 2$, where ν is number density and a is the half-length of a crack (Kachanov, 1992; Davis and Knopoff, 1995). In the circumstances of the numerical experiments, this noninteracting crack approximation is far more accurate than the self-consistent or differential methods (Davis and Knopoff, 1995).

In this paper we attempt to provide theoretical expressions for the overall properties of material with aligned cracks which will be valid at high crack densities but free from the rather heuristic assumptions underlying the self-consistent and differential methods. The method is based on results for a plane fault, consisting of areas of slip and areas of stick (Hudson et al., 1996b, 1997) allied to the method of combining the effects of parallel slip planes described by Schoenberg and Douma (1988). The end result is a model of a cracked material in the form of parallel faults of arbitrary spacing where the proportion of the area of each fault that slips and can be compressed (that is, acts as a crack) is high. In other words, there is a high density of cracking, but the cracks are aligned in parallel planes. Figure 1 shows schematically three fracture models. Model 1 portrays a plane distribution of small cracks; Model 2 is a plane distribution of contacts. Both models can be replaced with an equivalent fracture of constant aperture (thickness) (Model 3) with appropriate infill material.

In our model, aligned cracks may not take arbitrary positions in the matrix rock. However, first-order expressions for dilute crack densities are the same whether the cracks are aligned in planes or not (Schoenberg and Douma, 1988), so we may perhaps expect the same for first-order expressions for dense crack populations. If the cracks are randomly orientated as well, the effective material strength will go to zero and the material will fail at some point determined, presumably, by higher-order statistics. Crampin (1994) argues that, for this reason, coherent rock is constrained to have an upper bound to the crack number density. However, with the cracks confined to fault planes as in the present model, the material will not fail unless the fault planes fail. This point is reached when the number density of points of welded (nonfriction) contact goes to zero.

PARALLEL FAULTS WITH LOW CRACK DENSITY

The effect of a fault plane, consisting of an array of approximately circular areas of slip (cracks) lying in an otherwise welded interface, on waves of long wavelength is that of a slip plane, where the traction $\mathbf{t} = (t_1, t_2, t_3)$ is proportional to the displacement discontinuity $[\mathbf{u}] = ([u_1], [u_2], [u_3])$ across the faces of the fault (Hudson et al., 1996b). The continuity conditions for a fault with normal in the 3-direction are

$$[u_1] = \frac{\alpha}{\mu\omega} A t_1, \qquad [u_2] = \frac{\alpha}{\mu\omega} A t_2,$$

and

$$[u_3] = \frac{\alpha}{\mu\omega} B t_3, \tag{1}$$

where μ is the rigidity and α is the P-wave speed in the uncracked material, ω is the angular frequency of the wave,

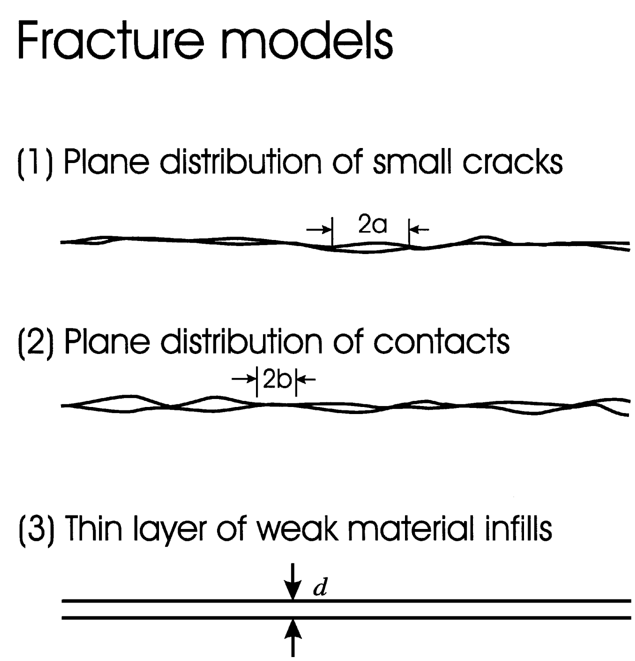

FIG. 1. Schematic of three fracture models. (1) A plane distribution of small cracks; (2) a plane distribution of contacts; and (3) a fracture with weak infill of a constant aperture.

$$A = \left(\frac{\omega a}{\alpha}\right)(\nu^s a^2)\bar{U}_{11}\left\{1 + (\nu^s a^2)^{\frac{3}{2}}\bar{U}_{11}\frac{\pi}{4}(3 - 2\beta^2/\alpha^2)\right\}$$

and (2)

$$B = \left(\frac{\omega a}{\alpha}\right)(\nu^s a^2)\bar{U}_{33}\left\{1 + (\nu^s a^2)^{\frac{3}{2}}\bar{U}_{33}\pi(1 - \beta^2/\alpha^2)\right\}.$$

The number density of cracks on the fault surface is ν^s, and the mean squared crack radius is a^2; β is the S-wave speed, and the quantities $\bar{U}_{11}$ and $\bar{U}_{33}$ depend on the internal crack conditions (dry, liquid filled, etc). For a dry crack (that is, a crack filled with inviscid gas with high compressibility)

$$\bar{U}_{11} = \frac{16}{3}\left(\frac{\lambda + 2\mu}{3\lambda + 4\mu}\right)$$

and (3)

$$\bar{U}_{33} = \frac{4}{3}\left(\frac{\lambda + 2\mu}{\lambda + \mu}\right),$$

where λ and μ are the Lamé parameters of the unfaulted material. If the crack is filled with an inviscid liquid of low compressibility, $\bar{U}_{11}$ is as given above and $\bar{U}_{33}$ is zero. Expressions for other crack conditions are given by Hudson (1981, 1988) and Hudson et al. (1996a). Equations (2) represent an expansion in ascending powers of ν^s and are valid for $(\nu^s a^2)$ small only.

An array of parallel faults of this type with average spacing H is described by Schoenberg and Douma (1988) in terms of a fracture system compliance matrix $\mathbf{Z}$, where

$$\mathbf{Z} = \begin{pmatrix} Z_T & 0 & 0 \\ 0 & Z_T & 0 \\ 0 & 0 & Z_N \end{pmatrix}$$

and (4)

$$Z_N = \frac{\alpha B}{\mu\omega H}, \qquad Z_T = \frac{\alpha A}{\mu\omega H}.$$

The normal to all the faults lies in the 3-direction as before, and the individual cracks are oriented the same way. The effective properties of the faulted material may be written in terms of the compliances as follows (see also Schoenberg and Sayers, 1995; Sayers and Kachanov, 1995):

$$\mathbf{s} = \mathbf{s}^0 + \Delta\mathbf{s}, \tag{5}$$

where $\mathbf{s}$ is the overall compliance tensor and $\mathbf{s}^0$ is the compliance tensor of the unfaulted material. The excess compliance $\Delta\mathbf{s}$ is calculated by Schoenberg and Douma (1988), using Backus's (1962) method, to be

$$\Delta s_{1313} = \Delta s_{2323} = Z_T$$

and (6)

$$\Delta s_{3333} = Z_N,$$

with similar equalities for the components linked to the above by symmetry and all other components zero.

The corresponding overall stiffnesses are

$$c_{1111} = \lambda + 2\mu = c_{2222}, \qquad c_{3333} = (\lambda + 2\mu)/(1 + E_N)$$
$$c_{1133} = \lambda/(1 + E_N) = c_{2233}, \qquad c_{1122} = \lambda, \tag{7}$$
$$c_{2323} = \mu/(1 + E_T) = c_{1313}, \qquad c_{1212} = \mu,$$

where $E_N = (\lambda + 2\mu)Z_N,\ E_T = \mu Z_T$; that is,

$$E_N = \left(\frac{\nu^s a^3}{H}\right)\left(\frac{\lambda + 2\mu}{\mu}\right) \times \bar{U}_{33}\left\{1 + (\nu^s a^2)^{\frac{3}{2}}\bar{U}_{33}\pi(1 - \beta^2/\alpha^2)\right\} \tag{8}$$

and

$$E_T = \left(\frac{\nu^s a^3}{H}\right)\bar{U}_{11}\left\{1 + (\nu^s a^2)^{\frac{3}{2}}\bar{U}_{11}\frac{\pi}{4}(3 - 2\beta^2\alpha^2)\right\}.$$

In this result, interactions between cracks on the same fault are taken into account through the second terms in the brackets, but interactions between individual cracks in different faults are neglected in favor of the interaction between averaged faults. In this sense, our approach differs from approaches where all crack–crack interactions are taken into account. If we put $\nu^s/H = \nu$, the number density of cracks overall, the present results agree to the first order in (νa^3) with earlier first-order results for aligned but randomly distributed cracks. The agreement fails, however, at the second order in (νa^3) (see Hudson, 1980) owing to the difference in the allowance for crack interactions and also in the two-crack distribution function. In earlier work the cracks were assumed to be distributed randomly, whereas here they are aligned in parallel planes. This is an illustration of a general principle: to the first order, it does not matter whether the cracks are randomly distributed or aligned in planes. At the second order, where crack-crack interactions are taken into account, the statistics of the crack distribution are crucial.

Figures 2 and 3 show the variations of normalized elastic constants c_{3333} and c_{2323} with crack density $\epsilon = \nu a^3 = (\nu^s a^3)/H$ for three different a/H ratios (ν is the overall number of cracks per unit volume). The variations are calculated using

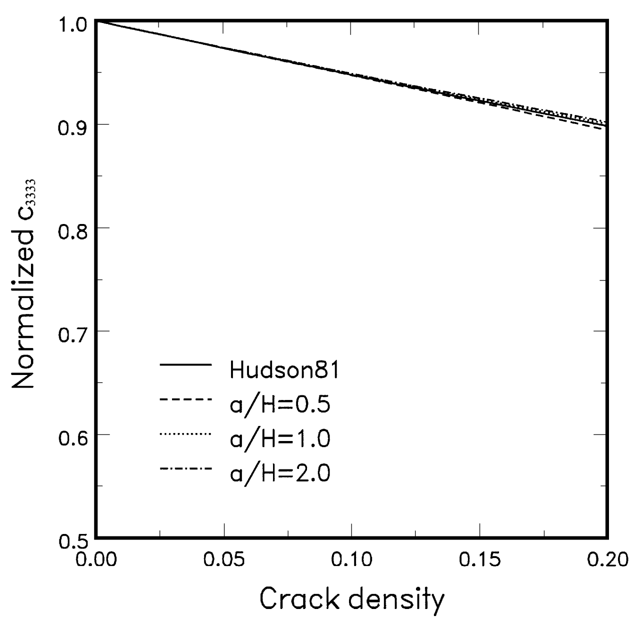

FIG. 2. Variation of normalized c_{3333} with crack density $\epsilon = \nu a^3$. The solid line is calculated from a volume distribution of small cracks (Hudson, 1980, 1981), and the dashed lines are from plane distributions of small cracks (this paper) for three different a/H ratios.

equations (7) and (8) for a solid with the background wave speeds $\alpha = 3.5$ km/s, $\beta = 2.3$ km/s, and density $\rho = 2.6$ g/cm^3. The cracks are filled with water (with acoustic velocity of 1.5 km/s, density of 1 g/cm^3, and viscosity of 0.01 poise) and have an average aspect ratio of 0.01. Compared with the equivalent variation for a volume distribution of small cracks computed using formulæ given in Hudson (1980, 1981, indicated by solid lines), we find that both models give very similar values for c_{3333} and there is little variation of c_{3333} with the a/H ratios. However, the variation of c_{2323} with crack density shows a strong dependence on the value of a/H; when $a/H = 2$, the results from both the volume distribution of small cracks and the plane distribution of small cracks are similar but the difference between the results from the two models increases sharply as a/H decreases. In general, the elastic constants decrease as crack density increases, as expected.

It is possible to construct a slightly different model of a fractured rock in terms of a random distribution of aligned circular faults on each of which the interface conditions are given by equation (1). This can be done by representing the fault conditions as a weak solid infill (Hudson et al., 1996b) and substituting the parameters of this equivalent solid material into the formula for the overall properties of such a model given by Hudson (1981). Once again, this gives an expression in terms of stiffness that agrees with the expansion of equation (7) to the first order but not to the second.

The equivalence of the fault conditions with a thin layer of weak material further shows that the model of aligned fault planes with which we started has the same overall properties as an aligned stack of randomly distributed thin planes of fluid or solid material whose properties are governed by the number and nature of the cracks on the fault planes (see Liu et al., 1996).

In the simplest case, where the fault is represented by a distribution of circular empty cracks, it may be replaced by

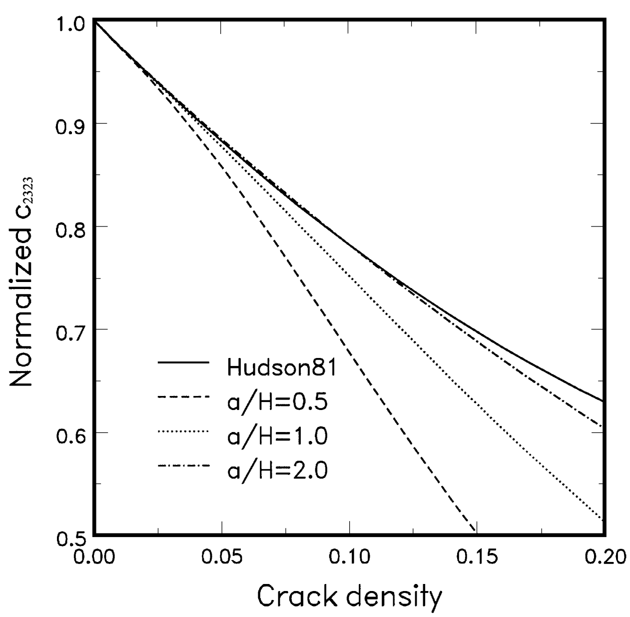

FIG. 3. Same as Figure 2 but for variation of c_{2323} with crack density ϵ.

a layer of thickness Δ and shear and bulk moduli μ^* and κ^* given by (Hudson et al., 1996b)

$$\mu^* = \frac{3\pi\Delta\mu}{16ar}\left(\frac{3\lambda+4\mu}{\lambda+2\mu}\right)\left\{1-\frac{4}{3}\left(\frac{r^3}{\pi}\right)^{\frac{1}{2}}\right\}$$

and (9)

$$\kappa^* = -\frac{\pi\Delta\mu}{4ar}\left(\frac{\mu}{\lambda+2\mu}\right)\left\{1-\frac{4}{3}\left(\frac{r^3}{\pi}\right)^{\frac{1}{2}}\right\},$$

where $r = \nu^s\pi a^2$ is the relative area of cracking on the fault. The value of Δ is, in fact, arbitrary, although it should be small.

This is not a realistic material, of course, as evidenced by the negative bulk modulus. It is, however, the correct expression for the representation of the fault. A more appropriate formula is obtained if we assume that the cracks within the fault are filled with material with elastic parameters μ' and κ'. (A viscous liquid is included by taking μ' to be imaginary or even complex.) In this case we have (Hudson et al., 1996b)

$$\mu^* = \mu' + \frac{3\pi\Delta\mu}{16ar}\left(\frac{3\lambda+4\mu}{\lambda+2\mu}\right)\left\{1-\frac{4}{3}\left(\frac{r^3}{\pi}\right)^{\frac{1}{2}}\right\}$$

and (10)

$$\kappa^* = \kappa' - \frac{\pi\Delta\mu}{4ar}\left(\frac{\mu}{\lambda+2\mu}\right)\left\{1-\frac{4}{3}\left(\frac{r^3}{\pi}\right)^{\frac{1}{2}}\right\},$$

where now $\Delta = rd$, d being the mean aperture (thickness) of the cracks. So $\Delta/ar = d/a$, the aspect ratio of the cracks.

Equivalent layers such as these can be included in the standard Backus (1962) averaging procedure to obtain results identical, to the given order of accuracy, to equations (7) and (8). [They will not be completely identical because to arrive at equations (9) from (8), the approximation $\{1 + (\nu^s a^2)^{3/2}4\pi/3\} \simeq \{1 - (\nu^s a^2)^{3/2}4\pi/3\}^{-1}$ was used.]

PARALLEL FAULTS WITH HIGH CRACK DENSITY

If the relative area of the fault that is welded is small (that is, the greater part of the fault acts like an open crack), the relationship expressed by equation (1) still holds but the formulæ for A and B are changed. Using a model in which the welded regions on the fault are approximately circular with mean squared radius b^2, Hudson et al. (1997) show that

$$A^{-1} = \nu^w\frac{b\alpha}{\omega}\cdot\frac{8(\lambda+\mu)}{(3\lambda+4\mu)}\left\{1+2(\nu^w b^2)^{\frac{1}{2}}\right\}$$

and (11)

$$B^{-1} = \nu^w\frac{b\alpha}{\omega}\cdot\frac{4(\lambda+\mu)}{(\lambda+2\mu)}\left\{1+2(\nu^w b^2)^{\frac{1}{2}}\right\}$$

for dry cracks. If the cracks are liquid filled, A is as given above and $B = 0$. The quantity ν^w is the number density of welded regions, and these expressions are valid for $(\nu^w b^2)$ small only.

Equation (6) still holds for the excess compliance, with Z_N and Z_T given by equation (4) with the new expressions for A and B.

Substituting from equations (11), we get new expressions for E_T and E_N:

$$E_N = \frac{(\lambda+2\mu)^2}{4\mu(\lambda+\mu)}\frac{1}{(\nu^w Hb)}\left(1+2\sqrt{\nu^w b^2}\right)^{-1}$$

and (12)

$$E_T = \frac{(3\lambda+4\mu)}{8(\lambda+\mu)}\frac{1}{(\nu^w Hb)}\left(1+2\sqrt{\nu^w b^2}\right)^{-1}.$$

Figures 4 and 5 show the variations of normalized elastic constants c_{3333} and c_{2323} for parallel dry fractures with a plane distribution of small contacts. The variations computed from equation (12) are plotted against parameter $r = 1 - \nu^w \pi b^2$;

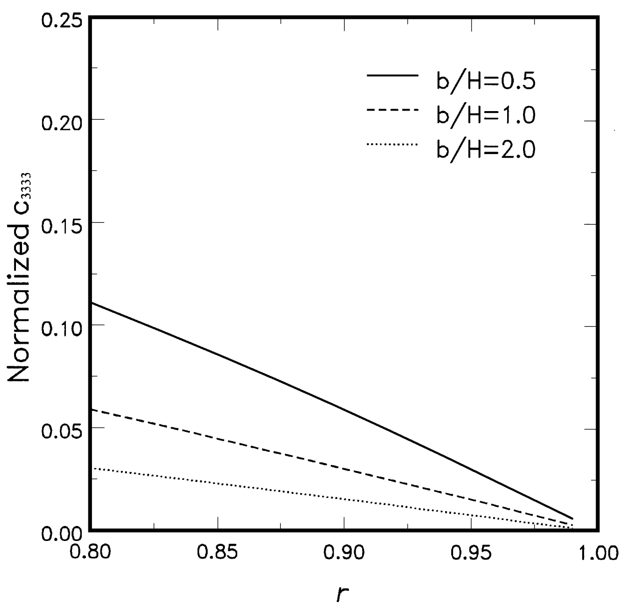

FIG. 4. Variation of normalized c_{3333} with the relative area of cracking $r = 1 - \nu^w \pi b^2$ computed for a plane distribution of contacts for three different b/H ratios. The value ν^w is the number density of welded regions on a fault surface.

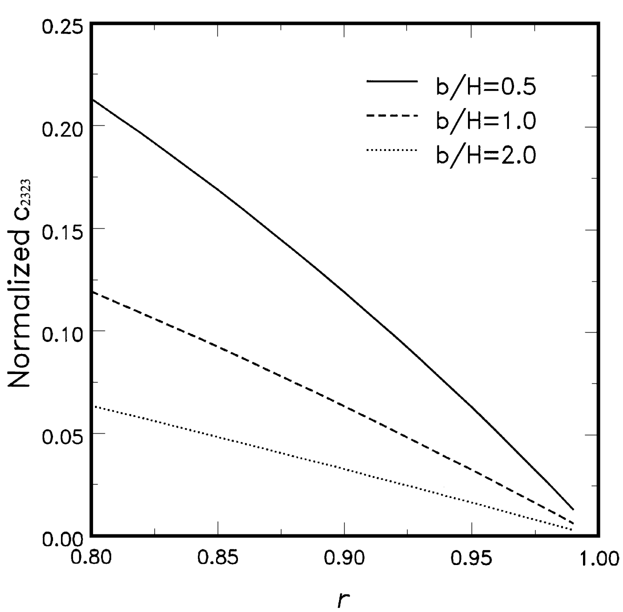

FIG. 5. Same as Figure 4 but for a variation of c_{2323} with parameter r, $r = 1 - \nu^w \pi b^2$.

r can be regarded as the proportion of the fault face, which consists of open cracks [see equation (15)]. As indicated earlier, the results are only valid for small $\nu^w \pi b^2$, say, $\nu^w \pi b^2 \le 0.2$, so that $0.8 \le r \le 1$. The values of normalized c_{3333} and c_{2323} decrease as r increases and are strongly dependent on the b/H ratios. Again, as expected, both c_{3333} and c_{2323} are much smaller than those from the plane distribution of small cracks shown in Figures 2 and 3, and both tend to zero as $r \to 1$, at which point the two faces of the crack lose all contact.

To lowest order in $(\nu^w b^2)$ we have from equations (12) is

$$(1+E_N)^{-1} = \frac{4\mu(\lambda+\mu)}{(\lambda+2\mu)^2}\nu^w\, Hb$$

and (13)

$$(1+E_T)^{-1} = \frac{8(\lambda+\mu)}{(3\lambda+4\mu)}\nu^w\, Hb,$$

and this may be compared with results derived for a related structure, i.e., one with a regular packing of spheres. The model of spheres in a regular cubic packing is broadly similar to the fault model we have been considering in that contacts between spheres are assumed to be small in area and lie in parallel planes. The number density of contacts in any plane is $\nu^w = 1/4R^2$, where R is the radius of a sphere and the separation between the planes is $H = 2R$. White (1983) gives expressions for two moduli, namely

$$c_{3333} = \frac{2\mu(\lambda+\mu)}{(\lambda+2\mu)}\cdot\frac{b}{R} = \frac{4\mu(\lambda+\mu)}{(\lambda+2\mu)}\cdot\nu^w\, Hb$$

and (14)

$$c_{1313} = \frac{4\mu(\lambda+\mu)}{(3\lambda+4\mu)}\cdot\frac{b}{R} = \frac{8\mu(\lambda+\mu)}{(3\lambda+4\mu)}\cdot\nu^w\, Hb,$$

where b is the radius of the contact areas, as before. These are exactly the same as the result of substituting equations (13) into equations (7).

Once again we see that, to the first order (this time in the number density of contacts), the spatial distribution of the contact areas is not important. In our model, they are randomly distributed in the fault planes in a cubical packing of spheres; the distribution is regular. A more surprising feature is that, for dry cracks, the shape of the cavity region between contact areas does not seem to matter either; in the one case they are assumed to be flat and thin and in the other high and pointed. This implies that, to the present order of approximation, the model of a random distribution of spheres (Digby, 1981; Walton, 1987) may be expected to give results applicable to an aggregate of nonspherical particles as long as the number density and orientation of contacts are the same.

In the same way as for the dilute cracking model, the fault modelled by a distribution of welded areas in an otherwise dry fault can be represented by a layer of thickness $\triangle$ of equivalent material with parameters given by (Hudson et al., 1997)

$$\mu^* = \frac{8\triangle\mu(1-r)}{\pi b}\left(\frac{\lambda+\mu}{3\lambda+4\mu}\right)\left\{1+2\left(\frac{1-r}{\pi}\right)^{\frac{1}{2}}\right\}$$

and (15)

$$\kappa^* = -\frac{4\triangle\mu(1-r)}{3\pi b}\frac{(\lambda+\mu)(4\mu-\lambda)}{(3\lambda+4\mu)(\lambda+2\mu)} \times\left\{1+2\left(\frac{1-r}{\pi}\right)^{\frac{1}{2}}\right\},$$

where $1 - r = \nu^w \pi b^2$ so that r is once again the proportion of the fault face that consists of open cracks.

Unfortunately, the theory formulated above does not allow for the cracked regions in this model to be filled with any material other than air (dry cracks) or an inviscid incompressible fluid. However, by analogy with the first model, it would be plausible to replace a fault with approximately circular welded regions which is otherwise filled with material whose elastic parameters are μ', κ' by a layer of thickness $\triangle$ and elastic constants

$$\mu^* = \mu' + \frac{8\triangle\mu(1-r)}{\pi b}\left(\frac{\lambda+\mu}{3\lambda+4\mu}\right)\left\{1+2\left(\frac{1-r}{\pi}\right)^{\frac{1}{2}}\right\}$$

and

$$\kappa^* = \kappa' - \frac{4\triangle\mu(1-r)}{3\pi b}\frac{(\lambda+\mu)(4\mu-\lambda)}{(3\lambda+4\mu)(\lambda+2\mu)} \times\left\{1+2\left(\frac{1-r}{\pi}\right)^{\frac{1}{2}}\right\}, \quad (16)$$

where once again $\triangle = rd$. However d/b is no longer the aspect ratio of the cracks since b is the radius of the welded regions, not of the cracks. It is essential for the validity of the theory, on the other hand, that d/b be small.

This equivalent layer may now be used in the Backus (1962) averaging procedure to obtain the response to a sequence of faults. We thus have a way of dealing with faults either singly or in sequences of parallel planes where the faults are either predominantly held together by friction or are welded, or else with faces largely separated by fluid (gas or liquid) or weak material—that is, the two cases $r \ll 1$ and $(1-r) \ll 1$. There is no valid formula for r in the region of 0.5.

The result $E_N = 0$ for an inviscid incompressible liquid is obtained from the second of equations (10) or (16) by taking κ' sufficiently large that

$$\frac{\omega d}{\alpha} \cdot \frac{\kappa}{\kappa'} \ll 1. \quad (17)$$

CONCLUSIONS

The main result of this paper is a formula for the overall elastic properties of a heavily cracked material. The cracks are aligned and lie in parallel fault planes, and the results are valid for the case where the welded regions form a small proportion of the area of the fault. Interactions between individual contact regions on the same fault are included. To the first order, where such interactions are ignored, it is expected, by analogy with models of sparse distributions of cracks, that the results will hold if the regions of fracture are randomly distributed yet aligned. Further comparison with the model of dilute cracking leads to a plausible construction for heavy faulting in which the open regions of each fault are filled with weak solid or fluid material.

Continuing to work to the first order, we have shown that the response of the heavily cracked faults to normal and tangential tractions on surfaces parallel to the faults is the same as for a cubical packing of spheres with the same size and number density of contact areas. This is another example of the indifference of first-order results to the actual distribution of contact regions or cracks. It also shows that the shape of the cavities between the welded areas does not affect the overall response to the first order. In fact, this was already apparent from the equivalence to the first order of the result for a single heavily cracked fault and formulæ derived from Hertzian contact theory (see Hudson et al., 1997). This implies that the model of a granular aggregate constructed from a random packing of spheres is better than might at first appear, in that it applies to situations in which the strict geometrical constraints of the model can be significantly relaxed.

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